# The Tractability of Model-Checking for LTL: The Good, the Bad, and the Ugly Fragments

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# The burning issue

### Problem

The model-checking problem for full Linear Temporal Logic (LTL) is PSPACE-complete [Sistla, Clarke 1985].

That is, this problem is (most probably) intractable. ☹

### Solution

Systematically restrict the propositional part of LTL.

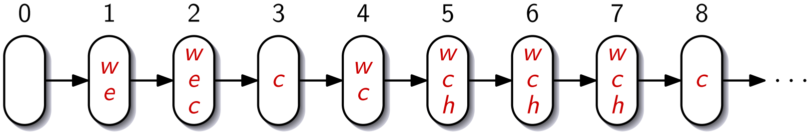
$\rightsquigarrow$ Many tractable (good) fragments ☺

$\rightsquigarrow$ Many intractable (bad) fragments

# What is Linear Temporal Logic?

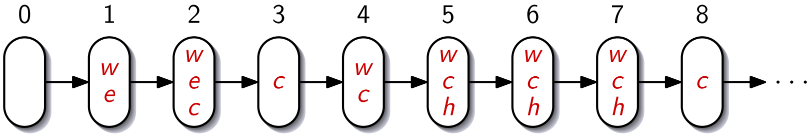
LTL = propositional logic plus temporal operators,
speaks about linear structures; for example:

A structure $P$



# The language and its interpretation

## The structure $P$



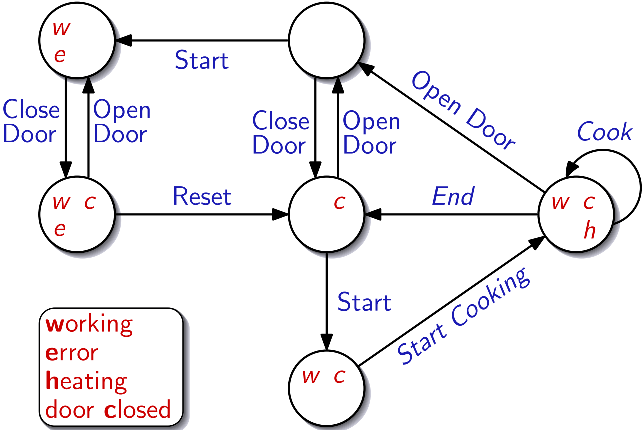

The following kinds of statements can be formulated in LTL.

- now $\quad P, 2 \models (w \land \neg e) \lor c$
- at some time in the **F**uture $\quad P, 0 \models \mathbf{F} h$
- always **G**oing to $\quad P, 3 \models \mathbf{G} \neg e$
- ne**X**t time $\quad P, 1 \models \mathbf{X}(w \longrightarrow e)$
- **U**ntil $\quad P, 5 \models c \mathbf{U} (\neg w)$
- **S**ince $\quad P, 3 \models c \mathbf{S} w$
- $P, 0 \models \mathbf{F}(c \land \neg e) \land \mathbf{G}\big[(c \land \neg w) \longrightarrow [\mathbf{X}(w \land \mathbf{X} h) \land (h \longrightarrow w \land c) \mathbf{U} c]\big]$

# A model and a structure

A model (cf. Clarke et al. „Model Checking“):

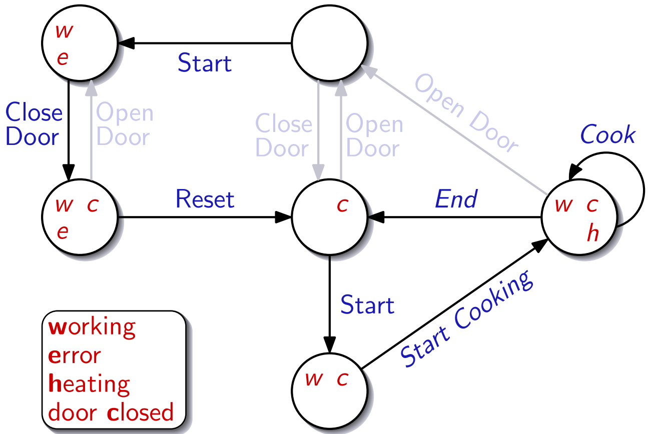
Possible behaviour of a microwave oven



# A model and a structure

A structure:

*Actual* behaviour of a microwave oven



# Summing up: Models and structures

### Model

A directed graph where every state has a successor.
States are marked with assignments to propositional variables.

