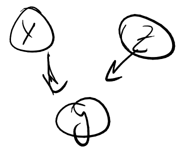
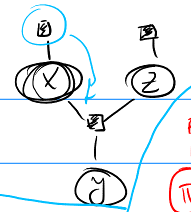
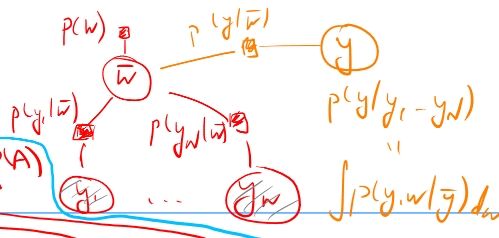


$$p(x)p(z)p(y|x,z) = p(x,y,z)$$

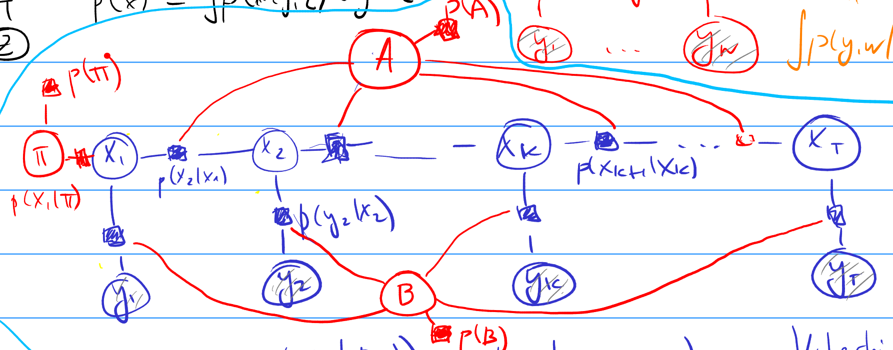


$$p(x) = \int p(x,y,z)\,dy\,dz$$



$p(w)$, $p(y|\bar{w})$, $p(y_1|\bar{w})$, $p(y_w|\bar{w})$

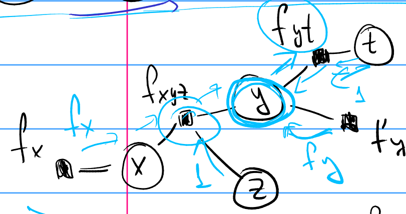
$$p(y|y_1 - y_N) = \int p(y,w|\bar{y})\,dw$$



$p(A)$, $p(\pi)$, $p(x_1|\pi)$, $p(y_2|x_1)$, $p(y_2|x_2)$, $p(x_{k+1}|x_k)$, $p(B)$

$$\sum\left[\cdot \sum [z]\right]$$

$p(x_k)$



$$p(x_k|D,\lambda) = p(x_k|y_1 - y_T) = \sum_{x \ldots x_k \ldots} p(x_1 - x_T | y_1 - y_T)$$

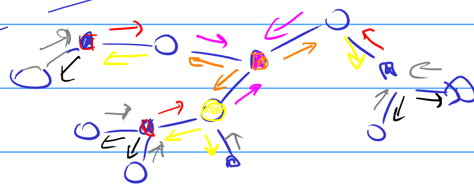
Viterbi

$$p(y) = \sum_{x,z,t} f_x f_{xyz} f_y f_{yt} = \left(\sum_{x,z} f_x f_{xyz}\right) \times \left(\sum_t f_{yt}\right) \times f_y$$

