Shared Memory Buffer Management for Heterogeneous Packet Processing

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Abstract-Packet processing increasingly involves heterogeneous requirements. We consider the well-known model of a shared memory switch with bounded-size buffer and generalize it in two directions. First, we consider unit-sized packets labeled with an output port and a processing requirement (i.e., packets with heterogeneous processing), maximizing the number of transmitted packets. We analyze the performance of buffer management policies under various characteristics via competitive analysis that provides uniform guarantees across traffic patterns [7]. We propose the Longest-Work-Drop policy and show that it is at most 2-competitive and at least $\sqrt{2}$ -competitive. Second, we consider another generalization, posed as an open problem in [10], where each unit-sized packet is labeled with an output port and intrinsic value, and the goal is to maximize the total value of transmitted packets. We show first results in this direction and define a scheduling policy that, as we conjecture, may achieve constant competitive ratio. We also present a comprehensive simulation study that validates our results.

I. INTRODUCTION

The modern network edge is required to perform tasks with heterogeneous complexity, including features such as advanced VPN services, deep packet inspection, firewalling, and intrusion detection (to list just a few). This trend is further exacerbated by the advent of new paradigms like Software-Defined Networking [17], [23], Fog Computing [6] which create new opportunities for advanced services.

Implementing such services, which give rise to different *traffic types*, requires considering various parameters of packets (different amounts of processing or intrinsic values) to achieve efficient implementation at the network processor. This leads to new challenges in performance and implementation for traditional architectures.

Because of the store-and-forward nature of packet processing — in contrast to OS scheduling — packets are processed in a "run-for-completion" manner where no two different cores can access the same packet during its processing to avoid expensive rescheduling. In this setting an efficient input buffer architecture and management become crucial. The best way to accommodate traffic burstiness is to adopt a single queue architecture, where the whole buffer is shared among all types of traffic and every core is able to process any type of traffic (see the top of Fig 1). In this architecture an online greedy policy that implements priority queuing (PQ), where packets are ordered in non-decreasing order of their processing requirement and can be pushed out upon arrival of new packets, has optimal throughput [11].

However this simple approach has important drawbacks. First, PQ order is required: with a simpler to implement FIFO processing order the competitive ratio degrades to $\Omega(\log k)$ with respect to the optimal clairvoyant algorithm [19], where k is a maximal processing per packet. Second, it can cause starvation of packets with higher processing requirements, which implicitly leads to setting priorities among traffic types and thus services so that they are rigged to the inverse of the processing requirements. Second, handling several traffic types on individual cores leads to increased complexity of the programs loaded onto these cores, as these must implement several services in a combined manner.

We thus consider a shared memory switch where the buffer is shared among all types of traffic but in contrast to the previous single queue model each core processes only one type of traffic. The single queue buffer architecture is in fact a special case of a shared memory switch (see the bottom of Fig 1). Since all arriving packets with the same output port label have the same traffic type, the implementation of advanced processing order is not required (regular FIFO suffices). Furthermore, there is no starvation of any specific type of traffic.

The paradigm of shared memory switch in general is very flexible since it can support both complete sharing, where the same buffer serves all output ports, and complete partition, where each output port gets a *de facto* dedicated buffer. Complete sharing utilizes the entire buffer space but can hamper fairness: a single output port may monopolize the shared memory and drop packets dedicated to other output ports. On the other hand, complete partitioning ensures fairness but may lead to significantly underutilized buffer space, losing more packets in case of congestion. But how to get the best of both worlds?

To study performance, we employ *competitive analysis* [7], [26]. An algorithm ALG is said to be α -competitive for some

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Fig. 1: Two different buffer management models. Left: each core can process any traffic type, IPsec, SSL, or firewall (FW); right: each core processes a specific traffic type from its own queue, but all queues share the same buffer.

 $\alpha \geq 1$ if for any arrival sequence, the number of packets transmitted by ALG is at least $1/\alpha$ times the number of packets transmitted by an optimal offline clairvoyant algorithm OPT.

Our contributions. In this paper we consider the crucial problem of throughput maximization. We generalize the previous shared memory switch model with uniform values and required processing [1] and consider two different packet characteristics that define a traffic type together with output port labels: (1) required processing per packet and (2) packet value. Our main contribution is to identify properties of an "ideal" online policy that maximizes throughput and achieve a constant competitive ratio [7]. We argue that required processing and packet values have different natures. In particular, we show that to achieve a constant competitive ratio in the case of required processing, it is enough to consider the total work per queue but the normalized work per queue cannot be used to achieve a constant competitiveness. In the value case however (where throughput is measured by total transmitted value) the total value per queue constitutes a poor choice but normalized value can potentially achieve constant competitiveness. Most considered algorithms are greedy (accept all arrivals if there is enough buffer space) and hence allow for simple implementations.

In the first part of this work (1) we consider heterogeneous processing requirements, as opposed to shared memory switches with a single processing cycle per packet [1], [16]. In Section III, we consider a shared memory architecture where an arriving packet has an output port label and amount of required processing in cycles, with the constraint that packets accepted to a same queue have the same processing requirements (two different queues still can accept packets with the same required processing, see Fig. 2). Our main result is to exhibit a novel Longest-Work-Drop (LWD) policy that is at most 2-competitive. When processing requirements are uniform, LWD emulates the well-known Longest-Queue-Drop (LQD) policy [1]. As result LWD is at least $\sqrt{2}$. In the special case when all queues accept packets with different processing requirements LWD is at least $\left(\frac{4}{3} - \frac{6}{B}\right)$ -competitive, where B is the buffer size. We also demonstrate a number of lower bounds for several policies that have good performance characteristics for homogeneous processing requirements. Namely, we show that: (1) LQD is at least $(\sqrt{k} - o(\sqrt{k}))$ -competitive, where k is the maximum processing required for packets; (2) the Harmonic policy [16], which is at most $O(\log n)$ -competitive in the model with a single processing cycle per packet and noutput ports, is at least $\left(\frac{1}{2}\sqrt{k \ln k} - o(\sqrt{k \ln k})\right)$ -competitive in our model; (3) the Biggest-Packet-Drop (BPD) policy that attempts to keep packets with lowest processing requirements is at least $(\ln k + \gamma)$ -competitive for $\gamma = 0.5772...$