### Structure

An infinite path in a model.

# The model-checking problem

**Model-Checking**

*Instance* $\langle \varphi, M, a \rangle$

*Question* Does $M$ contain a structure $P$ with initial state $a$ such that $P, a \models \varphi$?

**Theorem (Sistla, Clarke 1985)**

Model-checking for LTL is PSPACE-complete.

# When do LTL fragments suffice?

## Example

Properties of "microwave oven runs" expressible in LTL fragments:

| *Property* | *Formula* | *Operators used* |
|---|---|---|
| An error never occurs. | $\mathbf{G}\neg e$ | $\mathbf{G}, \neg$ |
| (Safety) | $\neg\mathbf{F}e$ | $\mathbf{F}, \neg$ |
| | $\mathbf{G}e'$ | $\mathbf{G}$ |
| Every error will | $\mathbf{GF}\neg e$ | $\mathbf{F}, \mathbf{G}, \neg$ |
| eventually be resolved. | $\mathbf{G}\neg\mathbf{G}e$ | $\mathbf{G}, \neg$ |
| (Liveness) | $\mathbf{GF}e'$ | $\mathbf{F}, \mathbf{G}$ |

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| (Liveness) | $\mathbf{GF}e'$ | $\mathbf{F}, \mathbf{G}$ |

# The model-checking problem for LTL fragments

### LTL fragment

Let $T \subseteq \{\mathbf{F}, \mathbf{G}, \mathbf{X}, \mathbf{U}, \mathbf{S}\}$ be a set of temporal operators and $B$ be a finite set of Boolean operators.*

$L(T, B)$ = set of all LTL formulas with operators in $T \cup B$.

*For instance, $\{\wedge, \vee\}$ — monotone formulae.

### Model-checking problem MC($T$, $B$) for LTL fragments

*Instance:* $\langle \varphi, M, a \rangle$ with $\varphi \in L(T, B)$

*Question:* Does $M$ contain a structure $P$ with initial state $a$ such that $P, a \vDash \varphi$?

# Known complexity results . . .

Theorem ([Sistla, Clarke 1985] and [Markey 2004])

1. $\mathsf{MC}(\{\mathbf{G}, \mathbf{X}\}, \{\wedge, \vee, \neg\})$ and $\mathsf{MC}(\{\mathbf{U}\}, \{\wedge, \vee, \neg\})$ are PSPACE-complete, even if negation is applied to atoms only.
2. $\mathsf{MC}(\{\mathbf{F}\}, \{\wedge, \vee, \neg\})$, $\mathsf{MC}(\{\mathbf{G}\}, \{\wedge, \vee, \neg\})$ and $\mathsf{MC}(\{\mathbf{X}\}, \{\wedge, \vee, \neg\})$ are NP-complete, even if negation is applied to atoms only.
3. $\mathsf{MC}(\{\mathbf{F}, \mathbf{X}\}, \{\wedge, \vee, \neg\})$ in general is PSPACE-complete, but NP-complete if negation is applied to atoms only.

# Known complexity results . . .

Consequences of results by [Sistla, Clarke 1985] and [Markey 2004]:

Hardness and completeness of MC($T$, $B$)

| $T$ \ $B$ | $\{\wedge, \vee\}$ | $\{\wedge, \vee, \neg\}$ |
|---|---|---|
| **X** | **NP** | **NP** |
| **G** | **NP** | **NP** |
| **F** | **NP** | **NP** |
| **FX** | **NP** | **PSPACE** |
| **GX** | **PSPACE** | **PSPACE** |
| **U** | **PSPACE** | **PSPACE** |

Bad fragments!

