On elementary theories of natural numbers equipped with Cantor parimg function.

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Georg Cantor has exhibited in 1873 a famous bijection from $\mathbf{N}^2$ onto $\mathbf{N}$, defined by:

$$C(x, y) = \frac{(x + y) \cdot (x + y + 1)}{2} + y.$$

We prove that the first order theory $Th(\mathbf{N}, \mathbf{C})$ is decidable using two methods: the first one is an application of Malcev's theorem on absolutely free algebras; the second is direct. Then we apply the second method to the theory $Th(\mathbf{N}, \mathbf{C} + 1)$, a more challenging problem.

Also we prove that theories $Th(\mathbf{N}, \mathbf{C}, \mathbf{P})$ where $\mathbf{P}$ is a predicate as $+$ or $<$, are undecidable.