$\bar{x} \sim p(\bar{x}) \propto p^*(\bar{x})$

$$p(\bar\theta \mid D) \propto \underbrace{p(\bar\theta)}_{\text{prior}} \underbrace{p(D\mid\bar\theta)}_{\text{likelihood}} = p(\bar\theta) \prod_{n=1}^{N} p(d_n \mid \theta)$$

$\bar\theta_r \sim p(\bar\theta \mid D)$

$$p(d \mid D) = \int p(d\mid\bar\theta)\, p(\bar\theta \mid D)\, d\bar\theta \approx \frac{1}{R}\sum_{r=1}^{R} p(d \mid \bar\theta^{(r)})$$

$\mathbb{E}_{\bar\theta \sim p(\bar\theta \mid D)}\left[p(d\mid\bar\theta)\right]$

$$Q(\theta, \theta^{(n)}) = \int p(X, Z\mid\theta)\, p(Z \mid \theta^{(n)})\, dZ = \mathbb{E}_{p(Z\mid\theta^{(n)})}\left[p(X, Z\mid\theta)\right]$$

$Z_r \sim p(Z \mid \theta^{(n)})$

$$Q \approx \frac{1}{R}\sum_{r=1}^{R} p(X, Z \mid \theta^{(n)}) \xrightarrow[\theta]{} \max$$

stochastic EM

k-means | Monte-Carlo EM

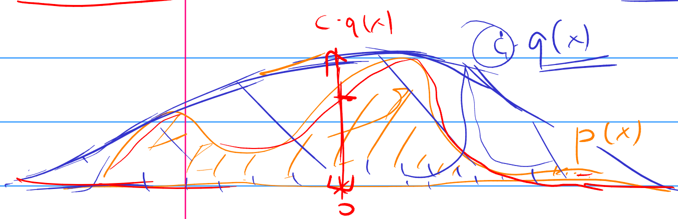
Imputation

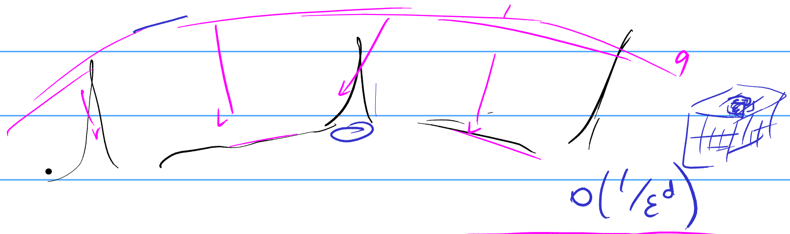
1) $Z_r \sim p(Z\mid X) = \int p(Z\mid\theta, X)\, p(\theta \mid X)\, d\theta \approx \frac{1}{R}\sum_r p(Z \mid \theta_r, X)$

2) $p(\theta \mid X) = \int p(\theta \mid Z, X)\, p(Z\mid X)\, dZ \approx \frac{1}{R}\sum_r p(\theta \mid Z_r, X)$

posterior $= \int p(\theta, Z \mid X)\, dZ$

[IP-algorithm]





$\bar{x}$, $p(\bar{x}_{t+1} \mid \bar{x}_t) = p(\bar{x}_{t+1} \mid \bar{x}_t, \bar{x}_{t-1}, \dots, \bar{x}_1)$

$x_1 \to x_2 \to \dots \to x_t \to \dots \to x_T$

$t=1 \quad t=2$

$p_1(\bar x)$ $\quad p_2(\bar x) = A \cdot p_1(\bar x)$

$p_t(\bar x) = A^{t-1} p_1(\bar x)$

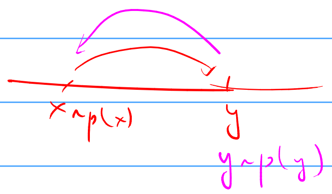
$p_2(\bar x) = \int T(\bar x', \bar x)\, p_1(\bar x')\, d\bar x'$

