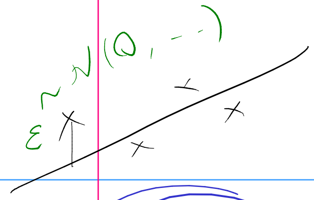
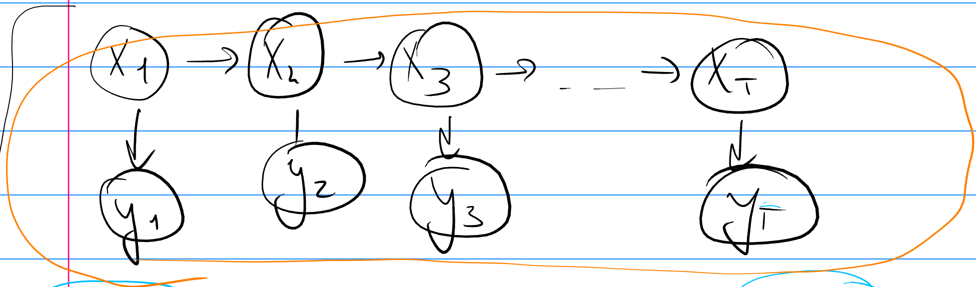
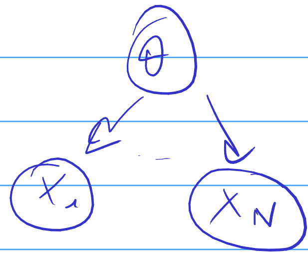



$$p(D|\bar w) = \prod p(d|\bar w) = \prod_n p(y_n|\bar x_n, \bar w)$$

$$p(\bar w|D) \propto p(\bar w)\prod_{n=1} p(y_n|\bar w, \bar x_n) \qquad = \prod_n \mathcal{N}(y_n|\bar w^T \bar x_n, \sigma^2)$$

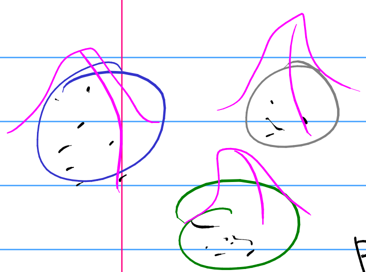
$$p(y|\bar x, D) = \int_{\bar w} p(y|\bar x, \bar w)\, \frac{p(\bar w)\prod_n p(y_n|\bar w, \bar x_n)}{p(D)}\, d\bar w$$




$$p(D|\bar\theta) = p(x_1|\theta)\,p(x_2|x_1,\theta)\,p(y_1|x_1,\theta)\,p(y_2|x_2,\theta)\ldots p(x_T|x_{T-1},\theta)\,p(y_T|x_T,\theta)$$

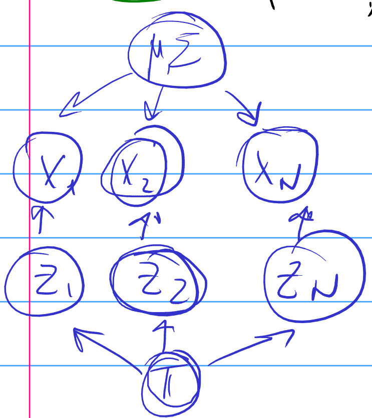
$$= p(y_1,\ldots,y_T, x_1,\ldots,x_T|\theta)$$



$$p(D|\bar\theta) = \prod_n p(\bar x_n|\bar\theta) = \prod_n \left[\sum_k \alpha_k\, p(\bar x_n|\mu_k, \Sigma_k)\right]$$

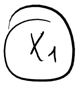
$$p(X, Z|\theta) = \prod_n p(\bar x_n, \bar z_n|\bar\theta) =$$

$$= \prod_n \prod_k \left(p(z_{nk}|\pi)\, p(x_n|\mu_k, \Sigma_k)\right)^{z_{nk}}$$



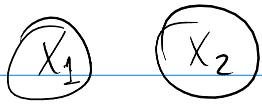

$$\bar\pi = (\pi_1 \ldots \pi_k)$$

$N=1$ $p(X_1)$



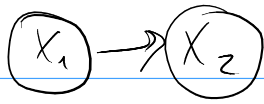
$$\boxed{p(x_1..x_n) = \prod p(x_i \mid par(x_i))}$$

