

| Class | Definition | Base(s) |
|------------------------------|--|--|
| BF | all Boolean functions | $\{and, not\}$ |
| R ₀ | $\{f \in \text{BF} \mid f \text{ is 0-reproducing}\}$ | $\{and, xor\}$ |
| R ₁ | $\{f \in \text{BF} \mid f \text{ is 1-reproducing}\}$ | $\{or, x \oplus y \oplus 1\}$ |
| R ₂ | $R_1 \cap R_0$ | $\{or, x \wedge (y \oplus z \oplus 1)\}$ |
| M | $\{f \in \text{BF} \mid f \text{ is monotonic}\}$ | $\{and, or, c_0, c_1\}$ |
| M ₁ | $M \cap R_1$ | $\{and, or, c_1\}$ |
| M ₀ | $M \cap R_0$ | $\{and, or, c_0\}$ |
| M ₂ | $M \cap R_2$ | $\{and, or\}$ |
| S ₀ ⁿ | $\{f \in \text{BF} \mid f \text{ is 0-separating of degree } n\}$ | $\{imp, dual(h_n)\}$ |
| S ₀ | $\{f \in \text{BF} \mid f \text{ is 0-separating}\}$ | $\{imp\}$ |
| S ₁ ⁿ | $\{f \in \text{BF} \mid f \text{ is 1-separating of degree } n\}$ | $\{x \wedge \bar{y}, h_n\}$ |
| S ₁ | $\{f \in \text{BF} \mid f \text{ is 1-separating}\}$ | $\{x \wedge \bar{y}\}$ |
| S ₀₂ ⁿ | $S_0^n \cap R_2$ | $\{x \vee (y \wedge \bar{z}), dual(h_n)\}$ |
| S ₀₂ | $S_0 \cap R_2$ | $\{x \vee (y \wedge \bar{z})\}$ |
| S ₀₁ ⁿ | $S_0^n \cap M$ | $\{dual(h_n), c_1\}$ |
| S ₀₁ | $S_0 \cap M$ | $\{x \vee (y \wedge z), c_1\}$ |
| S ₀₀ ⁿ | $S_0^n \cap R_2 \cap M$ | $\{x \vee (y \wedge z), dual(h_n)\}$ |
| S ₀₀ | $S_0 \cap R_2 \cap M$ | $\{x \vee (y \wedge z)\}$ |
| S ₁₂ ⁿ | $S_1^n \cap R_2$ | $\{x \wedge (y \vee \bar{z}), h_n\}$ |
| S ₁₂ | $S_1 \cap R_2$ | $\{x \wedge (y \vee \bar{z})\}$ |
| S ₁₁ ⁿ | $S_1^n \cap M$ | $\{h_n, c_0\}$ |
| S ₁₁ | $S_1 \cap M$ | $\{x \wedge (y \vee z), c_0\}$ |
| S ₁₀ ⁿ | $S_1^n \cap R_2 \cap M$ | $\{x \wedge (y \vee z), h_n\}$ |
| S ₁₀ | $S_1 \cap R_2 \cap M$ | $\{x \wedge (y \vee z)\}$ |
| D | $\{f \mid f \text{ is self-dual}\}$ | $\{x\bar{y} \vee x\bar{z} \vee y\bar{z}\}$ |
| D ₁ | $D \cap R_2$ | $\{xy \vee x\bar{z} \vee y\bar{z}\}$ |
| D ₂ | $D \cap M$ | $\{xy \vee yz \vee xz\}$ |
| L | $\{f \mid f \text{ is linear}\}$ | $\{xor, c_1\}$ |
| L ₀ | $L \cap R_0$ | $\{xor\}$ |
| L ₁ | $L \cap R_1$ | $\{eq\}$ |
| L ₂ | $L \cap R_2$ | $\{x \oplus y \oplus z\}$ |
| L ₃ | $L \cap D$ | $\{x \oplus y \oplus z \oplus c_1\}$ |
| V | $\{f \mid f \text{ is an } n\text{-ary } or\text{-function or a constant function}\}$ | $\{or, c_0, c_1\}$ |
| V ₀ | $\{\{or\}\} \cup \{\{c_0\}\}$ | $\{or, c_0\}$ |
| V ₁ | $\{\{or\}\} \cup \{\{c_1\}\}$ | $\{or, c_1\}$ |
| V ₂ | $\{\{or\}\}$ | $\{or\}$ |
| E | $\{f \mid f \text{ is an } n\text{-ary } and\text{-function or a constant function}\}$ | $\{and, c_0, c_1\}$ |
| E ₀ | $\{\{and\}\} \cup \{\{c_0\}\}$ | $\{and, c_0\}$ |
| E ₁ | $\{\{and\}\} \cup \{\{c_1\}\}$ | $\{and, c_1\}$ |
| E ₂ | $\{\{and\}\}$ | $\{and\}$ |
| N | $\{\{not\}\} \cup \{\{c_0\}\} \cup \{\{c_1\}\}$ | $\{not, c_1\}, \{not, c_0\}$ |
| N ₂ | $\{\{not\}\}$ | $\{not\}$ |
| I | $\{\{id\}\} \cup \{\{c_1\}\} \cup \{\{c_0\}\}$ | $\{id, c_0, c_1\}$ |
| I ₀ | $\{\{id\}\} \cup \{\{c_0\}\}$ | $\{id, c_0\}$ |
| I ₁ | $\{\{id\}\} \cup \{\{c_1\}\}$ | $\{id, c_1\}$ |
| I ₂ | $\{\{id\}\}$ | $\{id\}$ |

