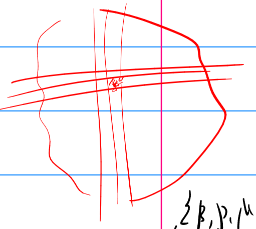
SIR -model

SEIR



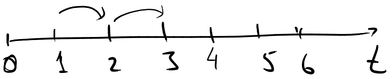
$\{\beta, p, \mu\}$

Susceptible → Infected $\overset{\mu}{\hookrightarrow}$ Recovered

$(1-\beta)^{I^{(t)}}$ $\quad 1-(1-\beta)^{I^{(t)}}$ $\quad \circlearrowleft, \mu$

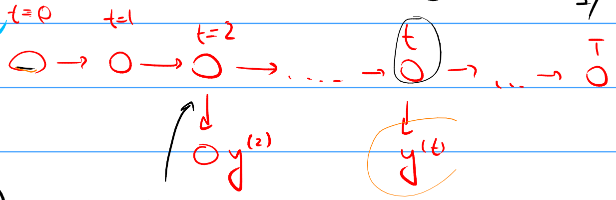
$$A = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cccc} S & I & R & \\ \end{array}$$

$$A = \begin{array}{cccc} S & I & R & \\ (1-\beta)^I & 1-(1-\beta)^I & 0 & S \\ 0 & 1-\mu & \mu & I \\ 0 & 0 & 1 & R \end{array}$$



$\bar{y} = (y_0, y_1, \ldots, y_T)$

π



$y^{(t)} \sim \text{Binomial}(I^{(t)}, p)$

$p(\beta) = \text{Beta}(\ldots, \ldots)$

$p(p) = \text{Bete}(\ldots, \ldots)$

$p(\mu) = \text{Beta}(\ldots, \ldots)$

$p(\bar{y} | \theta) \xrightarrow[\theta]{} \max$

$X = (\bar{x}_1, \ldots, \bar{x}_N)$

$x_{i,t}$

$\frac{\partial I}{\partial t} =$

I, S, R

$$p(\bar{y}, X | \theta) = \left[\prod_{i=1}^{N} \pi(x_{i,0})\right] \cdot p(y^{(0)} | X, \theta) \cdot \left[\prod_{t=1}^{T} \left(\prod_{i=1}^{N} p(x_{i,t} | x_{i,t-1}, \bar{X}_{-i}, \theta)\right) \cdot p(y^{(t)} | \bar{x}^{(t)}, \theta)\right]$$

EM/MCMC/HMM:

- fix θ, find X ⟶ $p(X | \cancel{\theta}, y)$ $\quad p(\bar{x}_j | \bar{x}_{-j}, \bar{y}, \theta)$ - Gibbs sampling
- fix X, find $p(\theta | X, \bar{y})$

$x_{j,0} \sim \pi$

$$p(\bar{x}_j | \bar{x}_{-j}, \bar{y}, \theta) =$$

$p(\bar{x}_j | \bar{x}_{-j}, \bar{y}, \theta)$

$$= p(x_{j1} \ldots x_{jT} | \ldots) = p(x_{jT} | \ldots) p(x_{j,T-1} | x_{j,T}, \ldots) \ldots p(x_{j1} | x_{j2}, \ldots)$$

$\alpha_i(t)$

$\beta_t(t)$

$\varepsilon_{ij}(t)$

$\delta_i(t)$

$p(x_{j,t+1}, x_{jt} | \bar{x}_{-j}, y, \theta)$

$$\begin{array}{cccc} & & & \\ S & p(\,) & p(-) & 0 \\ I & 0 & p(I, I | \ldots) & \ldots \\ R & 0 & 0 & \ldots \\ t-1 & S & I_{s'} & R \;\; t \end{array} = Q, \quad q^{(t)}_{j,s',s} = p(x_j^{(t+1)} = s', x_j^{(t)} = s | \ldots)$$

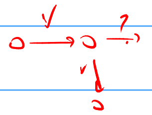
$q^{(t)}_{j,s',s}$

$\shortparallel$

$$p(x_{j,t-1} \overset{= s'}{}, x_{jt} \overset{= s}{} | \ldots) = p(x_{j,t-1} \overset{= s'}{} | \ldots) \cdot p(x_{jt} \overset{= s}{} | x_{j,t-1} \overset{= s'}{}, \ldots) \propto \left(\sum_{s''} q^{(t-1)}_{j,s'',s'}\right) \cdot (A_{s',s}(-j)) \cdot \text{Binomial}(y^{(t)}, I^{(t)}_{-j} + [x_{jt} = I], p)$$

$$\sum_{s''} q^{(t-1)}_{j,s'',s'} \qquad p(x_{jt} = s | x_{j,t-1} = s', x_{-j}, \theta) \cdot p(y^{(t)} | I^{(t)}_{-j}, x_{j,t}, \theta)$$

$$p(y^{(t)} | I^{(t)}_{-j}, \theta)$$

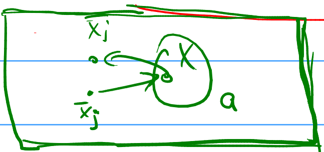


$$p(x_{jt} = s | x_{jt-1} = s', \bar{y}, \bar{x}_{-j}, \theta) \propto p(x_{jt} = s, y^{(t)} | x_{jt-1} = s', \bar{x}_{-j}, \theta) =$$

$$= p(x_{jt} = s | x_{jt-1} = s', \bar{x}_{-j}, \theta) \cdot p(y^{(t)} | x_{jt} = s, \theta, \bar{x}_{-j})$$

