1) Naive Bayes

$$p(d|\underline{\underline{c}}) = \prod_{n\in d} p(w_n|c) \quad \text{- bag of words}$$

2) Unsupervised NB

EM: $p(w|c) = \varphi_{wc}$

E: $z_d = p(c|d)$

M: $\varphi_{wc} = \dfrac{\#\{w\in d,\ d\in c\}}{\sum_{w'} \#\{w'\in c\}}$

3) pLSI

ARTM

$\theta_{td} = p(t|d)$

$\varphi_{wt} = p(w|t)$

$$p(d|\Theta,\Phi) = \prod_{w\in d} p(w|d,\Theta,\Phi) = \prod_{w\in d}\sum_{t\in T} p(w,t|d,\Theta,\Phi) = \prod_{w\in d}\sum_{t}\theta_{td}\varphi_{wt}$$

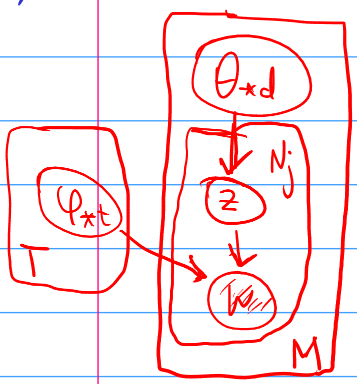
$$\boxed{p(D|\Theta,\Phi) = \prod_{d}\prod_{w\in d}\sum_{t}\theta_{td}\varphi_{wt}}$$

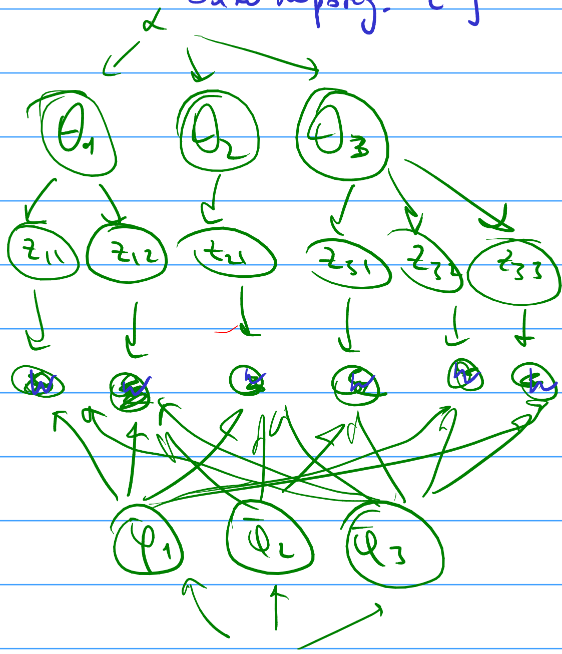
$n_{dwt} = \#\{w\in d,\ w\in t\}$

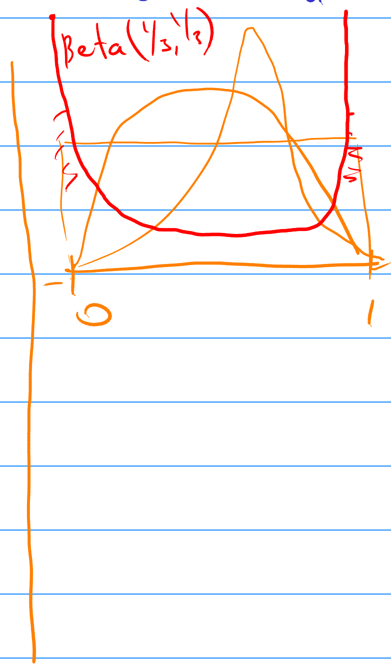
4) Latent Dirichlet Allocation LDA

$$\underset{w}{\left(\underset{d}{D}\right)} \approx \underset{w}{\left(\underset{t}{\Phi}\right)} \cdot \underset{t}{\left(\underset{d}{\Theta}\right)}$$

$[z_{jn}=t] \doteq [w_{jn}$ из j-го док. было порожд. $t]$







$p(W|\Theta,\Phi,\alpha,\beta) \xrightarrow[\Theta,\Phi]{} \max$

$p(W,\Theta,\Phi|\alpha,\beta) \xrightarrow[\Theta,\Phi]{} \max$

$= \sum_{Z} p(W,Z,\Theta,\Phi|\alpha,\beta)$

$$p(W,Z,\Theta,\Phi|\alpha,\beta) = p(\Phi|\beta)\cdot\prod_{j=1}^{M} p(\bar\theta_j|\alpha)\cdot\prod_{n=1}^{N_j} p(z_{jn},w_{jn}|\bar\theta_j,\Phi) =$$

$$= \prod_{t=1}^{T} p(\bar\varphi_t|\beta)\cdot\prod_{j=1}^{M}\left(p(\bar\theta_j|\alpha)\cdot\prod_{n=1}^{N_j} p(z_{jn}|\bar\theta_j)\, p(w_{jn}|z_{jn},\Phi)\right)$$