Figure 1: List of all Boolean clones with bases $(h_n = \bigvee_{i=1}^{n+1} x_1 \cdots x_{i-1} x_{i+1} \cdots x_{n+1})$ and $dual(f)(a_1, \dots, a_n) = \neg f(\bar{a}_1, \dots, \bar{a}_n)$.

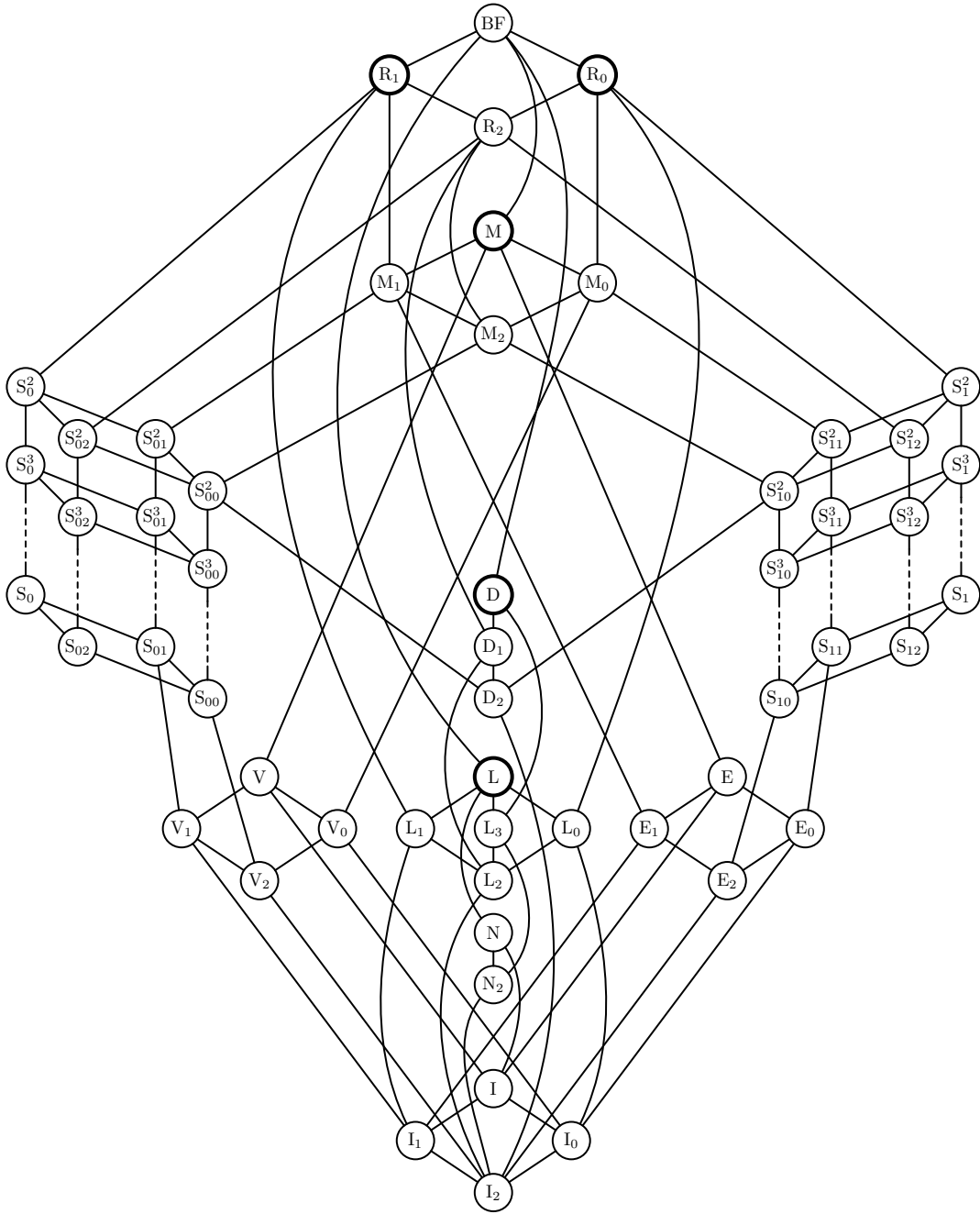


Figure 2: Graph of all Boolean clones.

