①

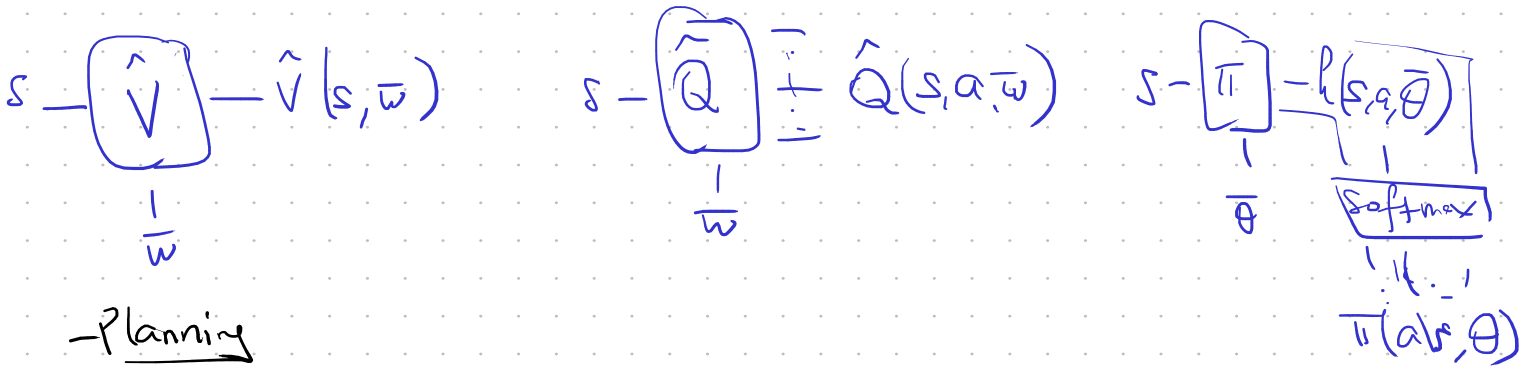
$p(r,s'|s,a)$ — динамика MDP

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 \ldots$$

$$V_\pi(s) = \mathbb{E}_\pi[G_t | S_t = s]$$

$Q_\pi(s,a)$ $\quad$ $Q_*(s,a)$

Eligibility traces



- Planning
- Decision-time planning

② Generative/discriminative models

Supervised learning $\quad D = \{(\bar{x}_n, y_n)\}_{n=1}^N$

$f: \bar{x} \mapsto y$ $\quad$ $p(y|\bar{x}) = ?$

Unsupervised learning

$p(\bar{x}) = ?$

$$p(\bar{\theta}|D) = p(\bar{\theta}|\bar{y}, X) \propto p(\bar{\theta}) \cdot \underbrace{p(\bar{y}|\bar{\theta}, X)}_{= \prod_{n=1}^N p(y_n|\bar{\theta}, \bar{x}_n)}$$

$\bar{\theta} \to \max$

$$p(y|\bar{x}, D) = \int p(y|\bar{\theta}, \bar{x}) \cdot p(\bar{\theta}|D) d\bar{\theta} = \mathbb{E}_{\bar{\theta} \sim p(\bar{\theta}|D)}[p(y|\bar{\theta}, \bar{x})]$$

Discriminative

$p(y|\bar{x})$

LR: $p(y|\bar{x}, \bar{w}) = \sigma(\bar{x}^T \bar{w})$

Generative

$p(\bar{x}, y)$ $\quad$ $p(y|\bar{x}) \propto p(\bar{x}, y)$

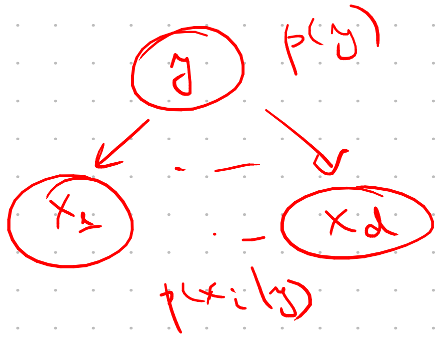
$= p(\bar{x}, y) / p(\bar{x})$

LDA/QDA $\quad p(\bar{x}|C_k) = \mathcal{N}(\bar{x}|\bar{\mu}_k, \Sigma_{(k)})$

$\forall k \quad p(\bar{x}, y=k)$

Naive Bayes

$$p(y,\bar{x}) = p(y)\cdot\prod_{i=1}^{d} p(x_i|y)$$



$y\in\{1,\dots,K\}$

$x_i\in\{1,\dots,M\}$

$$D=\{(\bar{x}_n,y_n)\}_{n=1}^{N}$$

$$\theta_k \overset{\log}{=} p(y=k)\ ; \qquad \theta_{imk} \overset{\log}{=} p(x_i=m\,|\,y=k)$$