In the second part (2) we consider a different yet related model of shared memory switch, where each unit-sized packet is labeled with an output port and an intrinsic value (Section IV). The objective is to optimize the total value of transmitted packets. The existence of a scheduling policy achieving a constant competitive ratio here was presented as an open question in SIGACT News [10, p. 22]. To the best of our knowledge, we provide the first results addressing this problem as follows. First we show that the above-mentioned LQD policy is at least $\left(\sqrt[3]{k} - o\left(\sqrt[3]{k}\right)\right)$ -competitive. Furthermore, we define a Minimal-Value-Drop (MVD) policy (that equivalent to BPD) that in the case of congestion drops a packet with minimal value. We show that MVD is at least $\frac{m-1}{2}$ -competitive for $m = \min\{k, B\}$, where k is the maximal packet value and B is the buffer size. As a result, an ideal policy should address a fundamental tradeoff between a number of "active" ports (like LQD) and maximization of total admitted value (like MVD) to achieve a constant competitiveness (similar to LWD in the first model considered in this paper). We thus define a Maximal-Ratio-Drop (MRD) policy that pushes out a packet from a queue with maximal ratio $\frac{|Q_j|}{a_j}$, where a_j is the average value in the *j*-th output queue Q_j and $|Q_j|$ is the size of Q_i . The competitiveness of MRD is at least $\sqrt{2}$. We also conduct a comprehensive simulation study validating our theoretical results in Section V.

II. RELATED WORK

The efforts most closely related to the present work are those of Aiello et al. [1] and of Kesselman and Mansour [16]. Aiello et al. [1] propose a non-push-out buffer management policy called Harmonic that is at most $O(\log n)$ -competitive and establish a lower bound of $\Omega(\frac{\log n}{\log \log n})$ on the performance of any online non-push-out deterministic policy, where *n* is the number of output ports. Kesselman and Mansour [16] demonstrate that the LQD policy is at most 2- and at least $\sqrt{2}$ -competitive. Note that both these works consider packets with an invariably single required processing cycle.

The additional line of research considers packets with heterogeneous processing [11], [18]–[20] but no one of them consider shared-memory switch.

Our current work can be viewed as part of a larger research effort concentrated on studying competitive algorithms for management of bounded buffers. A survey by Goldwasser [10] provides an excellent overview of this field. Initiated in [15], [21], this line of research has received tremendous attention over the past decade. Various models have been proposed and studied, including QoS-oriented models where packets have individual weights [2], [8], [15], [21] and models where packets have dependencies [12], [22]. A related field that has recently attracted much attention focuses on various switch architectures and aims to design competitive algorithms for multi-queued scenarios therein (cf. [3]–[5], [13], [14], [25]). However, these models do not cover the case of packets with heterogeneous processing requirements. Pruhs [24] provides a comprehensive overview of competitive online scheduling for server systems. Note that scheduling for server systems usually concentrates on average response time, while we focus mostly on throughput. Furthermore, scheduling of server systems does not allow jobs to be dropped, which is an inherent aspect of our model due to size limitations on buffers.

III. PACKET SCHEDULING WITH HETEROGENEOUS PROCESSING REQUIREMENTS

A. Model Description

We consider an $l \times n$ shared memory switch with a buffer of size B; l and n represent the number of input and output ports, respectively. We assume that $B \ge n$. Each i^{th} output port manages a single output queue Q_i , $1 \le i \le n$; the number of packets in Q_i is denoted by $|Q_i|$. Each Q_i implements first-infirst-out (FIFO) processing order. Each packet p(d, w) arriving at an input port is labeled with the output port number d and its required work w in processing cycles ($1 \le d \le n$ and $1 \le w \le k$), where k denotes the global upper bound on required work per packet. In what follows, we denote by w a packet with required work w; by $h \times w$, a burst of h packets with required work w each.

Time is slotted; we divide each time slot into two phases (see Fig. 2): During the (1) arrival phase a burst of new packets arrives at each input port, and the (*buffer management*) policy decides which ones should be admitted; we assume that during the arrival phase input ports are served in a fixed order from input port 1 to input port l with no restriction on the burst size (at the same time slot more than l arrivals are allowed); the arrivals are adversarial and do not assume any specific traffic distribution except a single constraint: all packets arriving with output port label *i* have the same required processing w_i , $1 \leq w_i \leq k$. Note that two different output queues Q_i and Q_j , $1 \leq i, j \leq n$ can still accept packets with the same processing requirement, $w_i = w_j$. Without loss of generality, we assume that queues are sorted in the order of their processing requirement: if i < j then $w_i \leq w_j$. An accepted packet can be later dropped from the buffer when another packet is accepted instead; in this case we say that a packet p is *pushed out* by another packet q, and a policy that allows this is called a *push-out* policy. During the (2) transmission phase, required work of the head-of-line packet in FIFO order at each non-empty queue is reduced by one, and every packet with zero residual work is transmitted.

B. Choosing the Right Properties for Buffer Management

Next we attempt to explore properties of a "perfect" policy that provides constant competitiveness — if it exists — in the generalized model with heterogeneous processing requirements and for any switch *configuration*, that is, any assignment of specific processing requirements to cores. We consider, in turn, several algorithms that are either simple, look like natural candidates, or have been proven to be efficient for the case of uniform processing [1], [16].

For each algorithm, we show a fast growing lower bound that indicates that this algorithm does not really extend competitively to our generalized model with heterogeneous processing. Rather than choosing respective corner-case configurations for each considered policy, i.e., assigning required processing values to output ports in a specific maximally adversarial way for every policy, we show all lower bounds in the same configuration of the model above: the case when there are exactly k output ports, and queue Q_i contains packets with required processing in different queues is ordered in a contiguous sequence from 1 to k. Even in the contiguous case, we show that established policies do not rise up to the challenges of heterogeneous processing, showing large lower bounds for each of them.