$$p(\lambda|D) \to \max$$

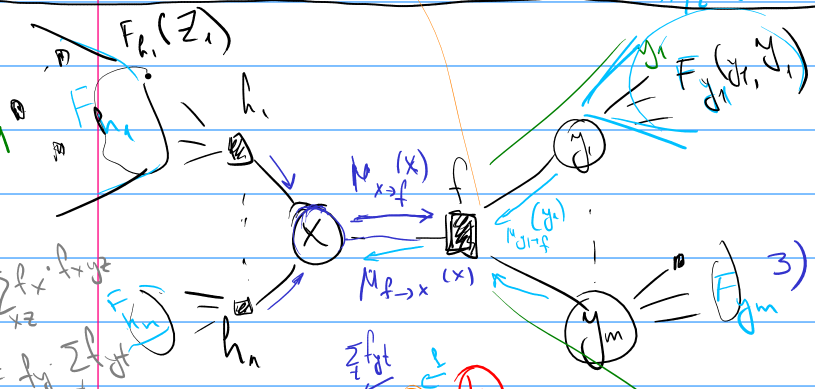
$$= p(A,B,\pi|D)$$

Baum-Welch



$$p(t) = \sum_{x,y,z} f_x f_{xyz} f_y f_{yt} = \sum_y \left[f_y f_{yt} \cdot \left(\sum_{x,z} f_x f_{xyz}\right)\right]$$

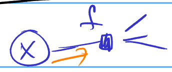
$\mu_{f_{xyz} \to y}$



$F_{h_1}(Z_1)$, Z_1, F_{h_1}, h_1, $\mu_{x \to f}(x)$, f, $\mu_{f \to x}(x)$, $\mu_{y_i \to f}(y_i)$, $F_{y_1}(y_1, Y_1)$, y_1, y_m, F_{y_m}, h_n, F_{h_n}

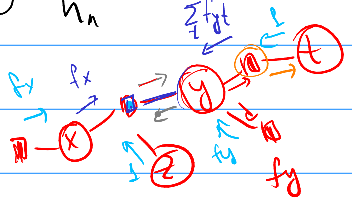
$$\mu_{x \to y} = \sum_{x,z} f_x \cdot f_{xyz}$$

$$\mu_{y \to} = f_y \sum_t f_{yt}$$

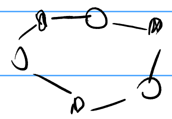
1)  $\mu_x = \underline{1}(x)$  $\mu_{f \to x} = f(x)$

2) $$\mu_{x \to f}(x) = \mu_{h_1 \to x}(x) \ldots \mu_{h_n \to x}(x)$$

3) $$\mu_{f \to x}(x) = \sum_{y_1 \ldots y_m} \left[f(x, y_1 - y_m) \cdot \prod_i \mu_{y_i \to f}(y_i) \right]$$



$\sum_t f_{yt}$, f_y, f_x



$$\mu_{y \to f_{yt}} = f_y \cdot \sum_{xz} f_x f_{xyz}$$

$$\mu_{f_{yt} \to t} = \sum_y f_{yt} f_y \sum_{xz} f_x f_{xyz}$$

Belief propagation

$$F = f \cdot h_1 \ldots h_n \cdot \prod F_{h_j} \cdot \prod F_{y_i}$$

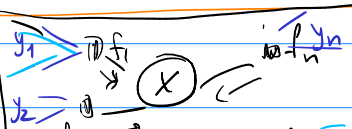
$$F(z_1 - z_n, x, y_1 - y_m, Y_1 - Y_m)$$

$$\mu_{f \to x}(x) = \sum_{y_1 \ldots y_m, Y_1 \ldots Y_m} f(x, y_1 - y_m)\, F_{y_1}(y_1, Y_1) \ldots F_{y_m}(y_m, Y_m)$$

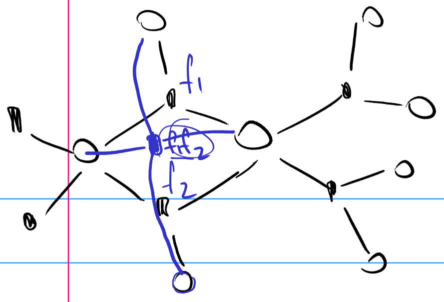
$$\mu_{x \to f}(x) = \sum_{z_1 \ldots z_m} F_{h_1} \ldots F_{h_n} h_1(z_1, x) \ldots h_n(z_n, x)$$

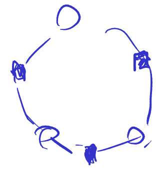
$$\mu_{f \to x}(x) = \sum_{y, Y} f(x, y) \cdot \prod_i F_{y_i}(y_i, Y_i) = \sum_y f(x, y) \cdot \prod_i \underbrace{\left[\sum_{Y_i} F_{y_i}(y_i, Y_i)\right]}_{= \mu_{y_i \to f}(y_i)}$$

