

Primal implication as encryption

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2014

Propositional Infon Logic (Y. Gurevich, I. Neeman, 2008)
Distributed-Knowledge Authorization Language DKAL

- Infon — a message as a piece of information.
- $\Gamma \vdash \varphi$ — “the principal can get (by herself, without any communication) the information φ provided she already has all infons $\psi \in \Gamma$ ”.

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General Infon Logic = intuitionistic propositional logic +
 quotation modalities $A_said()$, $B_said()$, ... (PSPACE)

Primal Infon Logic = its efficient fragment. (Linear TIME)

Primal implication \rightarrow_p

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$\Gamma, \varphi \vdash \varphi$, (but $\not\vdash \varphi \rightarrow_p \varphi$)

$$\frac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \rightarrow_p \psi} (\rightarrow_p I) \quad , \quad \frac{\Gamma \vdash \varphi \quad \Gamma \vdash \varphi \rightarrow_p \psi}{\Gamma \vdash \psi} (\rightarrow_p E).$$

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We propose a “**cryptographic**” interpretation:

- $\varphi \rightarrow_p \psi$ — “an infon, containing the information ψ encrypted by a symmetric key (generated from) φ ”.

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We propose a “**cryptographic**” interpretation:

- $\varphi \rightarrow_p \psi$ — “an infon, containing the information ψ encrypted by a symmetric key (generated from) φ ”.
- $(\rightarrow_p I)$ allows to encrypt any available message by any key.
- $(\rightarrow_p E)$ allows to extract the information from a ciphertext provided the key is also available.

Primal Infon Logic incorporated into communication protocols

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Example

Alice and Bob live in different places and communicate via a telephone line or by e-mail. They wish to play the following game distantly. Each of them picks a bit, randomly or somehow else. If the bits coincide then Alice wins; otherwise Bob wins. Both of them decide to play fair but don't believe in the fairness of the opponent. So they use cryptography.

To play fair means that they honestly declare their choice of a bit, independently of what the other player said.

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Policy: Alice

$\Gamma : A_said m_a, A_said k_a,$

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$A_said m_a \quad A_IsTrustedOn m_a$

$$\frac{m_a}{k_a \rightarrow_p m_a}$$

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$\Gamma := \Gamma, B_said(k_b \rightarrow_p m_b);$

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Policy: Bob

$\Gamma : B_said\ m_b, B_said\ k_b,$
 $B_IsTrustedOn\ m_b,$
 $B_IsTrustedOn\ k_b.$

$\Gamma \vdash k_b \rightarrow_p m_b; \text{SEND } k_b \rightarrow_p m_b.$

when gets $k_b \rightarrow_p m_b$:

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DKAL: $(\text{IF } p \text{ THEN } q) := \{q\}_{\{p\}} := p \rightarrow_p q$

Policy: Alice

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IF  $\{m_a\}_{\{k_a\}}$  THEN SEND  $\{m_a\}_{\{k_a\}}$ ;  
IF  $k_a \wedge \text{B\_said } \{m_b\}_{\{k_b\}}$  THEN SEND  $k_a$ ;  
IF  $\text{A\_said } m_a \wedge \text{B\_said } m_b$  THEN STOP.
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Translation:

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 $(k_a \rightarrow_p m_a) \rightarrow_p \text{SEND}(k_a \rightarrow_p m_a)$ 
 $k_a \wedge \text{B\_said}(k_b \rightarrow_p m_b) \rightarrow_p \text{SEND } k_a$ 
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(What is stored in the memory cells and sent?)*