$p(\bar\varphi_t|\bar\beta) = \frac{1}{\mathrm{Dir}(\bar\beta)}\prod_{v}\varphi_{tv}^{\beta_v-1}$ $\quad p(\bar\theta_j|\bar\alpha) \propto \prod_t \theta_{jt}^{\alpha_t-1}$ $\quad \theta_{jt} = p(z_{jn}=t)$ $\quad \varphi_{tv} = p(w_{jn}=v|z_{jn}=t)$

$$\cancel{\log}\, p(W|\alpha,\beta) = \log \int_{\Theta}\int_{\Phi}\sum_{Z} p(W,Z,\Theta,\Phi|\alpha,\beta)\, d\Theta\, d\Phi =$$

$$= \log \overbrace{\iint\sum_{Z}}^{E_{Z,\Theta,\Phi}} p(W,Z,\Theta,\Phi|\alpha,\beta)\cdot \frac{q(Z,\Theta,\Phi)}{q(Z,\Theta,\Phi)}\, d\Theta\, d\Phi \geq$$

$$\geq \iint\sum_{Z} \log \frac{p(W,Z,\Theta,\Phi|\alpha,\beta)}{q(Z,\Theta,\Phi)}\, q(Z,\Theta,\Phi)\, d\Theta\, d\Phi = \quad = \mathcal{L}(q)\to\max$$

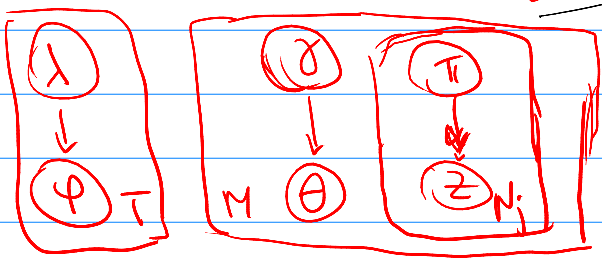
$KL(q\|p)\to\min$

$p(Z,\Theta,\Phi|W,\alpha,\beta)\cdot p(W|\alpha,\beta)$

$$= \iint\sum_{Z} \log\frac{p(Z,\Theta,\Phi|W,\alpha,\beta)}{q(Z,\Theta,\Phi)}\, q(Z,\Theta,\Phi)\, d\Theta\, d\Phi + \cancel{\log}\, p(W|\alpha,\beta)$$

$\mathcal{L}(q)\to\max$

$$q(Z,\Theta,\Phi|\Gamma,\Pi,\Lambda) = \left(\prod_{t=1}^{T} q_t(\bar\varphi_t|\bar\lambda_t)\right)\left(\prod_{j=1}^{N} q_{j\gamma}(\bar\theta_j|\bar\gamma_j)\cdot\prod_{n=1}^{N_j} q_{j\pi}(z_{jn}|\bar\pi_{jn})\right)$$



$$q_t(\bar\varphi_t|\bar\lambda_t) \propto \prod_{v} \varphi_{tv}^{\lambda_{tv}-1}$$

$$q_{j\gamma}(\bar\theta_j|\bar\gamma_j) \propto \prod_{t} \theta_{jt}^{\gamma_{jt}-1}$$

$$q_{j\pi}(z_{jn}|\bar\pi_{jn}) = \prod_{t} \pi_{jnt}^{[z_{jn}=t]}$$

1) Матрица Φ – парам. модели

$$q(Z,\Theta) \approx p(Z,\Theta|W,\Phi,\alpha,\beta)$$

$$KL(q_j\|p_j) = \int_{\bar\theta_j}\sum_{z_j} q(\bar z_j,\bar\theta_j|\bar\gamma_j,\bar\pi_j)\cdot\log\frac{q(\ldots)}{p(\bar z_j,\bar\theta_j|\bar w_j,\Phi,\alpha,\beta)}\, d\bar\theta_j$$

$$\mathcal{L}(\bar\gamma_j,\bar\pi_j) = \int_{\bar\theta_j}\sum_{z_j} \log\frac{p(\bar w_j,\bar z_j,\bar\theta_j|\Phi,\alpha,\beta)}{q(\bar z_j,\bar\theta_j|\bar\gamma_j,\bar\pi_j)}\cdot q(\bar z,\bar\theta|\gamma,\pi)\, d\bar\theta_j$$

$$= E_{q_j}[\log p(\bar w_j,\bar z_j,\bar\theta_j|\Phi,\alpha,\beta)] - E_{q_j}[\log q_j] =$$