Thus, a good policy has to account for the processing requirements explicitly; still, it remains to learn how to account for it. We first consider the BPD policy that drops packets with largest processing, but also show a fast growing lower bound for BPD. Then we proceed to the LWD policy for which we prove (in the general case, not only the contiguous case) a constant upper bound, the main result of this paper.

1) Non-Push-Out Policies: In the case of a single queue, non-push-out greedy policies perform poorly (they are k-competitive) on packets with heterogeneous processing regardless of processing order [11], [19]. In our model, we begin with two simple cases of greedy non-push-out buffer management policies with static thresholds. The first policy sets static thresholds on each queue inversely proportional to required processing, while the second simply uses identical thresholds for all queues. Interestingly, the second policy has a better competitive ratio. We denote $Z = \sum_{i=1}^{n} \frac{1}{r_i}$ (sum of inverse values of required processing for each port); also, in what follows we omit $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ for clarity, assuming that B divides all we need it to divide.

Non-Push-Out-Harmonic-Static-Threshold (NHST): during the arrival of a packet p with output port i, if $|Q_i| < \frac{B}{r_i Z}$, accept p into Q_i ; else drop p.

Theorem 1. *NHST is* (kZ + o(kZ))-competitive.

Proof: For the lower bound, consider a burst of $B \times \lfloor k \rfloor$. NHST will only accept $\frac{B}{kZ}$ of them, while OPT is free to accept all. After all packets are processed, the process repeats, getting the kZ lower bound. The matching upper bound in this case follows since all queues are isolated and process packets separately, so worst case analysis is reduced to a single queue architecture.

Non-Push-Out-Equal-Static-Threshold (NEST): during the arrival of a p with output port i, if $|Q_i| < B/n$ packets then accept p into Q_i , else drop p.

Theorem 2. NEST is (n + o(n))-competitive.

Proof: Each queue is simply a separate homogeneous (and



Fig. 2: A sample time slot of Non-Push-Out-Harmonic-Dynamic-Threshold (NHDT), Longest-Queue-Drop (LQD), Biggest-Packet-Drop (BPD), and Longest-Work-Drop (LWD) policies with maximal processing k = 3, 4 output ports (there are two different ports with the same processing requirement 2, denoted by rectangular and rhomboidal packets respectively), and a shared buffer of size B = 8.

hence optimal) queue with buffer size $\frac{B}{n}$.

Next we consider the Non-Push-Out-Harmonic-Dynamic-Threshold (NHDT) policy, previously considered in [16]. In the NHDT policy, thresholds for queues are dynamic and depend on the number of packets in the queues. The idea is that for each $1 \leq m \leq k$, m queues with most packets in them should contain at most $\frac{B}{H_k} \left(1 + \frac{1}{2} + \ldots + \frac{1}{m}\right)$ packets, where $H_k = 1 + \frac{1}{2} + \ldots + \frac{1}{k}$ is the k-th harmonic number.

Non-Push-Out-Harmonic-Dynamic-Threshold (NHDT): during the arrival of a packet p with output port i, denote by $j_1, j_2, \ldots, j_m = i$ the queues for which $|Q_j| \ge |Q_i|$; if $\sum_{s=1}^m |Q_{j_s}| < \frac{B}{H_k} \left(1 + \frac{1}{2} + \ldots + \frac{1}{m}\right)$ accept p into Q_i , else drop p.

For packets with homogeneous processing, NHDT was shown to be at most $O(\log n)$ -competitive for n output ports [16] (in our model, there are k output ports). It turns out that for heterogeneous processing requirements, NHDT is no better than NHST in the worst case.

Theorem 3. If B is asymptotically greater than k, the NHDT policy is at least $\left(\frac{1}{2}\sqrt{k \ln k} - o(\sqrt{k \ln k})\right)$ -competitive in the sequential case.

Proof: We denote $A = \frac{B}{\ln k}$. To show the lower bound, consider an input burst consisting of B(m+1) packets with processing requirements $k, k-1, \ldots, k-m$, and finally 1; here m is a number from 1 to k-1 that will be chosen below. The packets arrive in reverse order: first $B \times [k]$, then $B \times [k-1]$, and so on. As a result, NHDT accepts $A \times [k]$, $\frac{A}{2} \times [k-1]$, ..., $\frac{A}{k-m+1} \times [1]$. OPT, on the other hand, accepts only one packet for each of the queues from k to (k-m) and fills the rest of its buffer with [1]s, accepting $(B-k+m) \times [1]$. Then every i^{th} time slot, another [i] arrives for $k-m \leq i \leq k$, so OPT's queues are always busy. In $\frac{A}{k-m+1}$ rounds of processing, NHDT transmits all of its [1]s and then proceeds to process only higher queues. In $B - k + m - \frac{A}{k-m+1}$ more rounds, OPT also runs out of

1 s. By this time (B - k + m time slots in total), OPT has processed $(B-k+m)(1+H_k-H_m)$ packets, while NHDT has processed $(B-k+m)(H_k-H_m) + A/(k-m+1)$ packets; then another large burst arrives, and the process is repeated, getting the competitive ratio

$$\frac{1 + H_k - H_m}{H_k - H_m + \frac{A}{(B - k + m)(k - m + 1)}} \approx \frac{\ln k - \ln m + 1}{\ln k - \ln m + \frac{1}{(k - m)\ln k}}$$

(discarding asymptotically smaller terms and assuming that asymptotically *B* is greater than *k*). It now remains to optimize this ratio by choosing suitable *m*; setting $m = k - \sqrt{\frac{k}{\ln k}}$, since $-\ln\left(1 - \sqrt{\frac{1}{k\ln k}}\right) \approx \sqrt{\frac{1}{k\ln k}}$, we get the ratio of $\frac{-\ln\left(1 - \sqrt{\frac{1}{k\ln k}}\right) + 1}{-\ln\left(1 - \sqrt{\frac{1}{k\ln k}}\right) + \sqrt{\frac{1}{k\ln k}}} \approx \frac{1}{2}\sqrt{k\ln k} - o(\sqrt{k\ln k}).$

At this point, it is unclear how to generalize NHDT to heterogeneous processing better; this remains an interesting problem for future research.