$N=2$ $p(X_1, X_2)$



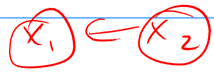
$p(x_1,x_2)=p(x_1)p(x_2)$

$x_1, x_2$ независимы



$p(x_1,x_2)=p(x_1)p(x_2|x_1)$

Нет предпочтений



$p(x_1,x_2)=p(x_2)p(x_1|x_2)$

Directed graphical models

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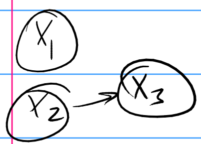
$N=3$ $p(X_1,X_2,X_3)$

1)

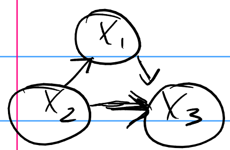
$p(x_1,x_2,x_3)=p(x_1)p(x_2)p(x_3)$

2)

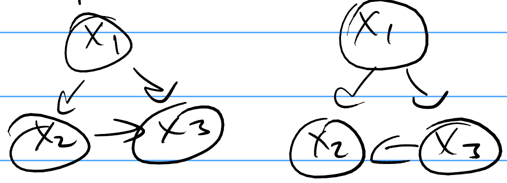


$p(x_1,x_2,x_3)=p(x_1)p(x_2)p(x_3|x_2)$

3)

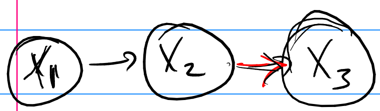


$p(x_1,x_2,x_3)= p(x_2)p(x_1|x_2)p(x_3|x_1,x_2)$



$= p(x_1)p(x_2|x_1)p(x_3|x_1,x_2)$

4)

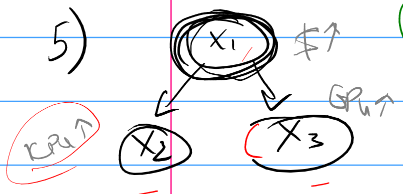


$p(x_1,x_2,x_3) = p(x_1)p(x_2|x_1)p(x_3|x_2) = p(x_2)p(x_1,x_3|x_2) =$

$p(x_1,x_3|x_2) = p(x_1|x_2)p(x_3|x_2) = \frac{p(x_1)p(x_2|x_1)p(x_3|x_2)}{p(x_2)}$

$x_1, x_3 \perp\!\!\!\perp x_2$

5)

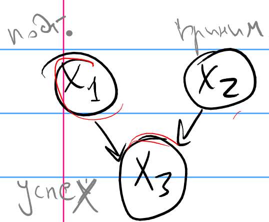



$p(x_1,x_2,x_3) = p(x_1)p(x_2|x_1)p(x_3|x_1) = p(x_1)p(x_2,x_3|x_1)$

$p(x_2,x_3|x_1) = p(x_2|x_1)p(x_3|x_1)$

$x_2, x_3 \perp\!\!\!\perp x_1$

6)