# What we would like to know ...

Goal

- classify the complexity of $\mathsf{MC}(T, B)$ for *all* LTL fragments
- separate LTL fragments into good (efficiently solvable) and bad (NP-hard)

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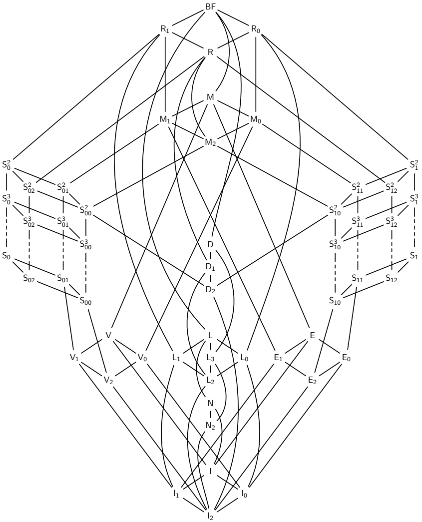
# Fragments of propositional logic: Clones



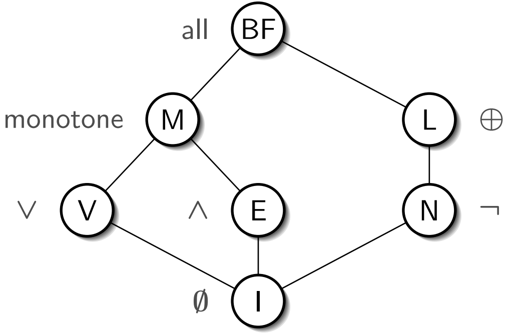

## Post's lattice

(est'd 1941 by Emil Post)

| | |
|---|---|
| $X_2$ | without constants |
| $X_{0,1}$ | with constant 0,1 |
| BF | all BF |
| M | monotone functions |
| $S_1$ | $x \wedge \overline{y}$ |
| $S_0$ | $x \longrightarrow y$ |
| D | $f(a_1, \ldots, a_n) = \overline{f(\overline{a_1}, \ldots, \overline{a_n})}$ |
| L | $x \oplus y$ (xor) |
| V | $x \vee y$ |
| E | $x \wedge y$ |
| N | $\neg x$ |
| I | identities |

# Clones with both constants

## All relevant sets of Boolean operators




Every other set of Boolean op's can be reduced to one of these.

# Tractability of model-checking: Fragments with **F**,**G**,**X**

Hardness and completeness of MC($T$, $B$)

| $B$ / $T$ | I | N $\neg$ | E $\wedge$ | V $\vee$ | M mon. | L $\oplus$ | BF all |
|---|---|---|---|---|---|---|---|
| **X** | **NL** | **NL** | **NL** | **NL** | NP | **NL** | NP |
| **G** | **NL** | **NL** | **NL** | **NL** | NP | | NP |
| **F** | **NL** | **NL** | NP | **NL** | NP | | NP |
| **FG** | **NL** | **NL** | NP | **NL** | NP | | NP |
| **FX** | **NL** | **NL** | NP | **NL** | NP | | PS |
| **GX** | **NL** | **NL** | **NL** | NP | PS | | PS |
| **FGX** | **NL** | **NL** | NP | NP | PS | | PS |

(PS = PSPACE)

# Tractability of model-checking: Fragments with **S**, **U**

## Hardness and completeness of MC($T$, $B$)

| $B$ | I | N | E | V | M | L | BF |
|---|---|---|---|---|---|---|---|
| $T$ | | $\neg$ | $\wedge$ | $\vee$ | mon. | $\oplus$ | all |
| **S** | L | L | L | L | L | L | L |
| **SX** | NP | NP | NP | NP | NP | NP | NP |
| **SG** | NP | NP | NP | NP | PS | NP | PS |
| **SF** | **NL** | NP | NP | **NL** | PS | NP | PS |
| **SFG** | NP | NP | NP | NP | PS | NP | PS |
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| *other* | NP | NP | NP | NP | PS | NP | PS |