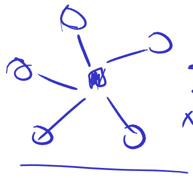
$$\mu_{x \to f}(x) = \sum_{z_{\ldots}} h_1(z_1, x) \ldots h_n(z_n, x) F_{h_1} \ldots F_{h_n} = \prod_j \underbrace{\sum_{z_j} h_j(x, z_j) \cdot F_{h_j}(z_j)}_{= \mu_{h_j \to x}(x)}$$



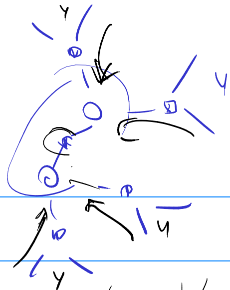
$$\sum_{\ldots x \ldots} F(\ldots) = \sum_{y_1 \ldots y_n} f_1 \ldots f_n \cdot F_1 \ldots F_n = \prod_i \underbrace{\left[\sum_{y_i} f_i(x, y_i)\, F_i(y_i)\right]}_{= \mu_{f_i \to x}(x)}$$

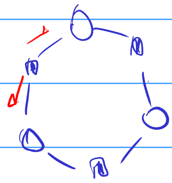






$$\sum_{x_1 \dots x_n} F(x_1 \dots x_n)$$

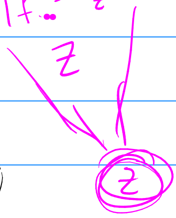


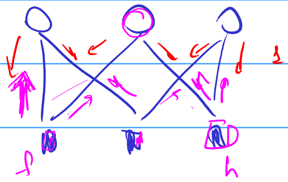


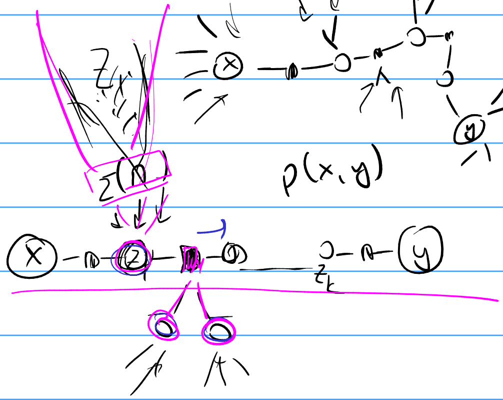
Loopy Belief Propagation

$$\sum_z F_z(z, \tilde{z})$$

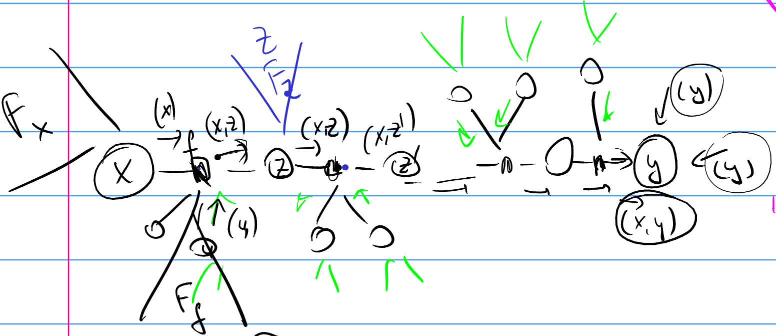
$$\prod f_{\dots} = F_z(z, \tilde{z})$$







$p(x, y)$



$$\sum F(z, \tilde{z}, x, \dots) = \sum_{\dots} \dots F_z(z, \tilde{z}) \dots =$$

$$= \sum \Big(\dots \Big[\sum_z F_z(z, \tilde{z}) \Big] \dots \Big)$$

$$\Big[\sum_{u_1 \dots u_k} F_f(u_1 \dots u_k) \cdot F_x(x) \cdot f(x, z, u_1 \dots u_k) \Big](x, z)$$