$p_t(\bar x) = \int T(\bar x', \bar x)\, p_{t-1}(\bar x')\, d\bar x'$

$p_t(\bar x) \xrightarrow[t\to\infty]{} ?$

Стационарное распред.: $p(\bar x)$

$$p(\bar x) = \int T(\bar x', \bar x)\, p(\bar x')\, d\bar x'$$



Условие баланса: $\boxed{p(\bar x)\, T(\bar x, \bar y) = p(\bar y)\, T(\bar y, \bar x)}$

$$p(x) = \int T(y, x)\, p(y)\, dy = \int p(x)\, T(x, y)\, dy = p(x)$$

$x \sim p(x)$

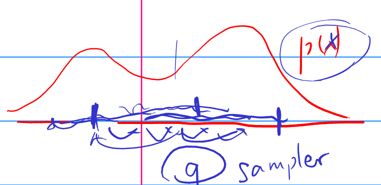
$p^*(\bar x) \propto p(\bar x)$

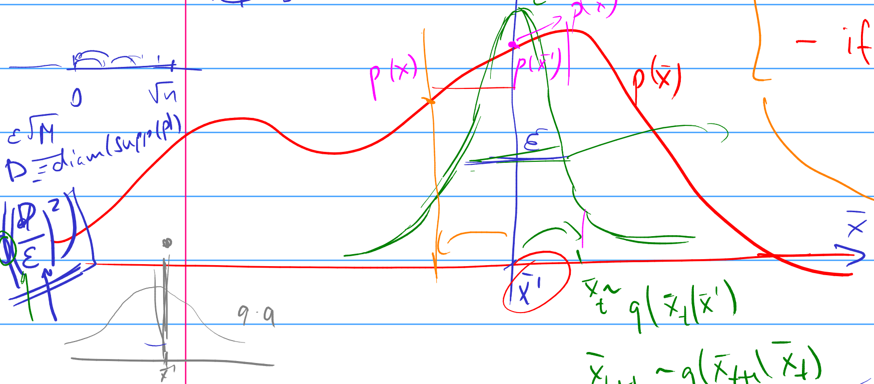
Metropolis-Hastings:

- $\bar x \sim q(\bar x \mid \bar x')$; $\quad a = \dfrac{p(\bar x)}{p(\bar x')} \cdot \dfrac{q(\bar x' \mid \bar x)}{q(\bar x \mid \bar x')}$
- if $a \geq 1$ then $x_t = x$
- if $a < 1$ then

$\bar x_t = \bar x$ с вероятн. a

$\bar x_t = \bar x'$ с вероятн. $1-a$





$\bar x_{t+1} \sim q(\bar x_{t+1} \mid \bar x_t)$

$$p(\bar x)\, T(\bar x, \bar x') \stackrel{?}{=} p(\bar x')\, T(\bar x', \bar x)$$

1) $a \geq 1$ $\quad \dfrac{p(\bar x)}{p(\bar x')} \dfrac{q(\ )}{q(\ )} \geq 1 \Rightarrow T(\bar x', \bar x) = q(\bar x \mid \bar x') \cdot 1$

2) $a < 1$ - упр

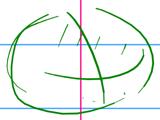
$$T(\bar x, \bar x') = \left(\frac{p(\bar x')}{p(\bar x)} \cdot \frac{q(\bar x \mid \bar x')}{q(\bar x' \mid \bar x)}\right) q(\bar x' \mid \bar x)$$

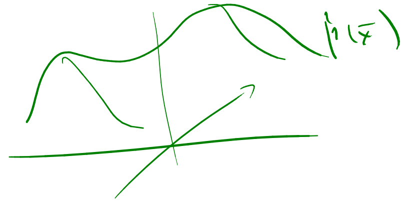
$$\cancel{p(\bar x)} \cdot \left(\frac{p(\bar x')}{\cancel{p(\bar x)}} \cdot \frac{q(\bar x \mid \bar x')}{\cancel{q(\bar x' \mid \bar x)}}\right) \cdot \cancel{q(\bar x' \mid \bar x)} = p(\bar x') \cdot q(\bar x \mid \bar x')$$

