Simplified models establishing some of Nezondet's results on the Erdös-Woods conjecture and related statements

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One year before, Nezondet described a model proving that finite arithmetics does not prove the Erdös-Woods conjecture. This model was constructed in two steps involving ultraproducts. The first of these steps uses some properties of the set of cosets of the $P(a)$ in an ultrapower of the natural numbers, where P is a (variable) polynomial with rational coefficients and a some (fixed) sequence increasing faster than exponentially and whose properties rest on involved arithmetical lemmas.

We first note that the positive properties (to be a model of finite arithmetics) follow just from the fact that a tends to infinity. We show afterwards that when this sequence is (for example) the identity, we can choose the ultrafilter so that it obeys some prescription, which can be given in advance, about the remainders of the (coset of the) identity modulo the standard prime numbers, and that this allows - using as main tool, just the chinese remainder theorem - to obtain directly a model refuting the Erdös-Woods conjecture.

Nezondet proved also that the complete theory of the natural numbers, in the langage of coprimarity and of the arithmetical FUNCTIONAL operations, but without equality, does not define the equality. We show that, renouncing to all what does not concern the coprimarity and the successor function, a model of the complete theory of the standard model in the langage of the coprimarity and the successor function without equality can be very simply described, which proves that the equality cannot be defined in the theory (unfortunately, none of the models alluded to above is a model of a theory able to formalize Woods proof of the equivalence of the definability of the equality with the Erdös-Woods conjecture).