# Tractability of model-checking: Fragments with **S**, **U**

## Hardness and completeness of MC($T$, $B$)

| $B$ | I | N | E | V | M | L | BF |
|---|---|---|---|---|---|---|---|
| $T$ | | $\neg$ | $\wedge$ | $\vee$ | mon. | $\oplus$ | all |
| **S** | L | L | L | L | L | L | L |
| **SX** | NP | NP | NP | NP | NP | NP | NP |
| **SG** | NP | NP | NP | NP | PS | NP | PS |
| **SF** | **NL** | NP | NP | **NL** | PS | NP | PS |
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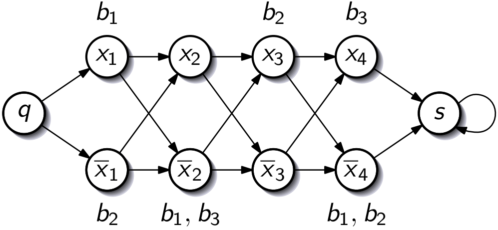
# An NP-hardness proof

Theorem (Sistla, Clarke 1985)

MC($\{\mathbf{F}\}, \{\wedge\}$) is NP-hard.

Proof sketch.

- Reduction from 3SAT
- From $(x_1 \vee \neg x_2 \vee \neg x_4) \wedge (\neg x_1 \vee x_3 \vee \neg x_4) \wedge (\neg x_2 \vee x_4)$ we obtain the model




and the $L(\{\mathbf{F}\}, \{\wedge\})$-formula $\mathbf{F}b_1 \wedge \mathbf{F}b_2 \wedge \mathbf{F}b_3$ . $\square$

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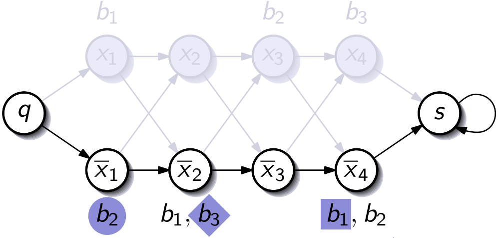


and the $L(\{\mathbf{F}\}, \{\wedge\})$-formula $\mathbf{F} b_1 \wedge \mathbf{F} b_2 \wedge \mathbf{F} b_3$. □

# An NP-hardness proof

### Theorem

MC($\{\mathbf{U}\}, \emptyset$) is NP-hard.

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- Reduction from 3SAT
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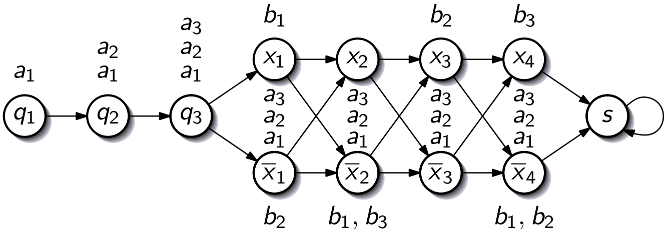


and the $L(\{\mathbf{U}\}, \emptyset)$-formula $((a_1 \mathbf{U} b_1) \mathbf{U} (a_2 \mathbf{U} b_2)) \mathbf{U} (a_3 \mathbf{U} b_3)$ . $\square$

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**Theorem**

$\mathrm{MC}(\{\mathbf{U}\}, \emptyset)$ is NP-hard.

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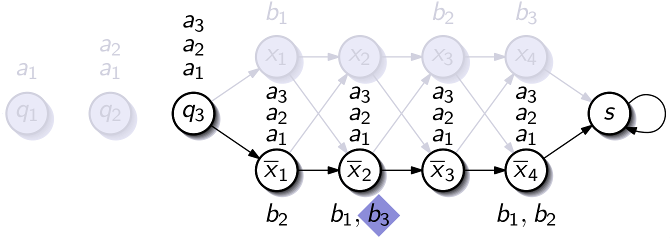


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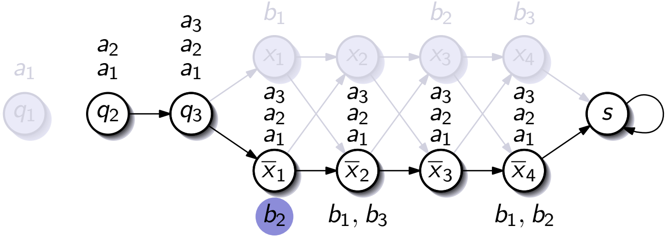
### Theorem

MC($\{\mathbf{U}\}, \emptyset$) is NP-hard.