$$\hat{\theta}_k = \frac{\#\{y=k\}+1}{N+K} \qquad e$$

$$\hat{\theta}_{imk} = \frac{\#\{m \text{ leg. ha } i\text{-ú ng. 6 ka. } k\}+1}{\#\{i\text{-ő ng. 6 ka. } k\}+M} \qquad e$$

$$p(D|\bar{\theta}) = \prod_n p(y_n)\prod_i p(x_{ni}|y_n) =$$

$$= \prod_{n=1}^{N}\prod_{k=1}^{K}\left(p(y_n=k)\cdot\prod_{i=1}^{d}\prod_{m=1}^{M} p(x_{ni}=m|y_n=k)^{[x_{ni}=m]}\right)^{[y_n=k]}$$

$$= \prod_{n=1}^{N}\prod_{k=1}^{K}\left(e^{\theta_k}\right)^{[y_n=k]}\cdot\prod_{i=1}^{d}\prod_{m=1}^{M}\left(e^{\theta_{imk}}\right)^{[x_{ni}=m]\cdot[y_n=k]}$$

$$\log p(D|\bar{\theta}) = \sum_n\sum_k [y_n=k]\cdot\theta_k + \sum_n\sum_k\sum_i\sum_m [\,.\,][\,.\,]\cdot\theta_{imk}$$

$$= \sum_{k=1}^{K}\left(\sum_{n=1}^{N}[y_n=k]\right)\theta_k + \sum_{k=1}^{K}\sum_{i=1}^{d}\sum_{m=1}^{M}\left(\sum_{n=1}^{N}[x_{ni}=m][y_n=k]\right)\cdot\theta_{imk}$$

$$\log p(D|\bar{\theta}) = \sum_k f_k(D)\cdot\theta_k + \sum_{i,m,k} f_{imk}(D)\,\theta_{imk}$$

$$f_k(D) = \sum_n [y_n=k]$$

$$\cancel{\theta\cdot[x_{ni}\,x_{nj}=1]}$$

$$f_{imk}(D) = \sum_{imk}[x_{ni}=m][y_n=k]$$

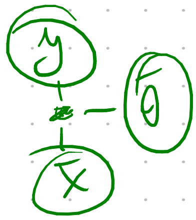
Logistic regression

$$p(y=k\mid\bar{x},\bar{\theta}) = \frac{e^{\bar{\theta}_k^T\bar{x}}}{\sum_s e^{\bar{\theta}_s^T\bar{x}}} = \frac{1}{Z(\bar{x})}\cdot e^{\sum_{i=1}^{d}\theta_{ki}\cdot x_i}$$

новые признаки

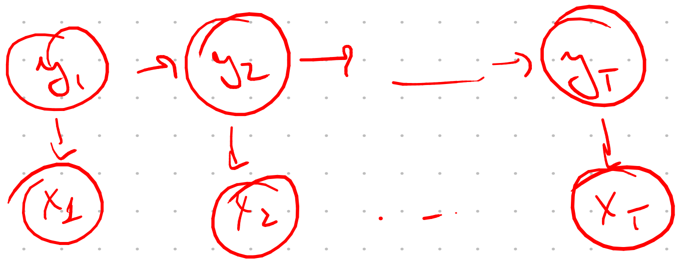
$$\log p(D\mid\bar{\theta}) = \sum_n \log(\_\_) = \overbrace{-\sum_n \underline{\log Z(\bar{x}_n)}}^{\text{Const}} + \sum_{n=1}^{N}\sum_{i=1}^{d}\sum_{k=1}^{K}[y_n=k]\cdot x_{ni}\cdot\theta_{ki}$$

$$\bar{x} \to \begin{pmatrix}\bar{x}\\ x_i x_j\end{pmatrix}$$



NB & LR – generative-discriminative pair

HMM:



$$p(\bar{x},\bar{y}) = p(y_1)p(x_1\mid y_1) \text{———} p(y_T\mid y_{T-1})p(x_T\mid y_T)$$

$$\log p(D\mid\bar{\theta}) = \sum_n \log p(\bar{x}_n,\bar{y}_n\mid\bar{\theta}) =$$

$$= \sum_n \log p(y_{n1}) + \sum_n\sum_{t=1}^{T-1}\log p(y_{n,t+1}\mid y_{n,t}) + \sum_n\sum_{t=1}^{T}\log p(x_{nt}\mid y_{nt})$$

$$= \sum_{n=1}^{N}\sum_{i=1}^{M}[y_{n1}=i]\cdot\underbrace{\log p(y_{n1}=i)}_{\theta_i} +$$

$$+ \sum_n\sum_{t=1}^{T-1}\sum_{i=1}^{M}\sum_{j=1}^{M}[y_{nt}=i,\ y_{n,t+1}=j]\cdot\overbrace{\log p(y_{n,t+1}=j\mid y_{n,t}=i)}^{\theta_{ij}}$$

$$+ \sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{K}[y_{n,t}=i,\ x_{nt}=k]\cdot\underbrace{\log p(x_{nt}=k\mid y_{nt}=i)}_{\theta_{ik}}$$