$p(\theta|D) \propto \boxed{p(\theta)}\, p(D|\theta)$

$\sim N(\theta|0, -)$

$\Sigma \|\theta\|^2 \qquad \Sigma|\theta|$

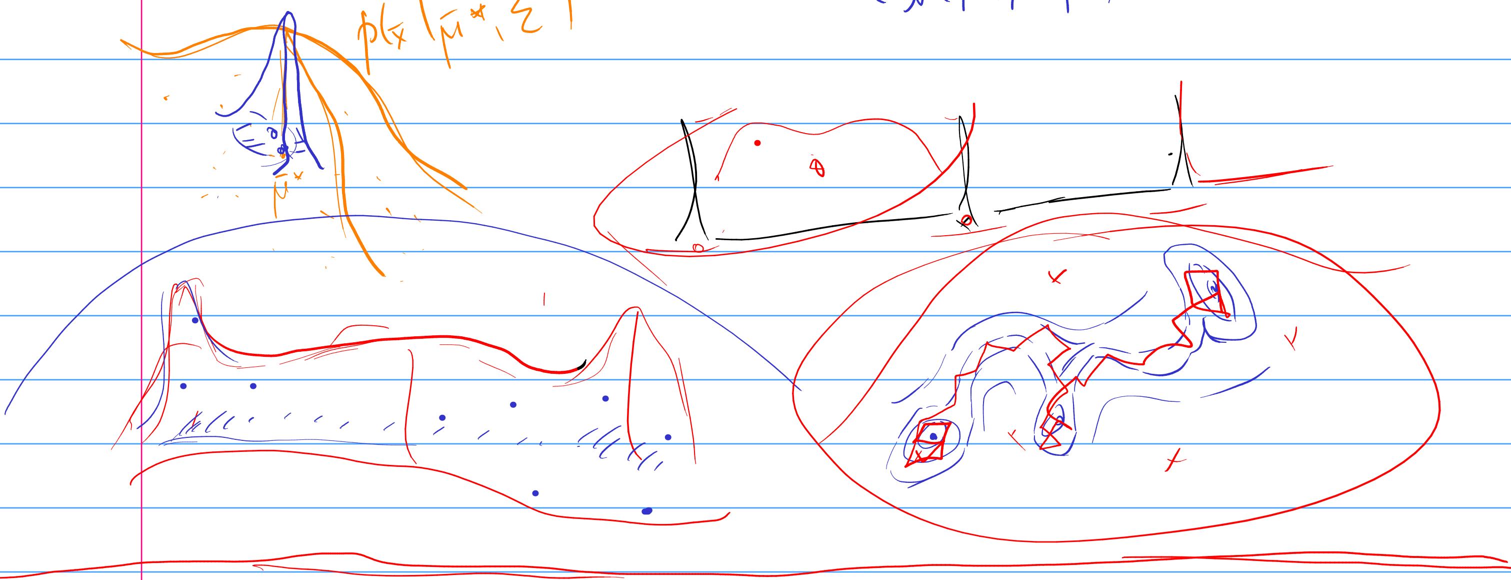




$p(\bar\mu|D) \propto p(\bar\mu)\, p(D|\bar\mu)$

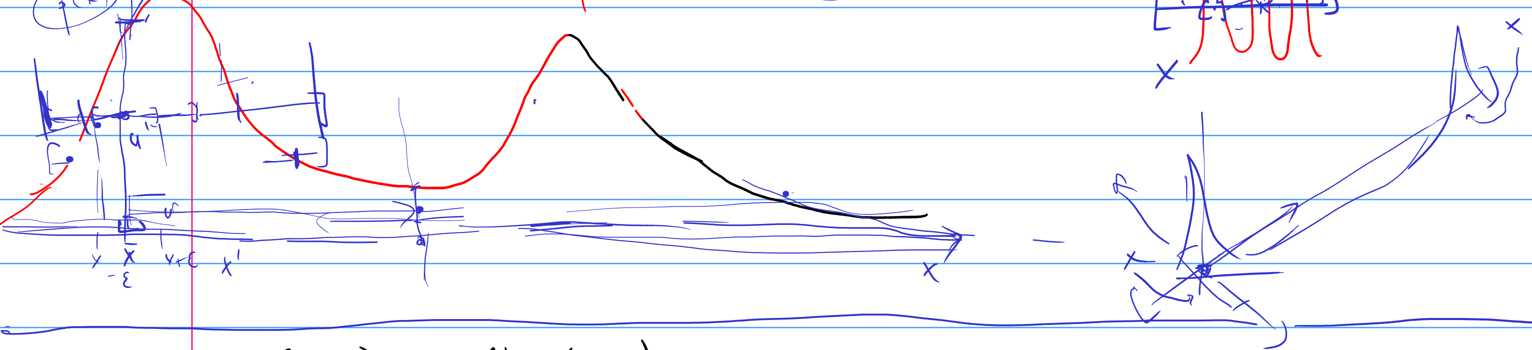
$= N(\bar\mu|\bar\mu^*, ?)$

$p(\mu|D)$



Slice sampling

$(\bar x, u) \mapsto (\bar x', u')$



$p(\theta|D) \propto p(\theta)\, p(D|\theta)$

$= \prod p(d|\theta)$

Gibbs sampling

$p(\bar x) = p(x_1, x_2, \dots, x_n)$

$p(x_i| x_1, x_2 \dots \cancel{x_i} \dots x_n)$

$\bar x = (x_1, x_2, \dots, x_n)$

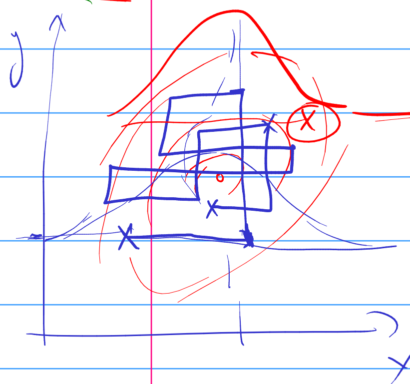
while True:

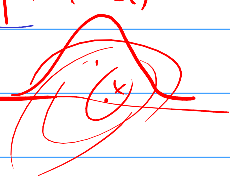
- for $i=1$ to n
  - $x_i \sim p(x_i|\bar x_{-i})$

$\bar X = (x_1 \dots x_{i-1}, x_i, x_{i+1}, \dots x_n) = (x_i, \bar x_{-i})$

$q(\bar x'|\bar x) = q(x_i', \bar x_{-i}|\bar x) =$

$= p(x_i'|\bar x_{-i}) \qquad x_i' \sim p(x_i|\bar x_{-i})$





$p(\bar x)\, T(\bar x, \bar x') = p(\bar x')\, T(\bar x', \bar x)$

$p(x_i, \bar x_{-i}) \cdot \underbrace{T_1(x_i, x_i')}_{p(x_i'|\bar x_{-i})} = p(x_i', \bar x_i) \cdot \underbrace{T_2(x_i', x_i)}_{p(x_i|\bar x_{-i})}$

$\underbrace{p(x_i, \bar x_{-i})}_{p(\bar x_{-i})\, p(x_i|\bar x_{-i})}\, \cancel{p(x_i'|\bar x_{-i})} = \underbrace{p(x_i', \bar x_{-i})}_{p(\bar x_{-i})\, p(x_i'|\bar x_{-i})}\, \cancel{p(x_i|\bar x_{-i})}$