

Professor Nikolai Shanin and Constructive Mathematics (abstract)

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Professor Shanin's approach to constructive mathematics builds on earlier work in the areas of intuitionism and recursive analysis, and it supplements them with a number of original philosophical and mathematical ideas. His research program and his enthusiasm have inspired many students and younger colleagues, including this author. Our papers on constructive analysis were a direct continuation of Shanin's work. We routinely referred the reader to Shanin's papers for basic definitions, notation and results. We worked hard to imitate our teacher's style of presentation, even in minute details.

It is interesting that Shanin's own work does not show the same respectful consistency with his past publications that is found in papers by his students. More than once, logical and philosophical analysis led him to drastic changes in his view of constructive mathematics. For instance, Shanin's 1973 paper *On the hierarchy of methods of understanding judgements in constructive mathematics* explains the ramifications of basic constructivist principles in a way that is very different from the one found in his 1958 paper *On constructive understanding of mathematical judgements*. Whenever Professor Shanin modified his approach to constructivism, his students had to decide which Shanin they would rather follow.

Shanin's attitude towards the development of constructive mathematics emphasizes its positive aspects—clarifying the constructive kernel of mathematical theories—rather than counterexamples. Young and technically skillful mathematicians were naturally attracted by the task of inventing tricky

recursion-theoretic constructions that would allow them to say about some famous old theorem: We proved that, in constructive mathematics, this theorem is incorrect! Our teacher whom we tried to emulate gave us an example of a healthier and more mature sense of values in logical and mathematical research.

The work on constructive mathematics done by Professor Shanin's school has not led to a revolution in which the mathematical community would agree to treat nonconstructive proofs as incorrect, but it has provided a wealth of valuable information about the constructive and nonconstructive elements of mathematical practice.