Two-sided contexts and how to parse them

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Overview

Introduction to two-sided context

Previous models of context

Grammars with two-sided context

Parsing two-sided context

Barash and Okhotin's algorithms

New algorithm

Open questions

Context in context

- Context-free grammars to model syntax of languages
- Are these sufficient?
  - No: cross-serial dependencies (e.g. Swiss German):

    Jan säit das mer d'chind em Hans es huus lönd hälfe aastriiche.

    John said that we let the children help Hans paint the house.
  - Also insufficient for programming languages

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Context in context

- First attempt to model context: context-sensitive grammars
  - Too powerful (= NLINSPACE)
  - Lack phrase structure
- Mildly context-sensitive grammar classes
- Regulated rewriting approach

A new approach

Barash and Okhotin's grammars with {one, two}-sided contexts model context directly.

$$ABC \to A\beta C$$

$$\rightsquigarrow$$

$$B \to \lhd A \;\&\; \beta \;\&\; \rhd C$$

Adding conjunction to context-free grammars

Context-free:

$$A \to BC$$

If B matches u and C matches v then A matches uv

Conjunction:

$$A \to B \;\&\; C$$

If B matches u *and* C matches u then A matches u

Or is implicit (or written $|$)

Adding context to conjunctive grammars

Context:

$$A \to \lhd B$$

If v is preceded by u, and u matches B, then v matches A

?

Strings in context

A string-in-context is a triple of strings, written $u\langle v \rangle w$.

Intuitively, v seen as a substring of uvw.

$$\frac{\text{he}\langle\text{llo }\rangle\text{world} \quad \text{hello }\langle\text{wor}\rangle\text{ld}}{\text{he}\langle\text{llo wor}\rangle\text{ld}}$$

but

$$\frac{\text{he}\langle\text{llo }\rangle\text{world} \quad \text{hello }\langle\text{ear}\rangle\text{th}}{\text{undefined}}$$

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We can improve the previous description of $\lhd$:

$$A \to \lhd B$$

If u matches B, then $u\langle v\rangle w$ matches A

or better
If $\varepsilon\langle u\rangle vw$ matches B, then $u\langle v\rangle w$ matches A

We also have $\unlhd$, $\rhd$ and $\unrhd$

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$$
\begin{aligned}
&\vdash_G [\varepsilon, u\langle\varepsilon\rangle w] && \\
&\vdash_G [a, u\langle a\rangle w] && a \in \Sigma \\
[\alpha, u\langle v\rangle w] &\vdash_G [A, u\langle v\rangle w] && A \to \alpha \in R \\
[\alpha, u\langle v_1\rangle v_2 w], [\beta, u v_1\langle v_2\rangle w] &\vdash_G [\alpha\beta, u\langle v_1 v_2\rangle w] && \\
[\alpha, u\langle v\rangle w], [\beta, u\langle v\rangle w] &\vdash_G [\alpha \mathbin{\&} \beta, u\langle v\rangle w] && \\
[\alpha, \varepsilon\langle u\rangle vw] &\vdash_G [\lhd\alpha, u\langle v\rangle w] && \\
[\alpha, \varepsilon\langle uv\rangle w] &\vdash_G [\trianglelefteq\alpha, u\langle v\rangle w] && \\
[\alpha, uv\langle w\rangle\varepsilon] &\vdash_G [\rhd\alpha, u\langle v\rangle w] && \\
[\alpha, u\langle vw\rangle\varepsilon] &\vdash_G [\trianglerighteq\alpha, u\langle v\rangle w] &&
\end{aligned}
$$

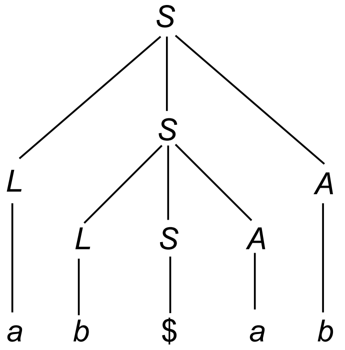
$$L_G(\alpha) = \{u\langle v\rangle w : u, v, w \in \Sigma^* \text{ and } \vdash_G [\alpha, u\langle v\rangle w]\}$$

$$L(G) = \{v \in \Sigma^* : \varepsilon\langle v\rangle\varepsilon \in L_G(S)\}$$