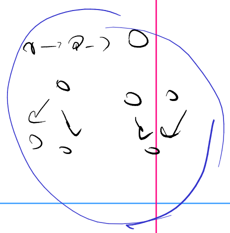



$\int p(x_1,x_2,x_3)\,dx_3 = \int p(x_1)p(x_2)p(x_3|x_1,x_2)\,dx_3$

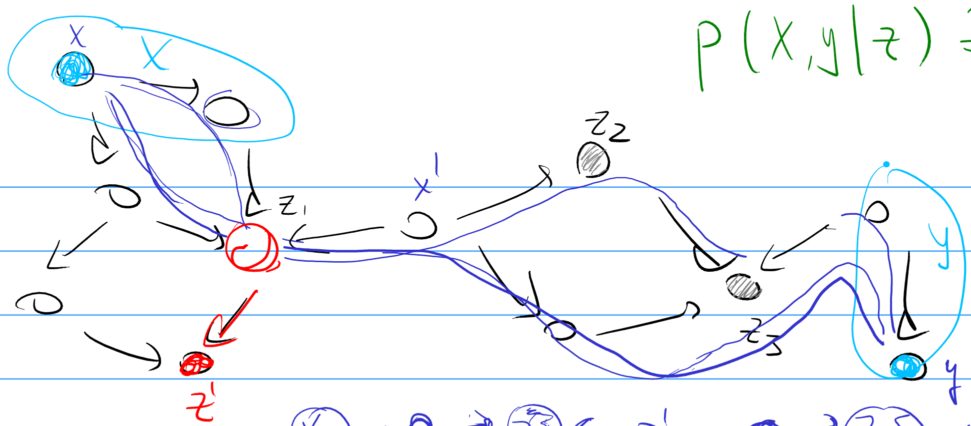
explaining away

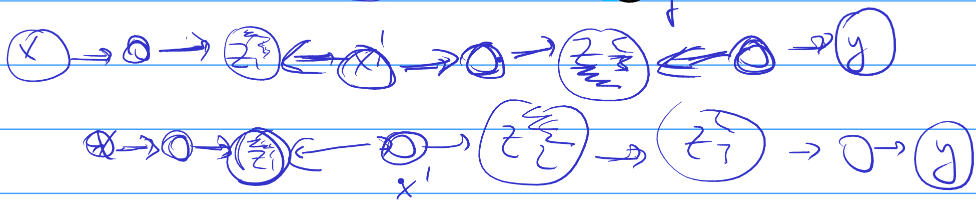
$p(x_1,x_2) = p(x_1)p(x_2)$

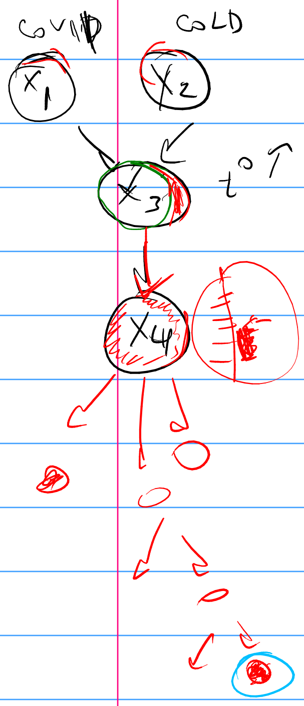
$p(x_1,x_2|x_3) \neq p(x_1|x_3)\,p(x_2|x_3)$



$$p(X,y|z) \overset{?}{=} p(X|z)\,p(y|z)$$



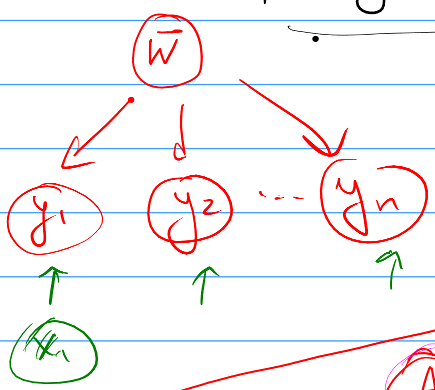




$$p(\bar{y}|X,\bar{w}) = \prod p(y_n|\bar{x}_n,\bar{w})$$

$$p(\bar{w},\bar{y},X) = p(\bar{w}) \prod_n p(y_n|\bar{x}_n,\bar{w}) \cdot \prod_n p(\bar{x}_n)$$