2) Push-Out Policies: We now proceed to push-out policies, starting with the well-known Longest-Queue-Drop that overlooks required processing and only considers queue sizes.

Longest-Queue-Drop (LQD): during the arrival of a packet p with output port i, denote by $j^* = \arg \max_j \{|Q_j| + [i = j]\}$ where [i = j] = 1 if i = j and 0 otherwise (i.e., Q_{j^*} is the longest queue once we virtually add p to Q_i ; we choose one with largest required processing if there are several); then do the following: (1) if the buffer is not full, accept p into Q_i ; (2) if the buffer is full and $i \neq j^*$, push out last packet from Q_{j^*} and accept p into Q_i ; else drop p.

In case of homogeneous processing, LQD is at least $\sqrt{2}$ and at most 2-competitive [1]. For heterogeneous required processing, the situation is worse.

Theorem 4. For sufficiently large B, LQD is at least $(\sqrt{k} - o(\sqrt{k}))$ -competitive in the sequential case.

Proof: We introduce a parameter m to be defined later. Over the first burst, there arrive B packets of each of the following kinds: $\boxed{1}$, \boxed{k} , $\boxed{k-1}$, ..., $\boxed{k-m+1}$. LQD evenly distributes the packets among queues and has B/m packets in each of its nonempty queues (throughout the proof we assume that B is large and divides everything we need it to divide). OPT accepts $(B-m+1) \times \boxed{1}$ and one each in the remaining queues. Packets with required processing from k-m+1 to k keep coming so OPT always has packets in these queues to work on, but there are no more $\boxed{1}$ s. Thus, after B/m time slots LQD runs out of $\boxed{1}$'s; by this time, both LQD and OPT have processed $\frac{B}{m} + \frac{B}{m}\beta_{k,m}$ packets for $\beta_{k,m} = \frac{1}{k} + \frac{1}{k-1} + \ldots + \frac{1}{k-m+1}$, and OPT still has $(B\frac{m-1}{m} - m) \times \boxed{1}$ in the first queue. Therefore, over the next $B\frac{m-1}{m}$ steps LQD processes $B\frac{m-1}{m}\beta_{k,m}$ packets while OPT processes $B\frac{m-1}{m}(1+\beta_{k,m}) - m$ packets. The total ratio is

$$\frac{\frac{B}{m} + \frac{B}{m}\beta_{k,m} + B\frac{m-1}{m}(1+\beta_{k,m}) - m}{\frac{B}{m} + \frac{B}{m}\beta_{k,m} + B\frac{m-1}{m}\beta_{k,m}} = 1 + \frac{\frac{m-1}{m} - \frac{m}{B}}{\frac{1}{m} + (1-\frac{m}{B})\beta_{k,m}} - O(1/Bm).$$

To optimize the bound with respect to m, we choose $m = k^{\alpha}$, approximate the bound for large B as $1 + \frac{k^{\alpha} - 1}{1 + k^{\alpha} \beta_{k,m}}$, and recall that $\beta_{k,m} = \frac{1}{k} + \ldots + \frac{1}{k-m+1} = \ln \frac{k}{k-m} + o(\frac{1}{m}) = \ln \frac{k}{k-k^{\alpha}} + o(k^{-\alpha})$. For $\alpha < 1$ and large k, $\ln \frac{k}{k-k^{\alpha}} = -\ln(1-k^{\alpha-1}) = k^{\alpha-1} + o(k^{\alpha-1})$, so we can roughly estimate the bound as $1 + \frac{k^{\alpha}}{1+k^{\alpha}k^{\alpha-1}} = 1 + \frac{k^{\alpha}}{1+k^{2\alpha-1}}$, and this fraction reaches its maximum of $\sqrt{k} - o(\sqrt{k})$ for $\alpha = 1/2$.

Next we propose a policy that aims to minimize total required processing in case of congestion by pushing out packets with maximal processing requirements.

Biggest-Packet-Drop (BPD): during the arrival of a packet p with output port i, denote by Q_j the nonempty queue with largest processing requirement; then do the following: (1) if the buffer is not full, accept p into Q_i ; (2) if the buffer is full and $i \leq j$, push out last packet from Q_j and accept p into Q_i ; (3) if the buffer is full and i > j, drop p.

Theorem 5. For $B \ge \frac{k(k+1)}{2}$, BPD is at least $(\ln k + \gamma)$ -competitive in the sequential case, where $\gamma = 0.5772...$ is the Euler–Mascheroni constant.

Proof: The counterexample is the following: every time slot, there arrive $B \times [1]$, $B \times [2]$, ..., $B \times [k]$ (a full set of packets); BPD accepts only $B \times [1]$ and keeps processing only 1 packet per time slot, i.e., k! packets per k! time slots, while OPT is free to accept the packets evenly and get $k! + k!/2 + \ldots + k!/k$ packets per k! time slots, getting the bound as $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} = H_k \ge \ln k + \gamma$.

As an improvement over LQD, we propose a policy that pushes packets out of the queue with most required work; we denote the total work in queue Q_i (sum of remaining work for packets in Q_i) by W_i . **Longest-Work-Drop** (LWD): during the arrival of a packet p with output port i, let $j^* = \arg \max_j \{W_j + [i = j]r_i\}$ (i.e., W_{j^*} is maximal once we virtually add p to Q_i ; we choose maximal among those queues if there are several); then do the following: (1) if the buffer is not full, accept p into Q_i ; (2) if the buffer is full and $i \neq j^*$, push out last packet from Q_{j^*} and accept p into Q_i ; else drop p.

Theorem 6. If $k \ge 6$, LWD is at least $\left(\frac{4}{3} - \frac{6}{B}\right)$ -competitive in the sequential case.