### Proof sketch.

- Reduction from 3SAT
- From $(x_1 \lor \neg x_2 \lor \neg x_4) \land (\neg x_1 \lor x_3 \lor \neg x_4) \land (\neg x_2 \lor x_4)$ we obtain the model

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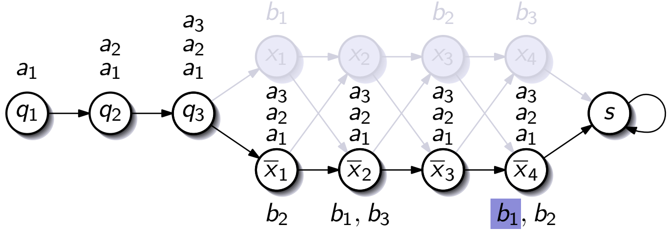


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# Tractability of model-checking: Fragments with **F**,**G**,**X**

## Hardness and completeness of MC($T$, $B$)

| $B$ | I | N | E | V | M | L | BF |
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| $T$ | | $\neg$ | $\wedge$ | $\vee$ | mon. | $\oplus$ | all |
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# An NL-completeness proof

Theorem

MC($\{\mathbf{F}, \mathbf{X}\}, \{\vee\}$) is NL-complete.

Proof sketch.

- NL-hardness: Reduction from the Graph Accessibility Problem
- NL-membership via a logspace computable normal form

  Given $\langle \varphi, M, a\rangle$, transform $\varphi$ into

$$\varphi' = \mathbf{F}\mathbf{X}^{i_1} y_1 \vee \cdots \vee \mathbf{F}\mathbf{X}^{i_n} y_n \quad \vee \quad \mathbf{X}^{i_{n+1}} y_{n+1} \vee \cdots \vee \mathbf{X}^{i_m} y_m \,.$$

  - Guess one of the disjuncts $(\mathbf{F})\mathbf{X}^{i_j}$.
  - Guess the initial section of a path in $M$ from $a$. (Its length is determined by $i_j$.)
  - Check the truth of $(\mathbf{F})\mathbf{X}^{i_j}$ at $a$.

□

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  - Guess the initial section of a path in $M$ from $a$. (Its length is determined by $i_j$.)
  - Check the truth of $(\mathbf{F})\mathbf{X}^{i_j}$ at $a$.

$\square$

# Another good fragment

Theorem

$\mathrm{MC}(\{\mathbf{G},\mathbf{X}\},\{\wedge\})$ is NL-complete.

Proof sketch.

- NL-hardness: as above
- NL-membership:
  Example:
  $(\mathbf{X}b \wedge \mathbf{GX}(\mathbf{G}a \wedge \mathbf{XGXGX}b)) \equiv \mathbf{X}b \wedge \mathbf{XG}a \wedge \mathbf{XXXG}b$
  goal: guess a path with following properties:

| | |
|---|---|
| in state 0: | nothing to check |
| in state 1: | $b$ and $a$ hold |
| in state 2: | $a$ holds |
| in state 3: | $a$ and $b$ hold |
| in state 4: | $a$ and $b$ hold |

$\square$

# Duality

- $\mathrm{MC}(\{\mathbf{F}, \mathbf{X}\}, \{\vee\})$ is NL-complete.
- $\mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, \{\wedge\})$ is NL-complete.

- $\mathrm{MC}(\{\mathbf{F}\}, \{\wedge\})$ is NP-complete.
- $\mathrm{MC}(\{\mathbf{G}\}, \{\vee\})$ is NL-complete,
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# A PSPACE-hardness proof

**Theorem**

For each finite $B$ with $[B] \supseteq \mathrm{M}$: $\mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, B)$ is PSPACE-hard.