$$p(\bar{w},\bar{y}|X) = p(\bar{w}) \prod_n p(y_n|\bar{w},\bar{x}_n)$$

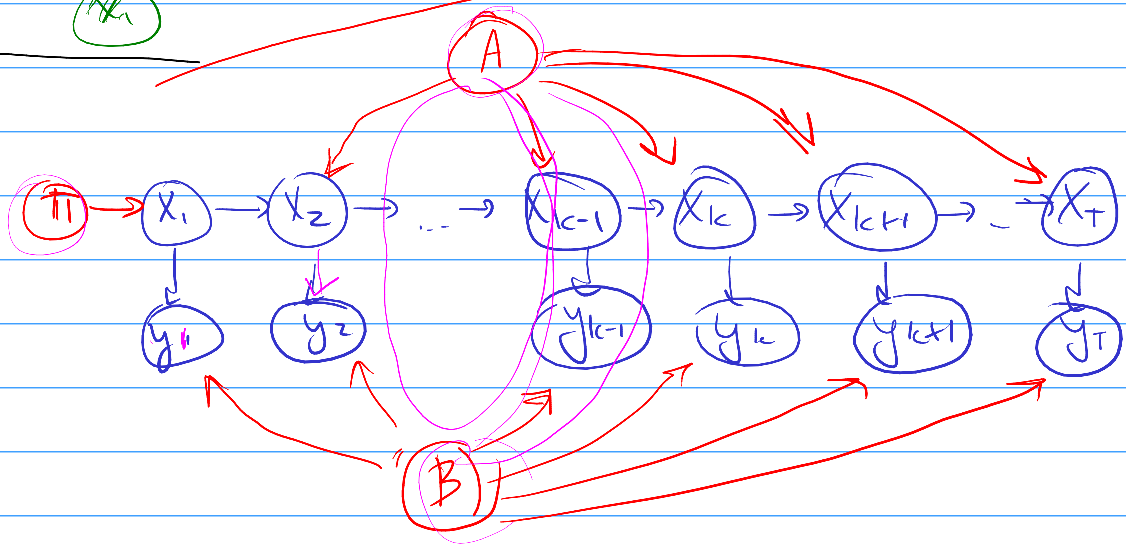


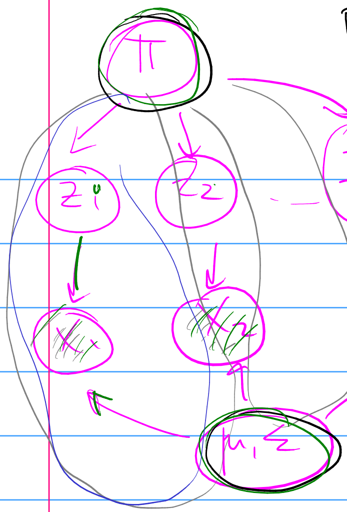

$$p(y_i|x_i)$$

$$\theta = (\pi, A, B)$$

$$\pi = p(x_1), \quad A = p(x_i|x_j)$$

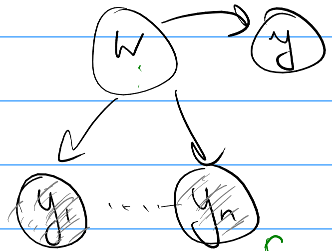
$$p(\bar{x},\bar{y}) = \prod p(x_i|x_{i-1})\,p(y_i|x_i)$$






$$p(\pi,\mu,\Sigma \mid X) = \int p(\pi,\mu,\Sigma,z \mid X)\,dz$$

$$p(w,D) = p(w)\cdot\prod p(y_n \mid w)$$



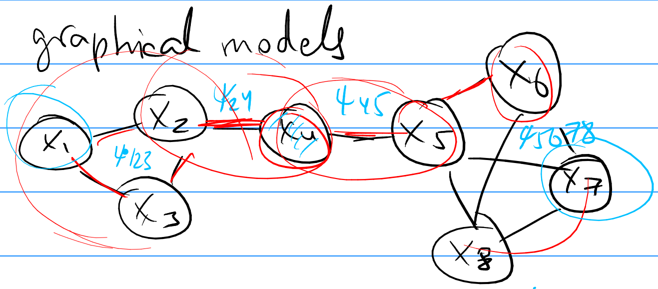

$$p(y \mid D) = \int p(y,w \mid D)\,dw$$

$$= p(y \mid w)\,p(w)\,\prod p(y_n \mid w)$$

$$p(x_1) = \int p(X)\,dx_2 \ldots x_m$$

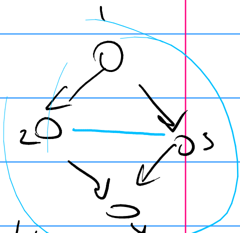
$$= \prod \ldots$$

## Undirected graphical models



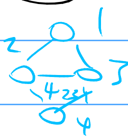

$$f(x_1 \ldots x_8) =$$

$$= \psi_{123}(x_1,x_2,x_3)\,\psi_{24}(x_2,x_4)\,\psi_{45}(x_4,x_5)\,\psi_{5678}(x_5,x_6,x_7,x_8)$$



