Extended abstract of preprint

Hunting zeros of Dirichlet series by linear algebra. III.

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In the third part we consider some non-evident ways to calculate numbers (2.3) and (2.4) where (1.2) are the non-trivial zeta zeros and maps P_n are defined in (2.2). The main objects of our study are numbers (2.20) where N is a natural parameter and a is a complex parameter. These numbers are defined by (2.23) where matrix $M_{N,n}(a)$ is the result of deleting the topmost row and the nth column from matrix (2.40). The entries (2.19) of the latter matrix are defined by (2.41) and conditions (2.18) where $D_{N,m}(a, s)$ are finite Dirichlet series (2.13).

Whenever a is close to some zero ρ of the zeta function, $\varrho_{N,n}(a)$ is close to $P_n(\rho)$ – see Tables 7–9, and 12, and inequalities (3.6)–(3.9).

For a fixed α functions $\mathrm{Re}(\varrho_{N,n}(\alpha + \mathrm{i}\tau))$ and $\mathrm{Im}(\varrho_{N,n}(\alpha + \mathrm{i}\tau))$ are almost stepwise functions of τ – see Figures 3–6.

When a is close neither to ρ_k nor to ρ_{k+1} the triangles with vertices $\varrho_{N,n}(\alpha+\mathrm{i}\tau)$, $P_n(\rho_k)$ and $P_n(\rho_{k+1})$ have almost similar form independent of the value of n – see Figure 10 and Table 13 (where r is defined by (4.4)). This allows us to calculate $P_n(\rho_{k+1})$ from $P_n(\rho_k)$ by (4.13) when $n^2 \le N$ – see Table 14.

When $n^3 \le N$, both $P_n(\rho_k)$ and $P_n(\rho_{k+1})$ can be calculated as the zeros of the quadratic equation (4.30) with the coefficients defined by (4.27)–(4.29) – see Tables 15 and 16.