Proof: Over the first burst, there arrive $B \times [1]$, $\frac{B}{4} \times [2]$, $\frac{B}{6} \times [3]$, and $\frac{B}{12} \times [6]$; LWD accepts $\frac{B}{2} \times [1]$ and all the other packets, while OPT accepts one of each larger packets and, correspondingly, $(B - 3) \times [1]$. Then packets with required processing 2, 3, and 6 keep arriving as needed to keep OPT's queues busy. Thus, over the first $\frac{B}{2}$ time slots both OPT and LWD process $\frac{B}{2}(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{6}) = B$ packets, but LWD has now run out of [1] packets while OPT still has $\frac{B}{2} - 3$ more, so over the next $\frac{B}{2} - 3$ time slots OPT processes $\frac{B}{2} - 3 + \left\lfloor \frac{B/2 - 3}{2} \right\rfloor + \left\lfloor \frac{B/2 - 3}{2} \right\rfloor + \left\lfloor \frac{B/2 - 3}{6} \right\rfloor \geq B - 9$ packets while LWD can only process $\left\lfloor \frac{B/2 - 3}{2} \right\rfloor + \left\lfloor \frac{B/2 - 3}{2} \right\rfloor + \left\lfloor \frac{B/2 - 3}{6} \right\rfloor = \frac{B}{2}$ packets. After that, the first burst arrives again, and the sequence repeats itself, getting the bound as $\frac{B+B-9}{B+B/2} = \frac{4}{3} - \frac{6}{B}$.

Since LWD is equivalent to LQD when packets in all queues have the same processing requirement, LWD is at least $\sqrt{2}$ competitive in this setting [1]. Note that our lower bound for LWD in the contiguous case is less than the $\sqrt{2}$ lower bound of LQD in the model with uniform processing requirements. LWD optimizes a "local" state by dropping packets from queues with maximal latency. On the other hand, OPT can leave these packets and keep these ports active when OPT knows that there will be no later arrivals to high-latency queues, whereas the corresponding ports of LWD will be idle; a similar phenomenon explains the non-optimality of LQD in the model with uniform processing. But in the contiguous case of the generalized model it is significantly harder for OPT to win much in this way since LWD will drop less packets from queues with the highest latencies (proportional to the required processing).

C. Upper Bound on the Competitiveness of LWD

Judging from the lower bounds presented in the previous section, LWD is the most promising policy among all we have considered. We now present the main result of this work: a constant upper bound on its competitive ratio. Note that although this bound is equal to the upper bound on LQD in the homogeneous processing model [1], the mapping routine is actually very different.

Theorem 7. LWD is at most 2-competitive.

Proof: The latency $\operatorname{lat}_t^{ALG}(p)$ of a packet p in a buffer is the number of time slots needed before p is transmitted (assuming it will not be pushed out). We define the latency of an already transmitted packet as -1 and the latency of a packet that has not yet arrived as ∞ . A port of ALG's buffer Consider the time t^* after transmission of a head-of-line packet p from Q_i by OPT or LWD, just before the mapping is updated.

- T0: If LWD transmits p from Q_j then its image in the OPT buffer becomes ineligible (we are to show that OPT never transmits eligible for mapping packet from Q_i if LWD does not transmit from Q_i at time t).
 - Consider the time t^* after acceptance of an *i*-packet *p* by OPT or LWD, just before the mapping is updated (if necessary).
- A0 (same queue): if p is accepted by OPT to the lth position (all ineligible packets are not accounted) of Q_i^{OPT} and there is a packet q (can be p) at the same position of Q_i^{LWD}, map p to q.
 A1 (other queue): if a packet p accepted by OPT is not mapped by A0, find any other packet q in LWD's memory that has no assignment of any OPT packet by step A1 and lat_{t*}^{OPT}(p) ≥ lat_{t*}^{LWD}(q); map p to q.
- A2 (push out): if LWD's packet p' of Q_j is pushed out by p, clear all mappings to p' and for each OPT packet q that was assigned to p' (at most one by step A0 and at most one by step A1) find a packet p'' in LWD's buffer that has no assignment by step A1 and map q to p''.
- A3 (release A1): if p is accepted by LWD and p is mapped to an OPT packet q by step A0 at time t and q was previously mapped by step A1, clear A1 mapping from q and its LWD pre-image.

Fig. 3: Mapping Routine for LWD policy.

is called *active* at time slot t if it transmits during t; otherwise, it is called *idle*. Since OPT is an offline optimal algorithm, we can assume that it never pushes out packets. To prove that LWD is 2-competitive, we will map each packet transmitted by OPT to a packet transmitted by LWD in such a way that at most two OPT packets correspond to each LWD packet. A packet p in OPT buffer mapped to an already transmitted LWD packet is called *ineligible* for mapping; else p is called *eligible*.

To avoid ambiguity during the arrival phase, a reference time t should be interpreted as the arrival of a single packet. If several packets arrive at the same time slot, we consider them independently, in the sequence in which they arrive. If several output ports are active during the same transmission phase, t should be interpreted as the processing and (if needed) transmission by a single output port in the following order: first, output ports with non-empty queues in LWD buffer, from 1 to n; then, remaining output ports with non-empty queues in OPT buffer, from 1 to n. A well-defined processing order of output ports during the transmission phase is necessary to process eligible OPT packets when mapping changes. The mapping routine is shown in Fig. 3.

Lemma 8. Let p be an eligible packet at the i^{th} -position (not counting ineligible packets) of Q_i^{OPT} at time t. If Q_i^{LWD} contains a packet q at the same position at time t, p is upped to q by step A0, and $\operatorname{lat}_{t}^{\operatorname{OPT}}(p) \geq \operatorname{lat}_{t}^{\operatorname{LWD}}(q)$. Otherwise, p is mapped to an LWD packet q' by step A1, and $\operatorname{lat}_t^{\operatorname{OPT}}(p) \geq$ $\operatorname{lat}_{t}^{\operatorname{LWD}}(q')$. Moreover, at most one OPT packet is mapped to an LWD packet by each of the steps A0 and A1, and all OPT packets are mapped.