Grammar for
$\{x\$x : x \in \{a, b\}^*\}$:
$G = (\{a, b, \$\},$
$\{S, J, A, L, C_a, C_b\}, R, S)$ where
R contains

$$\begin{aligned} S &\to \$ \\ S &\to LSA \\ J &\to \varepsilon \\ J &\to AJ \end{aligned}$$

$$\left.\begin{aligned} A &\to \sigma \\ L &\to \sigma \;\&\; \triangleright C_\sigma \\ C_\sigma &\to \$J\sigma \\ C_\sigma &\to AC_\sigma A \end{aligned}\right\} \sigma \in \{a, b\}.$$



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C_\sigma \to \$J\sigma \\
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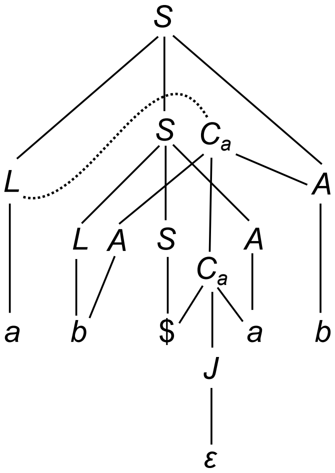

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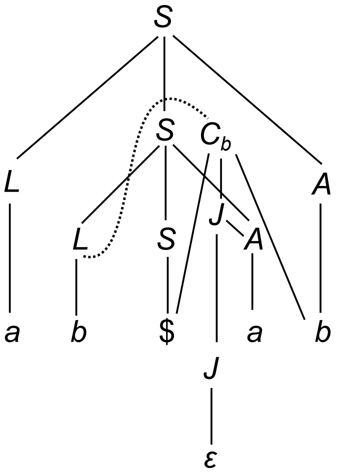

Parsing one-sided context

CKY-type algorithm for one-sided context: $O(n^3)$

Uses *binary normal form*

Can be sped up using matrix multiplication: $O(n^\omega)$
$(2 \leq \omega < 2.3727)$

Parsing two-sided context

CKY-type algorithm is still sound

but needs $O(|N| \cdot n)$ passes

so $O(n^{\omega+1})$

Example

Let $G = (\{a\}, \{A, S\}, R, S)$ where R contains the following rules:

$$S \to a$$
$$S \to AS$$
$$A \to a \mathbin{\&} \triangleright S$$

Example: $O(n^4)$ algorithm

$$S \to a$$
$$S \to AS$$
$$A \to a \;\&\; \triangleright S$$

$$aaa$$

Example: $O(n^4)$ algorithm

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⟨a⟩aa

$$[S, \varepsilon\langle a\rangle aa]$$

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$\langle$aa$\rangle$a

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There are only $O(|N| \cdot n^2)$ facts to learn
(one for each pair of non-terminal and substring)

but we consider each fact many times, before we can learn it.

Classic optimization: look at each fact when we are ready,
instead of looping over all facts

Our algorithm

Interpret

$$A \to BC \ \& \ \triangleright D$$

as a family of implications

$$[B, u\langle v_1\rangle v_2 w] \wedge [C, uv_1\langle v_2\rangle w] \wedge [D, uv_1 v_2\langle w\rangle \varepsilon] \Rightarrow [A, u\langle v_1 v_2\rangle w]$$

Each rule with a single concatenation $\rightsquigarrow O(n^3)$ implications

Forbid multiple concatenations (*separated normal form*):
$O(|G| \cdot n^3)$ implications

Our algorithm

Keep set of implications

Mark antecedents as we learn them

When we know all antecedents, learn the consequent

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An instance of Horn deduction (cf. Kowalski and Colmerauer; Shieber, Schabes and Pereira)

or hypergraph reachability (cf. Klein and Manning)

Analysis

There are $O(|N| \cdot n^2)$ facts:
each can appear many times.

But there are only $O(|G| \cdot n^3)$ symbols in the implications:
each one is marked once.

If we keep an index of each use of a fact,
we can do the marking in constant time for each appearance.

If we are careful, we only need to remember which facts we have learned,
so space usage is quadratic.

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Open questions

Can we improve parsing to $O(n^{\omega})$?

Better understanding of context and the languages it recognizes

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