On n-tupling polynomials of small degrees

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In this talk we shall consider n-tupling polynomials, i.e., polynomials that bijectively map $\mathbf{N}^n$ onto $\mathbf{N}$. Such functions are used in logic for arithmetic encoding.

The following problem naturally arises.

For given n and d, describe all n-tupling polynomials of degree d.

This problem (in general case) still remains open and it seems to be very difficult. The first result in this area is due to Fueter and Pólya. As they proved in 1923, there are only two quadratic pairing polynomials, namely, Cantor's polynomials

$$\begin{aligned} C_1(x,y) &= \frac{(x+y)^2+3x+y}{2}\,, \\ C_2(x,y) &= \frac{(x+y)^2+x+3y}{2}\,. \end{aligned}$$

But their proof was based on such deep and non-elementary results on transcendental numbers as Lindemann's theorem.

First, we present new elementary proof of the Fueter-Pólya theorem, which requires only arithmetics in $\mathbf{Z}/m\mathbf{Z}$ and simple estimates.

Second, we apply similar ideas to the problem of description of all cubic tripling polynomials.