Proof: We prove the lemma by induction on the number of mapping changes. Since all ports process packets at the same rate (one processing cycle per time slot for each head-of-line packet), the mapping may change only when algorithms accept or transmit packets. For the base case, consider the first packet p accepted by OPT, say to Q_j^{OPT} . If $|Q_j^{\text{LWD}}| > 0$, there is a head-of-line packet $q \in Q_j^{\text{LWD}}$ (maybe q = p) that could be accepted before p, and therefore may be partially processed, so p is mapped to q by step A0 and the lemma holds. If $|Q_i^{\text{LWD}}| =$ 0, p is not accepted by LWD, so LWD's buffer is full, and LWD's buffer must contain a packet q that satisfies the latency

constraint; thus, we map p to q by step A1, and the lemma holds again.

Assume by induction that the lemma holds for any t' < t. We are to show that it holds after the t^{th} mapping change. Let t^{-} be the time just before arrival or processing at timeslot t. Since packets cannot be accepted or transmitted during $(t-1,t^{-}]$, the induction hypothesis holds at time t^- . The following two cases are possible for a packet transmitted at time t by OPT

or LWD from Q_j . (1) Neither Q_j^{LWD} nor Q_j^{OPT} is empty at time t^- . Let pbe the first eligible packet in Q_j^{OPT} (if it exists) and q be the first packet in Q_j^{OPT} . If p does not exist then during tthe latency of a packet in Q_j^{LWD} may only decrease, and the induction hypothesis holds at the end of t. Otherwise, since the induction hypothesis holds at time t^- , p is mapped to q at time t^- by step A0 and $\operatorname{lat}_{t^-}^{\operatorname{OPT}}(p) \ge \operatorname{lat}_{t^-}^{\operatorname{LWD}}(q)$, so either both algorithms or LWD only may transmit at time t. In both cases, mapping for eligible packets does not change at the end of t. Since output ports are served in the same order, and each non-empty output port reduces required processing by a single cycle, after the t^{th} mapping change the latency constraint holds for all packets mapped with A0 and A1. Moreover, since there were no new arrivals all OPT packets are mapped at the end of t.

(2) OPT transmits an eligible packet p from Q_i^{OPT} but Q_i^{LWD} is empty at time t^- . Since at time t^- all OPT packets are mapped and Q_i^{LWD} is empty, it must be the case that p is mapped to an LWD packet q by step A1. Since output ports are processed in the same order, p cannot be eligible at time t^- , so this case is impossible.

The following two cases are possible when a packet is accepted at time t by OPT or LWD to Q_j .

(3) A packet is accepted at time t by OPT to the l^{th} position (ineligible packets are not counted) of Q_j^{OPT} , and there are at most $|Q_j^{\text{LWD}}|$ eligible packets in Q_j^{OPT} . So the l^{th} LWD packet q in Q_j^{LWD} is available for mapping by step A0. Moreover, since the latency constraint holds for each of the first i-1eligible OPT packets and all packets admitted to Q_j require the same processing, $\operatorname{lat}_t^{\operatorname{OPT}}(p) \ge \operatorname{lat}_t^{\operatorname{LWD}}(q)$. If p is accepted by LWD and pushes out another packet p' from Q_n^{LWD} then we need to find LWD's packets available for mapping by step

A1 for the pre-image of p' in OPT buffer. Since the induction hypothesis holds at time t^- , at most one packet is mapped to p' by each of the steps A0 and A1. Also, since p' is pushed out by LWD, its latency is highest in LWD's buffer, so any LWD packet (including p) that has no assignment by step A1 is available for mapping. Since LWD pushes out at time t, LWD's buffer was full at time t^- , so (since buffer sizes are equal) there are enough packets available to map the pre-image of p' by step A1.

(4) A packet q is accepted at time t by LWD to the l^{th} position, and there are more than $|Q_j^{\text{LWD}}|$ eligible packets in Q_j^{OPT} . Since latency constraints hold for the first l-1 eligible OPT packets mapped by step A0 and only j-packets are admitted to Q_j , the latency constraint holds for the l^{th} eligible OPT packet and the l^{th} LWD packet in Q_j . By step A3, the A1 mapping is cleared from the image of the l^{th} eligible OPT packet. Therefore, if a packet q is accepted by OPT and even if LWD's buffer is not full q can be mapped to p'' by step A1. If q is accepted by LWD and pushes out another packet p', LWD's buffer is already full at time t^- . Similar to (3), LWD has enough packets to map the image of p' by step A1.

Thus, the induction hypothesis holds at the end of *t*. ■ By Lemma 8, each packet transmitted by LWD is the image of at most two OPT packets (one by step A0 and another by step A1), and at any time all OPT packets are mapped. Theorem 7 immediately follows.

IV. PACKET SCHEDULING WITH HETEROGENEOUS VALUES

We now move to the packet scheduling problem with heterogeneous values in a shared memory switch architecture. In this model, each unit-sized packet with required work 1 has two parameters: an output port label and an intrinsic value. The objective function is to maximize the total transmitted value. It has been formulated as an open problem in SIGACT News [10, p. 22]: "What happens in the shared memory, multiple output queue model with general-valued packets? Is constant competitiveness achievable?" In this section we provide the first results in this direction. Note that here we consider a more general model and allow packets with heterogeneous values sojourn in the same queue. As result, in all policies below, we use the most favourable processing order in each queue corresponding to an output port: we assume they are priority queues (PQ) where most valuable packets are processed first. This can only be an improvement over FIFO processing or any other order since priority queue is optimal in the case of a single queue.