$Q^{(1)}_j, Q^{(2)}_j, \ldots, Q^{(T)}_j$

$$p(x_{jT} = s | \bar{x}_{-j}, \bar{y}, \theta) = \sum_{s'} q^{(T)}_{j,s',s}$$

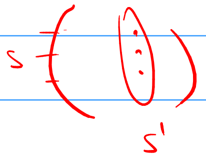


- sample $x_j | \bar{x}_{-j}, \theta, \bar{y}$:
  - find $Q^{(1)}_j \ldots Q^{(T)}_j$
  - sample $x_{jt} | x_{jt+1}$
- update $p(\theta | x, y)$

Mpy

$$p(x_{jt} = s | x_{j,t+1} = s', \bar{x}_{-j}, \bar{y}, \theta) \propto p(x_{jt} = s, x_{jt+1} = s' | \bar{x}_{-j}, \bar{y}, \theta)$$

$$x_{jt} \sim p(x_{jt}) \propto q^{(t+1)}_{j,s,s'}$$



$$a = \frac{p(x_{new})}{p(x_{old})} \cdot \frac{q(x_{old} | x_{new})}{q(x_{new} | x_{old})}$$

$p(\theta \mid \underline{\underline{X}}, \underline{\underline{y}})$ $\qquad \theta = \{p, \mu, \beta\}$

$$a_p \mathrel{+}= \sum_{t=1}^{T} y^{(t)}$$
$$b_p \mathrel{+}= \sum_{t=1}^{T} \left(I^{(t)} - y^{(t)}\right)$$

$$p(\mu) = \mathrm{Beta}(a_\mu, b_\mu)$$

$$\mu^*_{pri} = \frac{a_\mu}{a_\mu + b_\mu}$$

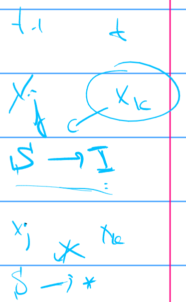
X

$$a_\mu \mathrel{+}= \#\{x_j^{(t-1)} = I,\ x_j^{(t)} = R\}$$
$$b_\mu \mathrel{+}= \#\{x_j^{(t-1)} = I,\ x_j^{(t)} = I\}$$

$$a_\beta =$$

$$b_\beta = \#\{x_j^{(t-1)} = S,\ x_j^{(t)} = S\} \cdot I^{(t-1)} + \dots$$

$I g^{(m)}$



$$a_\beta = E\{\text{кто-то заразился от кого-то}\}$$

$\bar{x}_j$ $\qquad \{x_j^{(t-1)} = S,\ x_j^{(t)} = I\} \Rightarrow$ было хотя бы 1 зараж. из $I^{(t-1)}$ возможных

$$p(x_j \text{ зараз. от } x_i : x_i^{(t-1)} = I \mid x_j \text{ зараз.}) = \frac{\beta'}{1-(1-\beta')^{I^{(t-1)}}} = \frac{p(x_j \text{ зараз. от } x_i)}{p(x_j \text{ зараз. от кого-то})}$$

$$E[\#\text{ заражн. } x_j] = \frac{\beta' I^{(t-1)}}{1-(1-\beta')^{I^{(t-1)}}} \ ; \quad E[\#\text{ не зар } x_j] = I^{(t-1)} -$$

$$a_\beta \mathrel{+}= \sum_{t, j:\ x_j^{(t-1)} = S,\ x_j^{(t)} = I} \left[\frac{\beta' I^{(t-1)}}{1-(1-\beta')^{I^{(t-1)}}}\right]$$

$$b_\beta \mathrel{+}= \sum_{t, j:\ x_j^{(t-1)} = S,\ x_j^{(t)} = S} I^{(t-1)} + \sum_{\substack{t, j:\ x_j^{(t-1)} = S \\ x_j^{(t)} = I}} \left[I^{(t-1)} - \frac{\beta' I^{(t-1)}}{1-(1-\beta')^{I^{(t-1)}}}\right]$$

$$p(S \to S) = (1-\beta)^I$$

$$\log p(S \to S) = I \log(1-\beta)$$

$$p(S \to I) = 1 - e^{\log p(S \to S)}$$

1 − np.~~exp~~(np.log. _)

np.expm1(x) = 1 − e^x

$$\bar{x}_j = (\overset{0}{S}, \overset{1}{S}, \overset{2}{S}, \overset{\downarrow}{I}, I, I, I, \overset{\downarrow}{R}, R)$$

T

нач. сост. self.pi[j] = (3, 7)

(3, −1)

(−1, −1)

пред. сост.

$$Q = \begin{array}{c|ccc} & S & I & R \\ \hline S & & & 0 \\ I & 0 & & \\ R & 0 & 0 & \end{array}$$

$$Q^{(0)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Q^{(t-1)} = \begin{pmatrix} & a & 0 \\ 0 & b & \cdot \\ 0 & 0 & \cdot \end{pmatrix}$$

S I

$$\log p(I)^{(t-1)} = \underline{\text{np.logaddexp}(a, b)}$$

$$a = \log p(S, I)^{(t-1)}$$
$$b = \dots (I, I)^{(t-1)}$$

$$Q^{(t)} = \begin{pmatrix} S & \boxed{\ } \cdot \boxed{\ } \end{pmatrix}$$

np.logaddexp(x, y) = $\ln(e^x + e^y)$