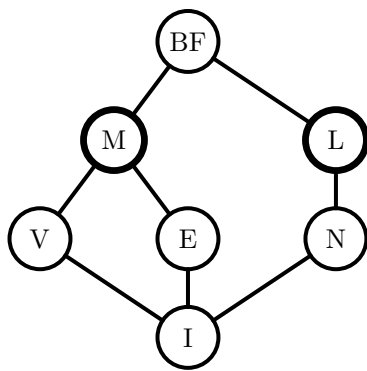


Figure 3: Graph of all Boolean clones that contain all constant functions

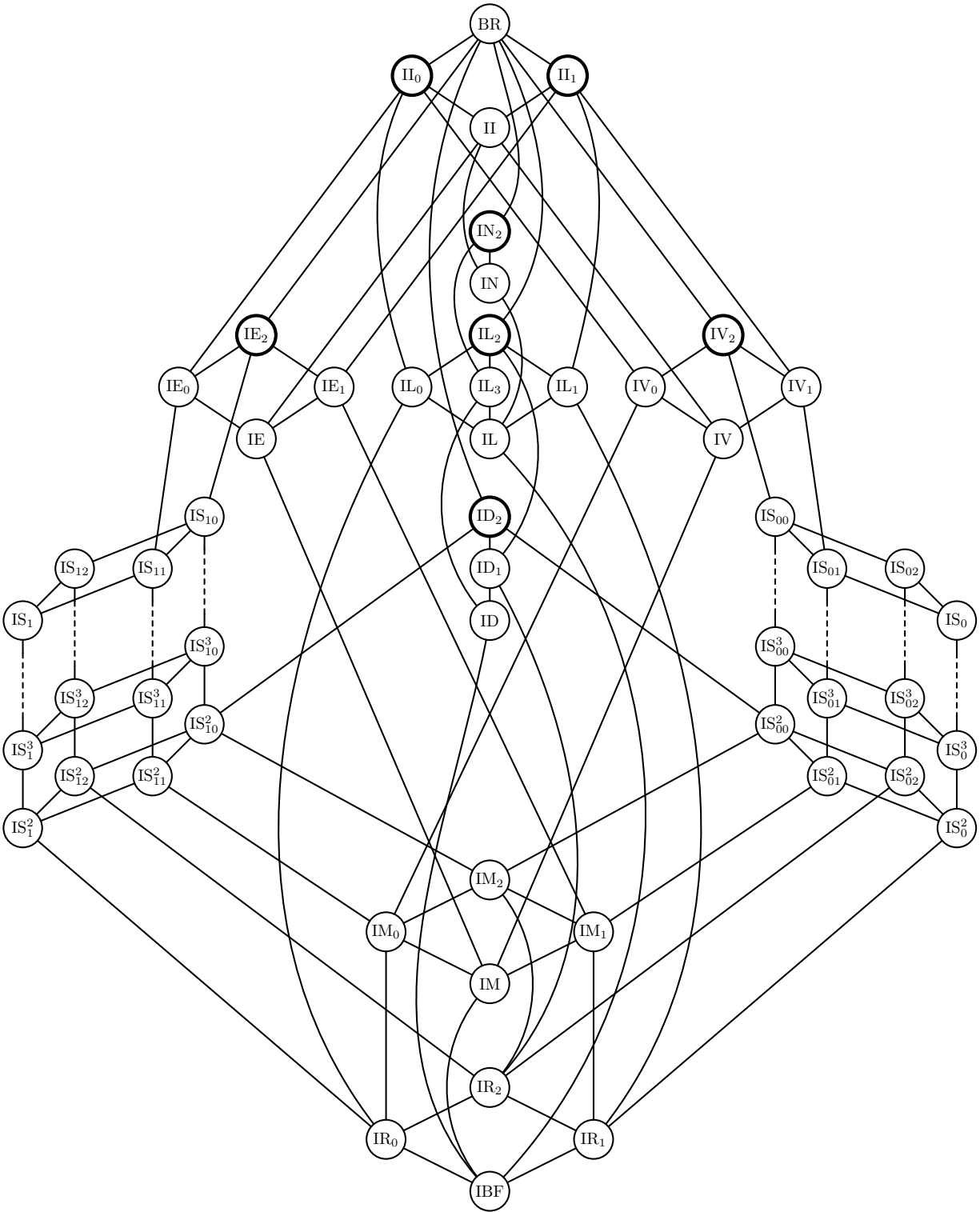


Figure 4: Graph of all Boolean co-clones

| Cl. | Or. | Remark | Base(s) of corresponding co-clone |
|------------------------------|-----|--|--|
| BF | 0 | | $\{=\}, \{\emptyset\}$ |
| R ₀ | 1 | dual of R ₁ | $\{\bar{x}\}$ |
| R ₁ | 1 | | $\{x\}$ |
| R ₂ | 1 | R ₀ ∩ R ₁ | $\{x, \bar{x}\}, \{x\bar{y}\}$ |
| M | 2 | | $\{x \rightarrow y\}$ |
| M ₁ | 2 | M ∩ R ₁ | $\{x \rightarrow y, x\}, \{x \wedge (y \rightarrow z)\}$ |
| M ₀ | 2 | M ∩ R ₀ | $\{x \rightarrow y, \bar{x}\}, \{\bar{x} \wedge (y \rightarrow z)\}$ |
| M ₂ | 2 | M ∩ R ₂ | $\{x \rightarrow y, x, \bar{x}\}, \{x \rightarrow y, \bar{x} \rightarrow \bar{y}\}, \{x\bar{y} \wedge (u \rightarrow v)\}$ |
| S ₀ ^m | m | | $\{\text{OR}^m\}$ |
| S ₁ ^m | m | dual of S ₀ ^m | $\{\text{NAND}^m\}$ |
| S ₀ | ∞ | $\bigcap_{m \geq 2} S_0^m$ | $\{\text{OR}^m \mid m \geq 2\}$ |
| S ₁ | ∞ | dual of S ₀ | $\{\text{NAND}^m \mid m \geq 2\}$ |
| S ₀₂ ^m | m | S ₀ ^m ∩ R ₂ | $\{\text{OR}^m, x, \bar{x}\}$ |
| S ₀₂ | ∞ | S ₀ ∩ R ₂ | $\{\text{OR}^m \mid m \geq 2\} \cup \{x, \bar{x}\}$ |
| S ₀₁ ^m | m | S ₀ ^m ∩ M | $\{\text{OR}^m, x \rightarrow y\}$ |