A. Model Description

We consider an $l \times n$ switch with shared memory of size B; l and n represent the number of input and output ports, respectively. The i^{th} output port has one corresponding output queue Q_i ; we assume that $B \ge n$. Packets arrive at input ports. Each packet is labeled with the destination output port and intrinsic value of at most $k, B \ge k \ge 1$. Note that unlike the previous model, each packet requires only a single processing cycle. In what follows, we denote by v a packet with value v;

by $x \times v$, a burst of x packets with value v each. All admitted packets in an output queue are ordered in non-increasing order of values; the number of packets in Q_i is denoted by $|Q_i|$. Time is slotted; we divide each time slot into two phases (see Fig. 4). During the (1) arrival phase a burst of new packets arrives at each input port, and the buffer management policy decides which packets should be admitted; we assume that during the arrival phase input ports are served in a fixed order from input port 1 to input port n, with no restriction on the burst size. During the (2) transmission phase, the head-of-line packet in each non-empty output queue is transmitted out. An accepted packet can again later be dropped from the buffer when another packet is accepted instead (push-out policy). Note that for unit values this reduces to the model of [1], so the 4/3 lower bound on any online policy shown there holds in our case too.

B. Choosing the Right Properties for Buffer Management

Even if processing order is fixed, it is unclear which traffic should be admitted; there may be a tradeoff between traffic that opens new active ports and traffic that increases the total admitted value. We do not consider non-push-out policies since it is straightforward that a greedy non-push-out policy that accepts a packet if there is available space is at least kcompetitive (fill the buffer with 1 s, then send in the k s). We define several push-out scheduling policies that greedily optimize the number of active ports, total admitted value, or a combination of both. The algorithms are illustrated on Fig. 4.

First, we consider the Longest-Queue-Drop (LQD) heuristic again; in this model, LQD drops the last (lowest value) packet from the longest queue when a buffer is congested, balancing the queue sizes. However, in the value-based model it again fails to reach constant competitiveness. Note that constructions in Theorems 9, 10, and 11 below all fall into an interesting special case of the general model, when a packet's value is uniquely defined by its output port label. This special case makes practical sense: often a different subset of cores is assigned to process specific types of traffic, and these types can be defined either by required processing, as in Section III, or by value, as in this special case. Obviously, the same bounds apply in the general case as well.

Theorem 9. The competitive ratio of LQD is at least $\left(\sqrt[3]{k} - o\left(\sqrt[3]{k}\right)\right)$.

Proof: In this construction, a packet's value is equal to its output port label. Fix $a, 1 \leq a \leq k$. On the first time slot, there arrive *B* packets of every value from 1 to *a* and *B* more packets of value *k*. LQD balances the queues, leaving $\frac{B}{a}$ in each of them (throughout the paper, we assume that *B* divides everything we need it to divide). At the same time, OPT takes in *B* packets of value *k*. Then, on each time slot packets with values from 1 to *a* arrive but packets of value *k* do not. Thus, over the next *B* time slots OPT transmits B(a+1)packets with total value $B(\frac{1}{2}a(a-1)+k)$ while LQD transmits $B(a+\frac{1}{a})$ packets with total value $B(\frac{1}{2}a(a-1)+k/a)$. Then the initial burst arrives again, and the construction repeats itself,



Fig. 4: A sample time slot of Longest-Queue-Drop (LQD), Minimal-Value-Drop (MVD), and Maximal-Ratio-Drop (MRD) with maximal value k = 4, 4 output ports, and a shared buffer of size B = 8.

getting a competitive ratio of $\frac{\frac{1}{2}a(a-1)+k}{\frac{1}{2}a(a-1)+k/a}$. This expression is maximized for $a \approx \sqrt[3]{k}$, with the resulting ratio of $\sqrt[3]{k}-o(\sqrt[3]{k})$.

The next policy greedily maximizes total admitted value by pushing out packets with minimal current value.

Minimal-Value-Drop (MVD): during the arrival of an \boxed{m} -packet p that is destined to the *i*-th port, denote by Q_j the nonempty queue that contains a packet with minimal value (if there are several such queues, choose the longest among them); then do the following: (1) if the buffer is not full, accept p into Q_i ; (2) if the buffer is full and the minimal admitted value is less than \boxed{m} , push out last packet from Q_j and accept p into Q_i ; (3) if the buffer is full and the minimal admitted value is at most \boxed{m} , drop p.

The MVD algorithm prioritizes packets of maximal value. Unfortunately, MVD has a linear lower bound on competitiveness in the worst case.

Theorem 10. MVD is at least $\frac{m-1}{2}$ -competitive for $m = \min\{k, B\}$.

Proof: Again, each packet's value equals its output port label. On each time slot, there arrive *B* packets of every value from 1 to *m*. As a result, MVD only processes packets of value *m* while the optimal algorithm processes each value from 1 to *m*, getting a competitive ratio of $\frac{m(m-1)/2}{m}$.

Note that a corresponding policy to MVD in the model with heterogeneous processing requirements is BPD that minimizes a total required processing. But in difference from MVD, BPD can achieve a relatively good competitive ratio that demonstrate a significant difference between required processing requirements and packet's values characteristic.

As a policy that we conjecture may reach constant competitive ratio, we propose a combination of these two characteristics, an idea similar to the LWD policy in the previous model that we have shown to be 2-competitive in Section III-C.

Maximal-Ratio-Drop (MRD): during the arrival of an m_{-} packet p that is destined to the *i*-th port, denote by Q_j the nonempty queue with a maximal value of $\frac{|Q_j|}{a_j}$, where a_j is an average value in Q_j (if there are several such queues, choose as Q_j a queue that contains a packet with a smaller value) then do the following: (1) if the buffer is not full, accept p into Q_i ;

(2) if the buffer is full and a minimal admitted value is less than m, push out last packet from Q_j and accept p into Q_i;
(3) if the buffer is full and a minimal admitted value in the buffer is bigger than m, drop p.

Note that MRD emulates LQD in case all packets have unit values, and the lower bound $\sqrt{2}$ shown in [1] applies. We can also show a constant lower bound for MRD in the special case when each packet's value is equal to its output port label.

Theorem 11. MRD is at least $\frac{4}{3}$ -competitive in case each packet's value is equal to its output port label.

