On parameter-free induction schemata.

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We study the fragments of Peano Arithmetic $I\Sigma_n^-$ and $I\Pi_n^-$ axiomatized by instances of the induction schema for Σ_n (respectively, Π_n) formulas *without parameters*. We give a precise characterization of $I\Sigma_n^-$ and $I\Pi_n^-$ in terms of reflection principles. This allows us to show that $I\Pi_{n+1}^-$ is conservative over $I\Sigma_n^-$ w.r.t. boolean combinations of Σ_{n+1} sentences, for $n \geq 1$. In particular, we give a positive answer to a question by R. Kaye, whether the provably recursive functions of $I\Pi_2^-$ are exactly the primitive recursive ones. We also obtain sharp results on the strength of bounded number of instances of parameter free induction in terms of iterated reflection.