$\mu_{f \to x}(x)$ $\qquad [\,\vdots\,]$ $\qquad \mathcal{N}(x; \mu, \sigma^2)$

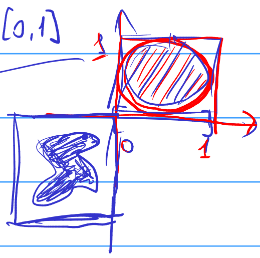
$$p(y|D) = \int p(y, \theta | D)\, d\theta = \int p(y|\theta) \cdot p(\theta|D)\, d\theta = \mathbb{E}_{p(\theta|D)}\big[p(y|\theta)\big]$$

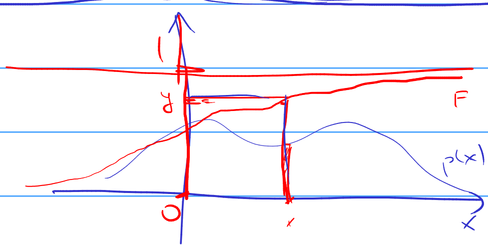
$$p(y|D) \approx \frac{1}{R} \sum_{r=1}^{R} p(y|\theta^{(r)}), \quad \theta^{(r)} \sim p(\theta|D)$$

Monte-Carlo EM

$$\mathbb{E}_{z|\theta_m}\big[\log p(X, z|\theta)\big] \approx \frac{1}{R} \sum_{r=1}^{R} \log p(X, z^{(r)}|\theta) \xrightarrow[\theta]{} \max \qquad z^{(r)} \sim p(z|X, \theta_m)$$