$$= E[\log p(\bar\theta_j|\alpha)] + E[\log p(\bar z_j|\bar\theta_j)] + E[\log p(\bar w_j|\bar z_j,\Phi)]$$

$$- E[\log q(\bar\theta_j|\bar\gamma_j)] - E[\log q(\bar z_j|\bar\pi_j)]$$

$$\mathbb{E}_q[\log p(\bar\theta_j|\alpha)] = \mathbb{E}_q\left[\log\left(\frac{\Gamma(\sum \alpha_t)}{\prod_t \Gamma(\alpha_t)} \prod_t \theta_{jt}^{\alpha_t-1}\right)\right] =$$

$$= \log\Gamma\left(\sum\alpha_t\right) - \sum \log\Gamma(\alpha_t) + \sum_{t=1}^{T} (\alpha_t-1)\,\underline{\mathbb{E}_q[\log\theta_{jt}]}$$

$$p(\bar x|\bar\eta) = \underline{h(\bar x)\, e^{\bar\eta^T t(\bar x) - a(\bar\eta)}}$$

$$\frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2\sigma^2}(x-\mu)^2 = \left(-\frac{1}{2\sigma^2}x^2 + \frac{1}{\sigma^2}x\mu\right) - \frac{\mu^2}{2\sigma^2}}$$

$$t(x) = \begin{pmatrix} -x^2/2 \\ x \end{pmatrix} \qquad \bar\eta = \begin{pmatrix} 1/\sigma^2 \\ \mu/\sigma^2 \end{pmatrix} = \begin{pmatrix} \tau \\ \mu\tau \end{pmatrix}$$

$$h(x) = 1$$

$$e^{-a(\eta)} \doteq \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{\mu^2}{2\sigma^2}}$$

$$\log p(\bar x|\bar\eta) = \log h(x) + \bar\eta^T t(\bar x) - a(\bar\eta)$$

$$\log Dir(\bar x|\bar\alpha) = \sum_i (\alpha_i - 1)\log x_i - \log B(\bar\alpha) = a(\bar\eta)$$

$$t(\bar x) = \begin{pmatrix} \vdots \\ \log x_i \\ \vdots \end{pmatrix}, \quad \bar\eta = \begin{pmatrix} \vdots \\ \alpha_i - 1 \\ \vdots \end{pmatrix}$$

$$\nabla_{\bar\eta} \int h(\bar x)\, e^{\bar\eta^T t(\bar x) - a(\bar\eta)}\, d\bar x = 0$$

$$\int h(\bar x)\, e^{\bar\eta^T t(\bar x) - a(\bar\eta)} \cdot \left(t(\bar x) - \nabla_\eta a(\bar\eta)\right) d\bar x = 0$$

$$\int t(\bar x)\, \underbrace{h(\bar x)\, e^{\bar\eta^T t(\bar x) - a(\bar\eta)}}_{p(\bar x)}\, d\bar x = \int \nabla_{\bar\eta} a(\bar\eta)\, \overbrace{h(\bar x)\, e^{\bar\eta^T t(\bar x) - a(\bar\eta)}\, d\bar x}^{1}$$

$$\mathbb{E}_{p(\bar x)}[t_i(\bar x)] = \frac{\partial a(\bar\eta)}{\partial \eta_i}$$

$$\boxed{\mathbb{E}_{p(x)}[t(\bar x)] = \nabla_{\bar\eta} a(\bar\eta)}$$

$$\frac{\partial \log\Gamma(x)}{\partial x} = \Psi(x)$$

digamma function

$$\mathbb{E}_q[\log\theta_{jt}] = \frac{\partial \log B(\bar\alpha)}{\partial \alpha_t} = \frac{\partial\left(\sum_{t'}\log\Gamma(\alpha_{t'}) - \log\Gamma\left(\sum\alpha_{t'}\right)\right)}{\partial \alpha_t}$$

$$\mathbb{E}_q[\log\theta_{jt}] = \Psi(\gamma_t) - \Psi\left(\sum\gamma_{t'}\right)$$

$$\mathbb{E}_q[\log p(\bar\theta_j|\bar\alpha)] = \log\Gamma\left(\sum\alpha_t\right) - \sum\log\Gamma(\alpha_t) + \sum_t(\alpha_t - 1)\left(\Psi(\gamma_{jt}) - \Psi\left(\sum_s \gamma_{js}\right)\right) \qquad (1)$$

$$\mathbb{E}_q[\log p(\bar z_j|\bar\theta_j)] = \mathbb{E}_q\left[\sum_{n=1}^{N_j}\sum_{t=1}^{T}[z_{jn}=t]\log\theta_{jt}\right] =$$

$$= \sum_n\sum_t \mathbb{E}_q\left[[z_{jn}=t]\cdot\log\theta_{jt}\right] = \sum_{n=1}^{N_j}\sum_{t=1}^{T} \pi_{jnt}\left(\Psi(\gamma_{jt}) - \Psi\left(\sum_s\gamma_{js}\right)\right) \qquad (2)$$