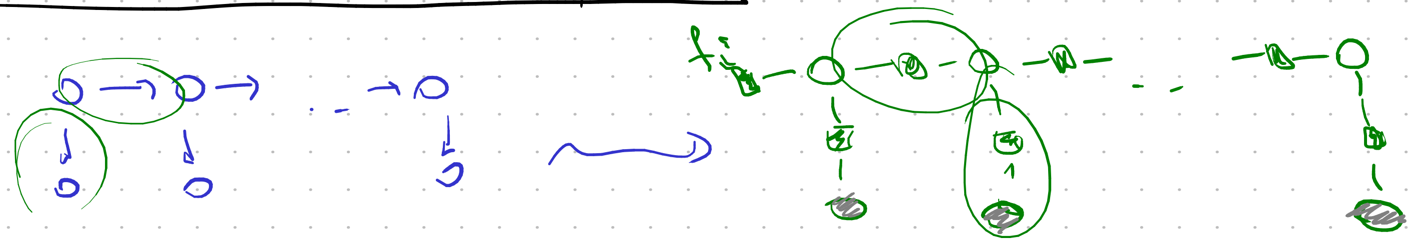
$$\log p(D|\bar\theta) = \sum_{i=1}^{M}\left(\sum_{n=1}^{N}[y_{n1}=i]\right)\theta_i \; +$$

$$+\sum_{i=1}^{M}\sum_{j=1}^{N}\left(\sum_{n=1}^{N}\sum_{t=1}^{T-1}[y_{nt}=i,\, y_{n,t+1}=j]\right)\cdot\theta_{ij}$$

$$+\sum_{i=1}^{M}\sum_{k=1}^{K}\left(\sum_{n=1}^{N}\sum_{t=1}^{T}[y_{n,t}=i,\, x_{n,t}=k]\right)\cdot\theta_{ik}$$

$\bar\theta \longrightarrow \max$

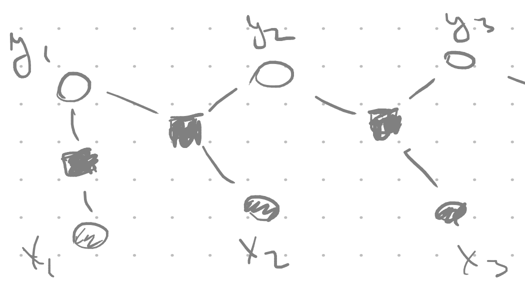
Conditional Random Field CRF



$$\log p(D|\bar\theta) = \sum_n \mathrm{Const}(\bar x_n) \; +$$

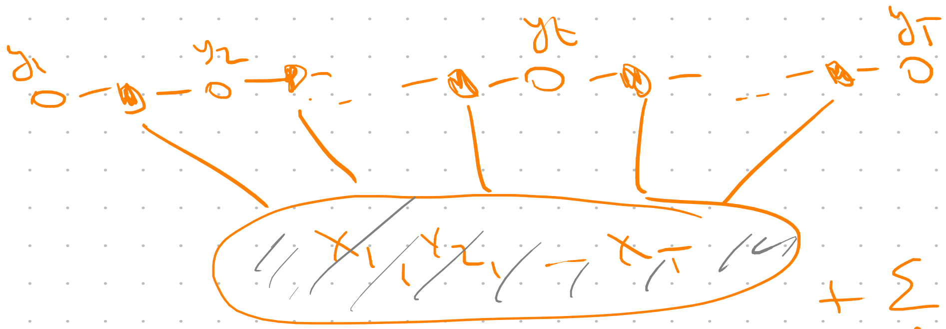
$$+\sum_{n=1}^{N}\left(\sum_{i=1}^{M} f_i(y_{n1})\,\theta_i + \sum_{i=1}^{M}\sum_{j=1}^{M}\left(\sum_{t=1}^{T-1} f_y(y_{nt}, y_{n,t+1})\right)\theta_{ij} \; + \right.$$

$$\left. + \sum_{i=1}^{M}\sum_{k=1}^{K}\left(\sum_{t=1}^{T} f_x(y_{nt}, x_{nt})\right)\theta_{ik}\right)$$



Const

$$\log p(D|\bar\theta) = \dots + \sum_{i,j,k}\left(\sum_t f(y_{t-1}, y_t, x_t)\right)\theta_{ijk}$$



$$\log p(D|\bar\theta) = \dots$$

$$+\sum_n \sum_{i,j} \sum_t f(y_t, y_{t+1}, \bar x)\,\theta_{ij}$$

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