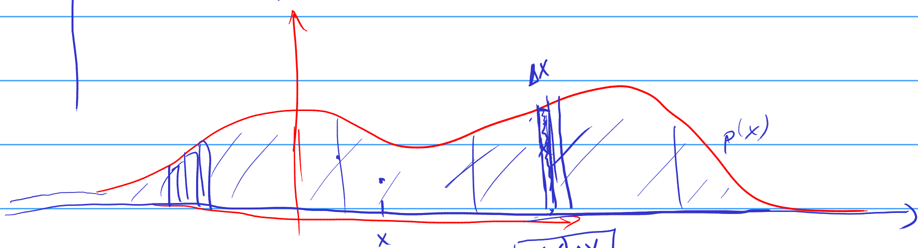
$x \sim \mathrm{Unif}[0,1]$





$y \sim \mathrm{unif}[0,1]$

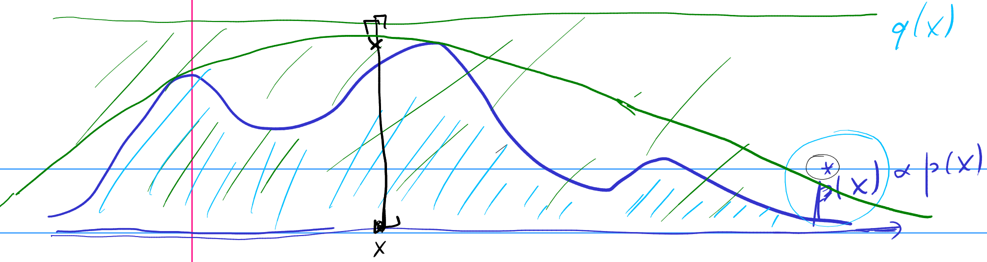
$$F^{-1}(y) \sim p(x)$$

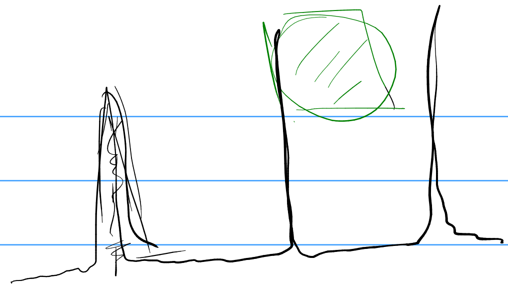


$\sim \boxed{p(x)\Delta x}$

$p(x)$

$p(\theta|D) \sim p(\theta) \prod p(d|\theta)$



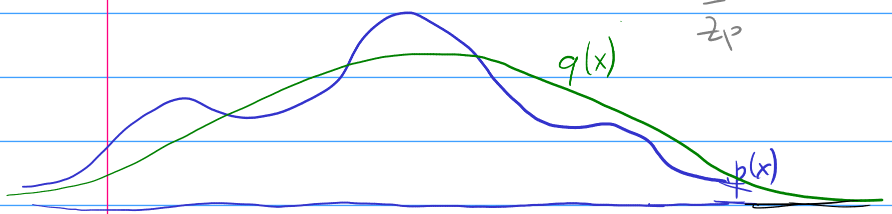


$c \cdot q(x) \geq \overset{*}{p}(x)$

Rejection sampling

$x \sim q(x)$
$r \sim \mathrm{Unif}[0, c \cdot q(x)]$
если $r > \overset{*}{p}(x)$ → выкид. x
$r \leq \overset{*}{p}(x)$ → оставляем x

$\frac{Z_q}{Z_p}$



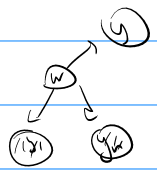
$x_1 \ldots x_n \sim q(x)$

$p^*(x), \; q^*(x)$

$\mathbb{E}_p[f] = \int f(x) p(x)\, dx =$

$= \int f(x) \boxed{\frac{p(x)}{q(x)}} q(x)\, dx$

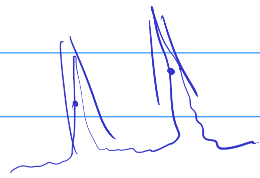
$\approx \frac{1}{k} \sum f(x^{(i)}) \frac{p(x^{(i)})}{q(x^{(i)})}$



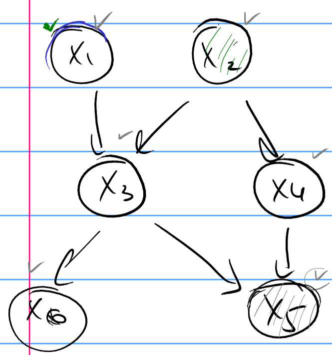
Importance sampling

$\mathbb{E}_p[f] = \int f(x) \cdot \frac{1}{Z_p} p^*(x) dx = \int f \frac{p^*(x)/Z_p}{q^*(x)/Z_q} \frac{1}{Z_q} q^*(x) dx$

$W(x) = \frac{p^*(x)}{q^*(x)} \cdot \frac{Z_q}{Z_p} \approx \frac{p^*}{q^*}$



$p(x_1 - x_6) = p(x_1) p(x_2) p(x_3|x_1 x_2)$
$p(x_4|x_2) p(x_5|x_3, x_4) p(x_6|x_3)$



$x_1 \sim p(x_1), \; x_2 \sim p(x_2)$
$x_3 \sim p(x_3|x_1, x_2)$

$p(x_1, x_3, x_4, x_6 | x_2, x_5)$

$q(x_1, x_3, x_4, x_6 | x_2, x_5)$
$\parallel$
$p(x_1) p(x_3|x_1, x_2) p(x_4|x_2)$
$p(x_6|x_3)$

$x_1, x_3, x_4, x_6 \mid x_2, x_5$

$\frac{p(x_1 - x_6)}{p(x_2, x_5)} = \frac{p(x_1, \ldots x_6 | x_2, x_5)}{\overset{*}{q}(x_1, x_3, x_4, x_6 | x_2, x_5)} = \frac{p(x_1 - x_6) = \cancel{p(x_1)} \_\_\_ \cancel{p(x_6|x_3)}}{p(x_2, x_5) \cancel{p(x_1)} \_\_\_ \cancel{p(x_6|x_3)}} =$

$= \frac{p(x_2) p(x_5|x_3, x_4)}{p(x_2, x_5)} \propto \prod_{x \in \text{Evidence}} p(x | \mathrm{par}(x))$

$\frac{p^*}{q^*} = \sum \frac{1}{q^*(\cdot)}$