Conflict Free Version of Covering Problems on Graphs: Classical and Parameterized

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June 5, 2018

Classical Problem

$\mathcal{Q}$
Input: An instance I.
Question:

Classical Problems: Running Examples

MINIMUM WEIGHT SPANNING TREE (MST)
Input: A Graph $G = (V, E)$, and a weight function $w : E \to \mathbb{Z}^+$
Output: A spanning tree T of G, of minimum weight.

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FEEDBACK VERTEX SET (FVS) **Parameter:** k
Input: A Graph $G = (V, E)$ and a positive integer k.
Question: Does there exist a set $S \subseteq V$ of size at most k such that $G - S$ does not contain any cycle?

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A lot of studies on problems that satisfy additional constraints.

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  - At least (most) k leaves.

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  - At least (most) k leaves.
  - At least (most) k internal nodes.

Conflict free Problem

Conflict Free-$\mathcal{Q}$ (CF-$\mathcal{Q}$)
Input: An instance I of classical problem $\mathcal{Q}$, a conflict graph H.
Output: A solution to I in $\mathcal{Q}$, which is also an independent set in H.

Running Examples

CF-MST

Input: A Graph $G = (V, E)$, a conflict graph $H = (E, E')$, and a weight function $w : E \to \mathbb{Z}^+$

Output: A spanning tree T of G, with minimum weight such that $E(T)$ is an independent set in H.

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- Maximum Flow

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By Kann, Pferchy , Epstein, Even. . . .

Why Conflict Free Problems?

- Take a detour and introduce vertex deletion graph problems.
- First introduce the notion of *graph properties*.

Graph Property

- A graph property Π is a family of graphs.

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- Π is Forest
- Π is Bipartite graphs
- Π is Chordal graphs
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- Π is Interval graphs - Interval Vertex Deletion (IVD).

CF-Π-VD

CONFLICT FREE Π-VERTEX DELETION (CF-Π-VD) **Parameter:** k
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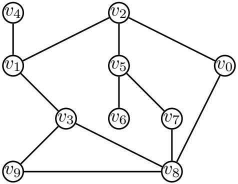
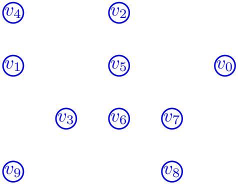
- Π is Forest - CF-FVS.
- Π is Bipartite graphs - CF-OCT.
- Π is Chordal graphs - CF-CVD.
- Π is Interval graphs - CF-IVD.

Motivation I: Generalizes Classical Graph Problem

When *H* is edgeless:



$G = (V, E)$



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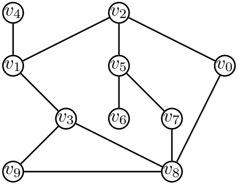

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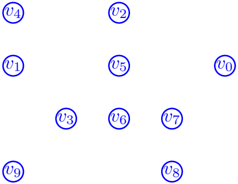

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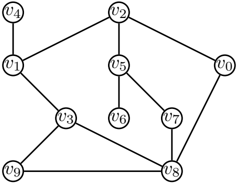
$\mathcal{Q}$=CF-$\mathcal{Q}$.

Motivation II: Generalizes Independence Constraint

When $G = H$



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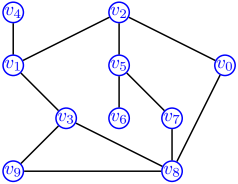

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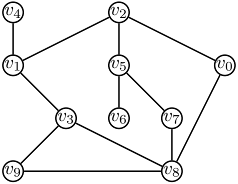

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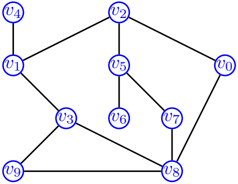

$H = (V, E')$

CF-$\mathcal{Q}$=Independent-$\mathcal{Q}$.

Models CSP like dichotomy for specific graph problem

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$(\mathcal{G}, \mathcal{H})$-CF-$\Pi$-VD
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Framework of Our Study – Parameterized Complexity

Parameterized Complexity : Quick Overview

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- W[t]-hard, for some $t \in \mathbb{N}$.

Our Results

Forbidden Characterization

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- FVS- $\mathcal{F}(\Pi)$ is set of all cycles (infinite).

Forbidden Characterization

- $\mathcal{F}(\Pi)$ is **finite**:
  - CF-Π-VD is FPT with running time $(\mathcal{O}(\alpha^k \cdot n \cdot T(m, n)))$, where $T(m, n)$ is time to recognize a graph in Π and α is the size of largest graph in $\mathcal{F}(\Pi)$.

