Processing Succinct Matrices and Vectors

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Definition MTDD (Fujita, McGeer, Yang 1997)

An MTDD is a triple $\mathbb{A} = (N, P, S)$ with N a finite set of variables, which is partitioned into levels $N_0, N_1, \ldots, N_h = \{S\}$ ($S =$ start variable).

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P contains for every $A \in N_i$ exactly one rule of the following form:

- $A \to \begin{pmatrix} B & C \\ D & E \end{pmatrix}$ with $B, C, D, E \in N_{i-1}$ (if $1 \leq i \leq h$)
- $A \to a$ with $a \in S$ (if $i = 0$)

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The height of $\mathbb{A}$ is h, and its size if $|N|$.

Multi-Terminal Decision Diagrams

Example: Hadamard matrix H_n

$$H_0 \to 1 \qquad H_0' \to -1$$

$$H_i \to \begin{pmatrix} H_{i-1} & H_{i-1} \\ H_{i-1} & H_{i-1}' \end{pmatrix} \qquad H_i' \to \begin{pmatrix} H_{i-1}' & H_{i-1}' \\ H_{i-1}' & H_{i-1} \end{pmatrix} \qquad (1 \le i \le n)$$

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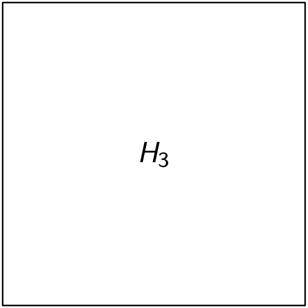

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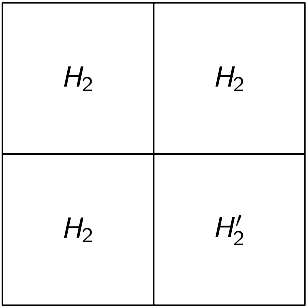

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H_1	H_1	H_1	H_1
H_1	H'_1	H_1	H'_1
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H_0	H_0	H_0	H_0	H_0	H_0	H_0	H_0
H_0	H_0'	H_0	H_0'	H_0	H_0'	H_0	H_0'
H_0	H_0	H_0'	H_0'	H_0	H_0	H_0'	H_0'
H_0	H_0'	H_0'	H_0	H_0	H_0'	H_0'	H_0
H_0	H_0	H_0	H_0	H_0'	H_0'	H_0'	H_0'
H_0	H_0'	H_0	H_0'	H_0'	H_0	H_0'	H_0
H_0	H_0	H_0'	H_0'	H_0'	H_0'	H_0	H_0
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1	1	1	1	1	1	1	1
1	−1	1	−1	1	−1	1	−1
1	1	−1	−1	1	1	−1	−1
1	−1	−1	1	1	−1	−1	1
1	1	1	1	−1	−1	−1	−1
1	−1	1	−1	−1	1	−1	1
1	1	−1	−1	−1	−1	1	1
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Matrix Multiplication

Observation

For any semiring with at least two elements there exist MTDDs $\mathbb{A}_n$ and $\mathbb{B}_n$ with:

- $\mathbb{A}_n$ and $\mathbb{B}_n$ have size $O(n)$.
- $\mathbb{A}_n$ and $\mathbb{B}_n$ have heigth n.
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Proof: (for the semiring $(\mathbb{N}, +, \cdot)$)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{pmatrix}$$

Multi-Terminal Decision Diagrams with Addition

Definition $MTDD_+$

An $MTDD_+$ is defined as an MTDD but in addition may contain variables, whose associated rules have the form

$$A \to B + C \quad \text{(matrix addition)}$$

Here A, B, C belong to the same level (and hence produce matrices of the same dimension).

The addition rules must be acyclic.

Multi-Terminal Decision Diagrams with Addition

Example

$$B_0 \to 1, \quad B_j \to \begin{pmatrix} B_{j-1} + B_{j-1} & B_{j-1} + B_{j-1} \\ B_{j-1} + B_{j-1} & B_{j-1} + B_{j-1} \end{pmatrix} \quad (1 \leq j \leq n)$$

$$A_0 \to 1, \quad A_j \to \begin{pmatrix} A_{j-1} & A_{j-1} \\ A_{j-1} + B_{j-1} & A_{j-1} + B_{j-1} \end{pmatrix} \quad (1 \leq j \leq n).$$

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A_j derives to the $(2^j \times 2^j)$-matrix

$$\begin{pmatrix} 1 & 1 & \dots & 1 & 1 \\ 2 & 2 & \dots & 2 & 2 \\ 3 & 3 & \dots & 3 & 3 \\ & & \vdots & & \\ 2^j & 2^j & \dots & 2^j & 2^j \end{pmatrix}$$

Matrix multiplication for MTDD_+

Proposition

For given MTDD_+ $\mathbb{A}$ and $\mathbb{B}$ of the same height one can compute in time $O(|\mathbb{A}| \cdot |\mathbb{B}|)$ an MTDD_+ $\mathbb{P}$ of size $O(|\mathbb{A}| \cdot |\mathbb{B}|)$ with

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$$A \cdot B \to \begin{pmatrix} A_{1,1}B_{1,1} + A_{1,2}B_{2,1} & A_{1,1}B_{1,2} + A_{1,2}B_{2,2} \\ A_{2,1}B_{1,1} + A_{2,2}B_{2,1} & A_{2,1}B_{1,2} + A_{2,2}B_{2,2} \end{pmatrix}.$$

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By eliminating linearly dependent equations we can bound the number of equations.

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  This allows to mimic Toda's proof for the fact that computing the determinant and matrix powering for explicit matrices is #**L**-complete.

Future work

- Compression of explicitly given matrices

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- Parallel algorithms