| S ₀₁ | ∞ | S ₀ ∩ M | $\{\text{OR}^m \mid m \geq 2\} \cup \{x \rightarrow y\}$ |
| S ₀₀ ^m | m | S ₀ ^m ∩ R ₂ ∩ M | $\{\text{OR}^m, x, \bar{x}, x \rightarrow y\}$ |
| S ₀₀ | ∞ | S ₀ ∩ R ₂ ∩ M | $\{\text{OR}^m \mid m \geq 2\} \cup \{x, \bar{x}, x \rightarrow y\}$ |
| S ₁₂ ^m | m | dual of S ₀₂ ^m | $\{\text{NAND}^m, x, \bar{x}\}$ |
| S ₁₂ | ∞ | dual of S ₀₂ | $\{\text{NAND}^m \mid m \geq 2\} \cup \{x, \bar{x}\}$ |
| S ₁₁ ^m | m | dual of S ₀₁ ^m | $\{\text{NAND}^m, x \rightarrow y\}$ |
| S ₁₁ | ∞ | dual of S ₀₁ | $\{\text{NAND}^m \mid m \geq 2\} \cup \{x \rightarrow y\}$ |
| S ₁₀ ^m | m | dual of S ₀₀ ^m | $\{\text{NAND}^m, x, \bar{x}, x \rightarrow y\}$ |
| S ₁₀ | ∞ | dual of S ₀₀ | $\{\text{NAND}^m \mid m \geq 2\} \cup \{x, \bar{x}, x \rightarrow y\}$ |
| D | 2 | | $\{x \oplus y\}$ |
| D ₁ | 2 | D ∩ R ₁ | $\{x \oplus y, x\}$, every $R \in \{(a_1, a_2, a_3), (b_1, b_2, b_3)\} \mid \exists c \in \{1, 2\}$ such that $\sum_{i=1}^3 a_i = \sum_{i=1}^3 b_i = c$ |
| D ₂ | 2 | D ∩ M | $\{x \oplus y, x \rightarrow y\}, \{x\bar{y} \vee \bar{x}yz\}$ |
| L | 4 | | $\{\text{EVEN}^4\}$ |
| L ₀ | 3 | L ∩ R ₀ | $\{\text{EVEN}^4, \bar{x}\}, \{\text{EVEN}^3\}$ |
| L ₁ | 3 | L ∩ R ₁ | $\{\text{EVEN}^4, x\}, \{\text{ODD}^3\}$ |