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Example

```
cp := CodePage(hash( $\varphi$ ))  
 $\varphi \rightarrow_p \psi := \text{convert } \psi \text{ to } cp$ 
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The “cryptographic” semantics gives some answer.

- In what follows we do not insist that the encryption is strong in some sense. One may assume that **the privacy is protected by the interface**: an agent simply has no tools that make the decryption of a ciphertext without key possible.
- We consider the **purely propositional** language and leave the modalities for the future.

P — the $\{\top, \wedge, \rightarrow_p\}$ -fragment.

$$\overline{\top} \quad \frac{\varphi_1 \quad \varphi_2}{\varphi_1 \wedge \varphi_2} \quad \frac{\varphi_1 \wedge \varphi_2}{\varphi_i} \quad \frac{\varphi_2}{\varphi_1 \rightarrow_p \varphi_2} \quad \frac{\varphi_1 \quad \varphi_1 \rightarrow_p \varphi_2}{\varphi_2}$$

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Theorem (L. Beklemishev, Y. Gurevich, 2012)

P is sound and complete w.r.t. quasi-boolean semantics.

\models is a quasi-boolean model iff

- $\models \top$,
- $\models \varphi_1 \wedge \varphi_2 \Leftrightarrow \models \varphi_1$ and $\models \varphi_2$,
- $\models \varphi_2 \Rightarrow \models \varphi_1 \rightarrow_p \varphi_2$,
- $\models \varphi_1 \rightarrow_p \varphi_2 \Rightarrow \not\models \varphi_1$ or $\models \varphi_2$.

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But it is not what we need.

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- $E \subset \Sigma^*$, $E \neq \emptyset$ — the information known by everyone.

Definition

A set $M \subseteq \Sigma^*$ is *closed* if $E \subseteq M$ and M satisfies the closure conditions:

- $a, b \in M \Leftrightarrow \pi(a, b) \in M$,
- $a \in \Sigma^*, b \in M \Rightarrow \text{enc}(a, b) \in M$.
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She has access to the encryption tool enc , so she can convert a plaintext into a ciphertext. The backward conversion (by dec) is also available provided she has the encryption key.

Definition

A *model* is a triple $\langle \mathcal{A}, M, v \rangle$ where \mathcal{A} is an infon algebra, $M \subseteq \Sigma^*$ is a closed set and $v: Fm \rightarrow \Sigma^*$ is an evaluation,

- $v(\top) \in E$,
- $v(\varphi_1 \wedge \varphi_2) = \pi(v(\varphi_1), v(\varphi_2))$,
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Theorem (Soundness and Completeness)

$\Gamma \vdash \varphi$ in **P** iff $v(\varphi) \in M$ for every model $\langle \mathcal{A}, M, v \rangle$ with $v(\Gamma) \subseteq M$.

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Theorem (Uniform model)

There exists an interpretation $\langle \mathcal{A}, v \rangle$ with the following property: for any context Γ there exists a model $\langle \mathcal{A}, M, v \rangle$ with $v(\Gamma) \subseteq M$, such that $\Gamma \not\vdash \varphi$ implies $v(\varphi) \notin M$ for all infons φ .

Constant \perp and backdoors

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$P[\perp]$:

$$\frac{\perp}{\varphi} (\perp E)$$

\perp as **superuser permissions**,
makes communications and all
other tools useless for the
owner.

$P[\perp_w]$:

$$\frac{\perp \quad \varphi \rightarrow_p \psi}{\psi} (\perp_w E)$$

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$$crack(\mathbf{f}, enc(a, b)) = b$$

Theorem

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Theorem

" $\Gamma \vdash \varphi$ in $\mathbf{P}[\perp_w]$ " is linear time decidable.

- if $\Gamma \vdash \varphi$ in \mathbf{P} , return “yes”;
- else if $\Gamma \not\vdash \perp$ in \mathbf{P} , return “no”;
- else return $At^+(\varphi) \subseteq At^+(\Gamma)$.

where $At^+(\varphi)$ is the set of all atoms that occur “positive” in φ ;
 $At^+(\varphi \rightarrow_p \psi) = At^+(\psi)$.

Primal disjunction \vee_p

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$\mathbf{P}[\vee_p]$ is the purely propositional part of **PPIL** (the recent stable formulation of the primal infon logic, C. Cotrini, Y. Gurevish, 2012)

$$\frac{\varphi}{\varphi \vee_p \psi} \quad \frac{\psi}{\varphi \vee_p \psi} \quad (\text{no elimination rules for } \vee_p)$$

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$(\varphi_1 \rightarrow_p \psi) \wedge (\varphi_2 \rightarrow_p \psi)$ makes the same in **P**, but here ψ is repeated twice.

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- $(\varphi \rightarrow_p \psi)^* = (\perp \vee_p \varphi^*) \rightarrow_p \psi^*$.

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The translation $\varphi \mapsto \varphi^*$ is a linear time reduction.

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Theorem

$\Gamma \vdash \varphi$ in $\mathbf{P}[\perp_w]$ *iff* $\Gamma^* \vdash \varphi^*$ in $\mathbf{P}[\vee_p]$.

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Theorem

$\Gamma \vdash \varphi$ in $\mathbf{P}[\perp_w]$ iff $\Gamma^* \vdash \varphi^*$ in $\mathbf{P}[\vee_p]$.

It is also possible to reduce $\mathbf{P}[\perp_w]$ to \mathbf{P} , but it requires exponential space and time:

- 1 $\varphi \mapsto \varphi^*$;
- 2 replace $(\perp \vee_p \psi) \rightarrow_p \eta$ with $(\perp \rightarrow_p \eta) \wedge (\psi \rightarrow_p \eta)$.