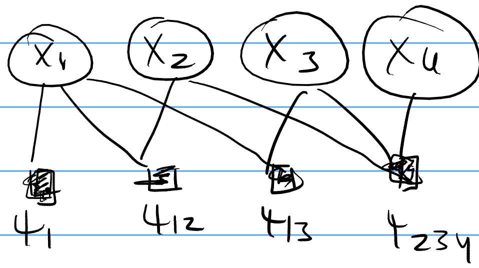

$$f_{1234} = \psi_1\,\psi_{12}\,\psi_{13}\,\psi_{234}$$

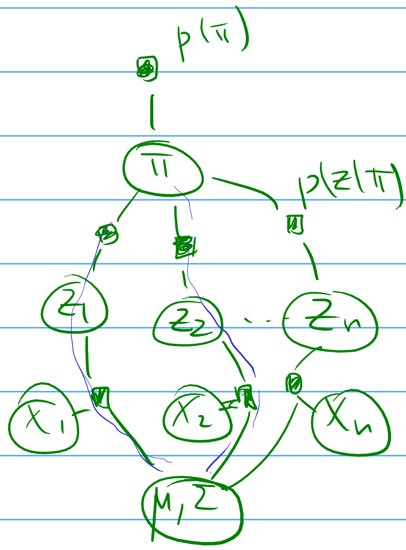
$p(x_4 \mid x_3, x_2)$

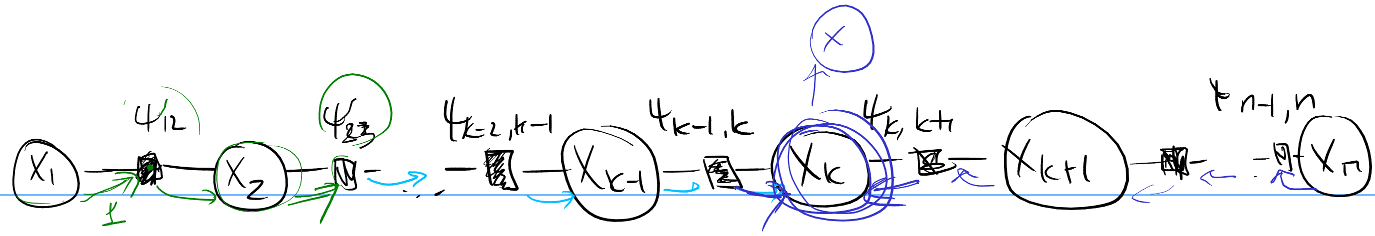



$\psi_{123}\,\psi_{234}$

## Factor graphs






$f(x_1, x_2, \dots, x_n)$ $\qquad f(x_k) = \sum\limits_{\dots \cancel{x_k} \dots} f(x_1 \dots x_n) =$

$$= \sum_{x_1} \dots \sum_{x_{k-1}} \sum_{x_{k+1}} \dots \sum_{x_n} \left[ \psi_{12}(x_1, x_2) \psi_{23}(x_2, x_3) \dots \psi_{n-1,n}(x_{n-1}, x_n) \right]$$

$2^{n-1}$

$$= \sum_{x_1 \dots x_{n-1}} \cdot \left[ (\psi_{12} \dots \psi_{n-2,n-1}) \cdot \underbrace{\sum_{x_n} \psi_{n-1,n}}_{x_{n-1}} \right] = \sum_{\dots} \left[ \psi_{12} \cdot \psi_{n-3,n-2} \underbrace{\sum_{x_{n-1}} \left( \psi_{n-2,n-1} \left[ \sum_{x_n} \psi_{n-1,n} \right] \right)}_{x_{n-2}} \right]$$

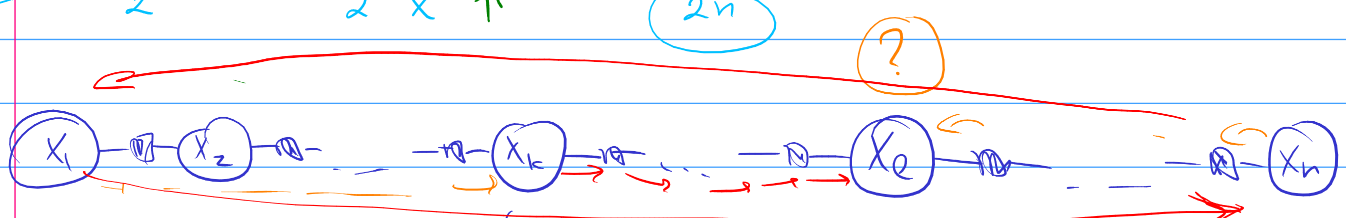
$\psi_{12}\psi_{23} - \psi_{k-1,k}$

$$= \sum_{x_1 \dots x_{k-1}} \left[ \psi_{12} \dots \psi_{k-1,k} \underbrace{\left( \sum_{x_{k+1}} \psi_{k,k+1} \left( \sum_{x_{k+2}} \psi_{k+1,k+2} \dots \left( \sum_{x_n} \psi_{n-1,n} \right) \dots \right) \right)}_{(x_k)} \right]$$

$$\sum_{x_1 \dots x_{k-1}, x_{k+1} \dots x_n} \psi_{12} \dots \psi_{n-1,n} = \left[ \sum_{x_1 - x_{k-1}} \psi_{12} - \psi_{k-1,k} \right] \left[ \sum_{x_{k+1} \dots x_n} \psi_{k,k+1} \dots \psi_{n-1,n} \right]$$

$$= \left[ \sum_{x_{k-1}} \psi_{k-1,k} \underbrace{\sum_{x_{k-2}}}_{2} \dots \psi_{34} \underbrace{\sum_{x_2}}_{2} \psi_{23} \underbrace{\sum_{x_1}}_{x} \psi_{12} \right] \times \left[ \sum_{x_{k+1}} \psi_{k,k+1} \dots \left[ \sum_{x_n} \psi_{n-1,n} \right] \right]$$

$2n$




$p(x_k)$ $\qquad p(x_e)$

$O(n)$ вместо $O(n^2)$