| L ₂ | 3 | L ∩ R ₂ | $\{\text{EVEN}^4, x, \bar{x}\}$, every $\{\text{EVEN}^n, (1)\}$ where $n \geq 3$ is odd |
| L ₃ | 4 | L ∩ D | $\{\text{EVEN}^4, x \oplus y\}, \{\text{ODD}^4\}$ |
| V | 3 | | $\{x \vee y \vee \bar{z}\}$ |
| V ₀ | 3 | V ∩ R ₀ | $\{x \vee y \vee \bar{z}, \bar{x}\}$ |
| V ₁ | 3 | V ∩ R ₁ | $\{x \vee y \vee \bar{z}, x\}$ |
| V ₂ | 3 | V ∩ R ₂ | $\{x \vee y \vee \bar{z}, x, \bar{x}\}$ |
| E | 3 | dual of V | $\{\bar{x} \vee \bar{y} \vee z\}$ |
| E ₁ | 3 | dual of V ₀ | $\{\bar{x} \vee \bar{y} \vee z, x\}$ |
| E ₀ | 3 | dual of V ₁ | $\{\bar{x} \vee \bar{y} \vee z, \bar{x}\}$ |
| E ₂ | 3 | dual of V ₂ | $\{\bar{x} \vee \bar{y} \vee z, x, \bar{x}\}$ |
| N | 3 | | $\{\text{DUP}^3\}$ |
| N ₂ | 3 | N ∩ L ₃ | $\{\text{DUP}^3, \text{EVEN}^4, x \oplus y\}, \{\text{NAE}^3\}$ |
| I | 3 | L ∩ M | $\{\text{EVEN}^4, x \rightarrow y\}$ |
| I ₀ | 3 | L ∩ M ∩ R ₀ | $\{\text{EVEN}^4, x \rightarrow y, \bar{x}\}, \{\text{DUP}^3, x \rightarrow y\}$ |
| I ₁ | 3 | L ∩ M ∩ R ₁ | $\{\text{EVEN}^4, x \rightarrow y, x\}, \{x \vee (x \oplus z)\}$ |
| I ₂ | 3 | L ∩ M ∩ R ₂ | $\{\text{EVEN}^4, x \rightarrow y, x, \bar{x}\}, \{1\text{text-}IN-3\}, \{x \rightarrow (y \oplus z)\}$ |

Figure 5: Bases for all Boolean co-clones