$$\mathbb{E}_q\left[\log p(\bar w_j \mid \bar z_j, \Phi)\right] = \mathbb{E}_q\left[\sum_{n=1}^{N_j}\sum_{t=1}^{T}\sum_{v=1}^{V}[z_{jn}=t][w_{jn}=v]\log\varphi_{tv}\right] \quad (3)$$

$$= \sum_n\sum_t\sum_v [w_{jn}=v]\cdot\pi_{jnt}\cdot\log\varphi_{tv}$$

$$\mathbb{E}_q\left[\log q(\theta_j \mid \gamma_j)\right] = \log\Gamma\left(\sum\gamma_{js}\right) - \sum_t \log\Gamma(\gamma_{jt}) + \sum_t(\gamma_{jt}-1)\left(\psi(\gamma_{jt})-\psi\left(\sum_s\gamma_{js}\right)\right) \quad (4)$$

$$\mathbb{E}_q\left[\log q(\bar z_j \mid \bar\pi_j)\right] = \mathbb{E}_q\left[\sum_n\sum_t[z_{jn}=t]\log\pi_{jnt}\right] = \sum_{n=1}^{N_j}\sum_{t=1}^{T}\pi_{jnt}\log\pi_{jnt} \quad (5)$$

$$\mathcal{L}(\gamma_j,\bar\pi_j) = (1)+(2)+(3)-(4)-(5) \xrightarrow[\gamma_j,\bar\pi_j]{} \max \qquad \boxed{\forall j,n \quad \sum_s \pi_{jns}=1}$$

$$L(\pi_{jnt}) = \pi_{jnt}\left(\psi(\gamma_{jt})-\psi\left(\sum_s\gamma_{js}\right)\right) + \pi_{jnt}\log\varphi_{t,w_{jn}} - \pi_{jnt}\log\pi_{jnt} +$$

$$+\lambda\left(\sum_s\pi_{jns}-1\right) \xrightarrow[\pi_{jn*}]{} \max$$

$$\frac{\partial L}{\partial\pi_{jnt}} = \psi(\gamma_{jt})-\psi\left(\sum_s\gamma_{js}\right) + \log\varphi_{t,w_{jn}} - \log\pi_{jnt} - 1 + \lambda = 0$$

$$\pi_{jnt} = e^{\psi(\gamma_{jt})-\psi(\sum_s\gamma_{js})+\log\varphi_{t,w_{jn}}}\cdot e^{\lambda-1}$$

$$\boxed{\pi_{jnt} \propto e^{\psi(\gamma_{jt})-\psi(\sum_s\gamma_{js})+\log\varphi_{t,w_{jn}}}}$$

$$\frac{\partial L}{\partial\gamma_{jt}} = (\alpha_t-1)\cdot\psi'(\gamma_{jt}) - \left[\sum_s(\alpha_s-1)\right]\psi'\left(\sum_s\gamma_{js}\right) + \sum_n\left(\pi_{jnt}\psi'(\gamma_{jt}) - \left(\sum_s\pi_{jns}\right)\psi'\left(\sum_s\gamma_{js}\right)\right)$$

$$\alpha_t-\cancel{1}+\sum_n\pi_{jnt}-\gamma_t+\cancel{1}$$

$$-(\gamma_t-1)\psi'(\gamma_{jt}) + \left[\sum_s(\gamma_s-1)\right]\psi'\left(\sum_s\gamma_{js}\right) =$$

$$= \psi'(\gamma_{jt})\left[\alpha_t+\sum_n\pi_{jnt}-\gamma_{jt}\right] - \psi'\left(\sum_s\gamma_{js}\right)\cdot\sum_{s=1}^{T}\left[\alpha_s+\sum_n\pi_{jns}-\gamma_{js}\right] = 0$$

$$\forall j,t \qquad \boxed{\gamma_{jt} = \alpha_t + \sum_n\pi_{jnt}}$$

EM

- E-war: fix Φ, $\pi_{jnt} \propto \ldots$, $\gamma_{jt} = \ldots$
- M-war: fix π_{jnt}, γ_{jt}

$$\varphi_{tv} \propto \sum_{j=1}^{M} \sum_{n=1}^{N_j} [w_{jn} = v] \, \pi_{jnt}$$

Bayes - LDA

$$p(Z, \Theta, \Phi \mid \alpha, \beta, W) \approx q(Z, \Theta, \Phi)$$

$$\lambda_{tv} = \beta_{tv} + \sum_{j=1}^{M} \sum_{n=1}^{N_j} [w_{jn} = v] \, \pi_{jnt}$$

$\pi_{jnt} \propto$ __

$\gamma_{jt} =$ __

$\lambda_{tv} =$ __