A few basic definitions (5 minuntes)

A brief sketch of the first 25 years of history of the subject (15 minutes)

A few more definitions (5 minutes)

Infinite Ramsey Theory (10 minutes)

Around the sine-principle (10 minutes)

About braids and graph minors (10 minutes)

Density A set X is $0-$dense$(2,2)$ if $|X| > \min X + 3$, X is $(n+1)-$dense$(2,2)$ if for every $f\colon [X]^2 \to 2$, there is an $n-$dense$(2,2)$ f-homogeneous subset.

Consider a theory $T = I\Sigma_1 + \cup_{i\in\omega} \forall a \exists b\ [a,b]$ is $n-$dense$(2,2)$.

Theorem (joint with A. Weiermann)
The set of Π_2-consequences of $\mathrm{WKL}_0 + \mathrm{RT}^2_2$ coincides with the set of Π_2-consequences of T.

This result may shed some light on the strength of RT^2_2 and on whether RT^2_2 proves totality of the Ackermann function.

METHOD: inside a nonstandard model of T, we build an initial segment satisfying $\mathrm{WKL}_0 + \mathrm{RT}^2_2$.

Nonstandard chunks of first-order definable sets play the role of infinite sets.

Canonical Ramsey Theorem for pairs CanRT^2 is the statement "for any function $f\colon [\mathbb{N}]^2 \to \mathbb{N}$, there is an infinite set $X \subseteq \mathbb{N}$ such that one of the following cases occurs:

1. X is f-homogeneous;
2. f is injective on X;
3. f on $[X]^2$ depends only on the first coordinate;
4. f on $[X]^2$ depends only on the second coordinate.

Regressive Ramsey Theorem for pairs is the statement "for any $f\colon [\mathbb{N}]^2 \to \mathbb{N}$ such that $f(x,y) \leq x$, there is an infinite set X such that on $[X]^2$, f depends only on the first coordinate".

With a similar definition of T, we can prove the following theorem:
Theorem (joint with A. Weiermann)
The Π_2 consequences of T are the same as of $\mathrm{WKL}_0 + \mathrm{CanRT}^2$ (and a similar theorem for RegRT^2).

Density theorems have also been developed for

- Kruskal's theorem;
- binary Kruskal's theorem;
- Erdös-Moser principle;
- ascending sequeneces of natural numbers.

Several other attempts (e.g. to Hindman's theorem) failed.

Many other conjectures are beeing tried at the moment.

Using the model-theoretic machinery of J. Paris and L. Kirby (**strong cuts**), it is possible to prove that

- the first-order consequences of RegRT^2 coincide with Peano Arithmetic (the recursion-theoretic approach was developed in the 1980s);
- the first-order consequences of CanRT^2 coincide with Peano Arithmetic (the recursion-theoretic proof was first obtained by J. Mileti in 2004).

METHOD: show that a semi-regular initial segment is strong if and only if it satisfies CanRT^2, using a definable ultarpower construction inside a model of arithmetic.

Friedman's sine principle (proposed in the internet forum FOM):
"for any k there exists N such that whenever $x_1 < x_2 < \cdots x_N$ are rational numbers, there exist $p_1 < p_2 < \ldots < p_{k+2}$ such that

$$|\sin(x_{p_1} \cdot x_{p_2} \cdot \ldots \cdot x_{p_k}) - \sin(x_{p_1} \cdot x_{p_3} \cdot \ldots \cdot x_{p_{k+1}})| < 4^{-p_1}$$

$$|\sin(x_{p_2} \cdot x_{p_3} \cdot \ldots \cdot x_{p_{k+1}}) - \sin(x_{p_2} \cdot x_{p_4} \cdot \ldots \cdot x_{p_{k+2}})| < 4^{-p_2}\text{''}$$

is unprovable in PA.

My sharp versions are:

For every $n \geq 1$ and every function F of one argument, we introduce the statement SP^n_F: "for all m, there is N such that for any increasing sequence $A = \{a_1, a_2, \ldots, a_N\}$ of rational numbers, there is $H \subseteq A$ of size m such that for any two n-element subsets $a_{i_1} < a_{i_2} < \cdots < a_{i_n}$ and $a_{i_1} < a_{k_2} < \cdots < a_{k_n}$ in H, we have

$$|\sin(a_{i_1} \cdot a_{i_2} \cdots a_{i_n}) - \sin(a_{i_1} \cdot a_{k_2} \cdots a_{k_n})| < F(i_1)\text{''}.$$

For $n \geq 2$ and any function $F(x)$ eventually dominated by $(\frac{2}{3})^{\log^{(n-1)}(x)}$, the principle SP^{n+1}_F is not provable in $I\Sigma_n$. In particular, the statement $\forall n \mathrm{SP}^n_{(\frac{2}{3})^{\log^{(n-1)}}}$ is not provable in PA.

METHOD: a combinatorial argument showing that the principle implies KM. I used the Rhin-Viola theorem on irrationality measure of π. (It is possible to use diophantine approximation instead.)

Further developments (see on the board):

- complex exponent;
- Gauss map (by Weiermann);
- zeta-function and beyond;
- dynamical system.

Threshold results for PH (model-theoretic approach):
(see my webpage or my Logic Colloquium 2006 paper).

Braids (see on the board).

Graph Minors (see on the board).

Rainbows and max-homogeneity (see on the board).