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- $\mathcal{F}(\Pi)$ is (well behaved) **infinite**: CF-Π-VD is W[1]-hard.

Can we give better than $\mathcal{O}(\alpha^k \cdot n \cdot T(m, n))$ running time algorithm?

Faster Running Time

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	Polynomial time- G has degree at most one, or both G and H have a perfect matching

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FVS IN TOURNAMENTS	$\mathcal{O}^\star(1.618^k)$ time algorithm	$\mathcal{O}^\star(2^k)$ time algorithm

Finite Forbidden Characterization

Theorem

CF-VC *admits a* $2k$*-vertex kernel, a factor* 2*-approximation algorithm, an* $\mathcal{O}^{\star}(1.2738^k)$ FPT *algorithm and a* $\mathcal{O}^{\star}(1.1996^n)$ *exact algorithm.*

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- Polynomial time parameter preserving reduction from CF-VC to Min Ones 2-SAT.
- Min Ones 2-SAT can be solved as fast as VC.
- In polynomial time we can test whether there exists a solution to CF-VC for pair (G, H).

Finite Forbidden Characterization

MIN ONES 2-SAT
Input: A 2-CNF formula ϕ
Output: A satisfying assignment for ϕ formula that minimizes the number of variables that are set to 1

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$$\Phi = \bigwedge_{uv \in E(G)} (u \vee v) \bigwedge_{uv \in E(H)} (\bar{u} \vee \bar{v}).$$

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CF-FVS *is* W[1]*-hard, when parameterized by the solution size.*

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Theorem

CF-FVS *is* W[1]*-hard, when parameterized by the solution size.*

- Polynomial time parameter preserving reduction from Multicolored Independent Set to CF-FVS.
- Multicolored Independent Set is W[1]-hard.
- A similar result holds for Conflict Free Odd Cycle Transversal (CF-OCT), Conflict Free Chordal Vertex Deletion (CF-CVD) and Conflict Free Interval Vertex Deletion (CF-IVD)

Infinite Forbidden Characterization

CF-FVS **Parameter:** k

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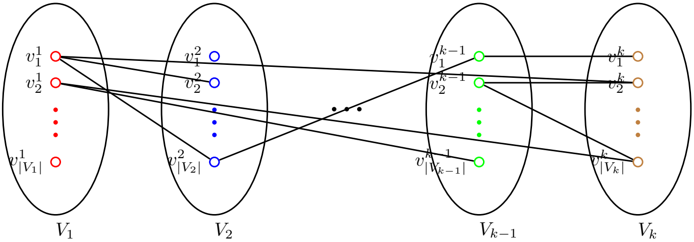
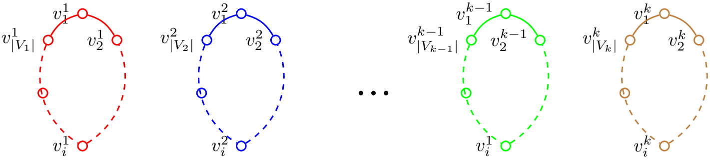
Multicolored Independent Set **Parameter:** k
Input: A graph $G = (V, E)$, and a partition $V_1, V_2, \cdots, V_k$ of V.
Question: Is there a set $S \subseteq V$ such that S is an independent set in G, and for each $i \in [k]$, we have $|S \cap V_i| = 1$?

Infinite Forbidden Characterization



Conflict graph $H = G$



Graph G'

When does CF-Π-VD admits an FPT algorithm?

Infinite Forbidden Characterization

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CF-FVS, CF-OCT, Chordal Vertex Deletion and Interval Vertex Deletion are FPT

Independence Covering Lemma [Lokshtanov et al., SODA 2018]

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k-Independence Covering Family ($\mathcal{F}(H^\star, k)$):

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Independence Covering Lemma

- Given a d-degenerate graph H^*, integer k.

Independence Covering Lemma [Lokshtanov et al., SODA 2018]

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A similar lemma exist for Nowhere Dense graphs.

FPT algorithm for $(\star, \mathcal{D}_d)$-CF-FVS

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- For each $Y \in \mathcal{F}(H^\star, k)$, find a solution of FVS contained in Y
  - Solve weighted version of FVS
  - Assign weight $k+1$ to all $v \in V(G) \setminus Y$ and 1 to $v \in Y$.
  - WEIGHTED FVS can be solved in time $\mathcal{O}(3.618^k n^{\mathcal{O}(1)})$

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- For what graph classes $\mathcal{G}, \mathcal{H}$ and graph property Π does $(\mathcal{G}, \mathcal{H})$-CF-$\Pi$-VD admits polynomial kernel?

Questions?