Proof: Consider the first burst with B packets of value 1, 2, 3, and 6 each. Balancing the size-value ratio, MRD will accept $\frac{B}{2} \times [6]$, $\frac{B}{4} \times [3]$, $\frac{B}{6} \times [2]$, and $\frac{B}{12} \times [1]$, while OPT accepts $(B-3) \times [6]$ and one packet of each other value. Then packets of value 1, 2, and 3 keep coming so that OPT always has something to do but packets of value 6 do not arrive any more. Thus, in B-3 steps OPT will have transmitted for a total value 12(B-3), while MRD will have total value $12\frac{B}{2} + 6(\frac{B}{2} - 3) = 9B - 18$, getting the bound.

It remains an interesting open problem to show whether MRD has a constant competitive ratio in the worst case.

V. SIMULATIONS

A. Experimental Setting

In this section, we present the results of a simulation study performed to validate our theoretical results for both models. To the best of our knowledge, publicly available traffic traces (e.g., CAIDA [9]) do not contain information on processing requirements of packets, and such requirements are difficult to extract since they depend on specific hardware and network processor configuration. Another handicap of such traces is that they provide no information about time scale, namely how long a timeslot should last. This information is essential in our model in order to determine both the number of processing cycles per timeslot and traffic burstiness. We therefore perform three series of experiments on synthetic traces, studying the dependence of the competitive ratio on the maximal packet size (i.e., the number of queues) k, the buffer size B, and the number of processing cores for each queue (speedup) C.



Fig. 5: Simulation results: competitive ratio in the required processing model as a function of (1) k, (2) B, (3) C; in the value model with uniform distributions of output port and value as a function of (4) k, (5) B, (6) C; in the value model with values uniquely assigned to ports as a function of (7) k, (8) B, (9) C. Specific simulation parameters are shown in graph captions.

Since it is computationally prohibitive to compute the true optimal policy, we used a single priority queue that first processes the smallest packets (resp., packets with largest value) and has kC cores. This algorithm has been proven optimal in the single queue model, so in case of congestion it may perform even better than optimal in our model, as our experiments show. The traffic is generated as the interleaving of 500 independent sources. Each source is an on-off bursty process modeled by a Markov-modulated Poisson process (MMPP); it has packet rate λ_{on} in the "on" state and does not emit packets in the "off" state. We ran all experiments for $2 \cdot 10^6$ time slots with periodic "flushouts".

B. Results in the Heterogeneous Processing Model

Experimental results are shown on Fig. 5. The top row shows results in the model with unit packet value and variable processing requirements. In the first set of experiments, we study competitive ratio as a function of the maximal packet size k. The performance of all algorithms decreases as k grows, but non-preemptive algorithms clearly deteriorate faster. BPD turns out to be a very poor heuristic: since it pushes out large packets independently of queue size it consistently underuses available cores and thus loses much ground. To avoid artificially reducing the number of active ports, we introduce the BPD₁

algorithm that works as BPD but does not push out the last packet in a queue; BPD_1 does better but remains a poor fit.

In the second set of experiments, we study how the competitive ratio changes as a function of the buffer size B, progressing from small to very large buffer sizes to show the transition into a situation with nearly no congestion. Here, non-preemptive algorithms become worse at first (OPT can make better use of a larger buffer) but then come back when OPT stops improving. Preemptive algorithms do better than non-preemptive ones, with BPD and BPD₁ outperforming non-preemptive algorithms as congestion reduces, and LWD retains best throughout.

In the third set of experiments, we look at competitive ratio as a function of the speedup C imposed on each core. Again, preemptive algorithms pick up on this advantage quicker than non-preemptive ones, and again, LWD is the best algorithm.

C. Results in the Heterogeneous Value Model

The bottom two rows of Fig. 5 show results in the model with variable packet values and unit required processing. The middle row (graphs 4-6) presents the results with both output port and value chosen uniformly at random, so the distribution of values in each queue is also uniform. One difference with Section V-B is that now growing k indicates less congestion and more throughput, and graph (4) shows that at first the

optimal algorithm can make better use of it, but then congestion reduces, and suboptimal algorithms catch up; the same is true about graph (6) that shows that a single priority queue can make better use of speedup until congestion begins to resolve. Similar to Section V-B, we see that the MRD algorithm outperforms all other algorithms, but the difference with LQD is rather small. Both MVD and MVD₁ (that does not push out single packets in a queue) trail relatively far behind.

One important special case is when the value uniquely corresponds to the output port; note that all lower bounds in Section IV actually deal with this special case. Non-preemptive algorithms in this case translate without change, except for NHST where it is now more desirable to have high-value packets, so we reverse the thresholds to $\frac{B}{(k-i+1)H_k}$ for queue with value *i*. In this special case, MRD performs noticeably better than LQD; in general, our experiments suggest that MRD is never explicitly worse than LQD, and its advantage grows for distributions that prioritize certain values at specific queues. Again, preemptive algorithms outperform non-preemptive ones, with the exception of MVD, even in its enhanced MVD₁ version. One interesting case appears on the graph (6): as speedup grows, MVD begins to outperform both LQD and MRD. This is caused by situations when a burst can be processed almost entirely in a single time slot (due to large speedup) but cannot fit in the buffer size (due to high intensity λ); in this case MVD is indeed the best policy.

VI. CONCLUSION

Over the recent years, there has been a growing interest in understanding the impact of buffer architecture on network performance. In this work, we study the tradeoffs inevitable on the path to a "perfect" policy in a shared memory switch, both analytically and with simulations. Recent research advocates the use of smaller buffers in routers, aiming to reduce queueing delay in the presence of (mostly) TCP traffic; however, it sidesteps the issue that as buffers get smaller, the effect of processing delay becomes much more pronounced. The majority of currently deployed admission control policies do not take into account (at least explicitly) the importance of heterogeneous packet processing. In the first part, we study the impact of heterogeneous processing on throughput in the shared memory switch architecture. We demonstrate that policies such as LQD or NHDT - very attractive under uniform processing requirements - perform poorly in the worst case. We believe that this observation will provide new insights to the practice of admission control policies. In addition, we consider an open problem that optimizes transmission of unit-sized packets with heterogeneous values in a shared memory switch architecture. We provide the first results in this direction and define the most promising direction for further studies.

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