**Proof sketch.**

- PSPACE-hardness of $\mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, \{\wedge, \vee\})$ follows from [Markey 2004].
- Every operator in $B$ can be represented by a *short* $\wedge, \vee$-formula.
- Hence, $\mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, \{\wedge, \vee\}) \leq_{\mathrm{m}}^{\log} \mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, B)$.

□

# General results: lower bounds

**Lemma (lower bounds are inherited to larger clones)**

Let $B \subseteq \{\wedge, \vee, \neg\}$ and $B \subseteq [C]$.
Then $\mathrm{MC}(T, B) \leq_m^{log} \mathrm{MC}(T, C)$.

specific: $\mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, \{\vee\})$ is NP-hard.

general: Let $C$ be a finite set of Boolean functions such that $\{\vee\} \subseteq [C]$.
Then $\mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, C)$ is NP-hard.

specific: $\mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, \{\vee, \wedge\})$ is PSPACE-complete.

general: Let $C$ be a finite set of Boolean functions such that $\{\vee, \wedge\} \subseteq [C]$.
Then $\mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, C)$ is PSPACE-hard.

# General results: upper bounds

### Fear

Upper bounds are not necessarily inherited to smaller clones.

Does $\mathrm{MC}(\{\mathbf{G}, \mathbf{X}\}, C) \in \mathrm{PSPACE}$ hold for every $C$ ?

Some upper bounds can be generalized. For example:

specific: $\mathrm{MC}(\{\mathbf{F}, \mathbf{X}\}, \{\vee\})$ is in NL.

general: Let $C$ be a finite set of Boolean functions such that $[C] \subseteq [\{\vee\}]$.
Then $\mathrm{MC}(\{\mathbf{F}, \mathbf{X}\}, C)$ in NL.

# Tractability of model-checking: Fragments with **F**,**G**,**X**

## Hardness and completeness of MC($T$, $B$)

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| **G** | **NL** | **NL** | **NL** | **NL** | NP | | NP |
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# Tractability of model-checking: Fragments with **S**, **U**

## Hardness and completeness of MC($T$, $B$)

| $B$ | I | N | E | V | M | L | BF |
|---|---|---|---|---|---|---|---|
| $T$ | | $\neg$ | $\wedge$ | $\vee$ | mon. | $\oplus$ | all |
| **S** | L | L | L | L | L | L | L |
| **SX** | NP | NP | NP | NP | NP | NP | NP |
| **SG** | NP | NP | NP | NP | PS | NP | PS |
| **SF** | **NL** | NP | NP | **NL** | PS | NP | PS |
| **SFG** | NP | NP | NP | NP | PS | NP | PS |
| **SFX** | NP | NP | NP | NP | PS | NP | PS |
| **SGX** | NP | NP | NP | NP | PS | NP | PS |
| **SFGX** | NP | NP | NP | NP | PS | NP | PS |
| *other* | NP | NP | NP | NP | PS | NP | PS |

# Conclusion

## Achieved

- Separated model-checking problems for almost all LTL fragments into

  good (efficiently solvable) and bad (NP-hard).

- Established the exact complexity of all good fragments.

## Open questions

- LTL fragments with $\oplus$ (ugly)
- upper bounds e.g. for $\mathrm{MC}(\{\mathbf{U}\}, \emptyset)$
- exact complexity of bad fragments
- CTL ...

# Conclusion

## Achieved

- Separated model-checking problems for almost all LTL fragments into

  good (efficiently solvable) and bad (NP-hard).
- Established the exact complexity of all good fragments.

## Open questions

- LTL fragments with $\oplus$ (ugly)
- upper bounds e.g. for $\mathrm{MC}(\{\mathbf{U}\}, \emptyset)$
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- CTL ...

# Related work

## Achieved

- Complete classification of satisfiability for all fragments of CTL*

- Partial classification of reasoning in fragments of default logic
  - existence of a stable extension
  - credulous reasoning
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# Thanks

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**Thank you!**