

Name: \_\_\_\_\_

Pid: \_\_\_\_\_

1. (40 points) Check all the correct statements.

- The statements  $\neg(p \wedge (q \vee p))$  and  $\neg p$  are equal.
- The negation of the statement  $(p \vee q) \wedge (q \vee \neg r)$  is equal to  $(\neg p \wedge \neg q) \vee (\neg q \wedge r)$ .
- The sets  $\{2k, -2k \mid k \in \mathbb{N}\}$  and  $\{2k \mid k \in \mathbb{Z}\}$  are equal.
- The sets  $\{2k \mid k \in \mathbb{Z}\} \cup \{3k \mid k \in \mathbb{Z}\}$  and  $\{6k \mid k \in \mathbb{Z}\}$  are equal.

2. (10 points) Let us consider three-points geometry, it is a theory with undefined terms: point, line, is on, and axioms:
1. There exist exactly three points.
  2. Two distinct points are on exactly one line.
  3. Not all the three points are collinear i.e. they do not lay on the same line.
  4. Two distinct lines are on at least one point i.e. there is at least one point such that it is on both lines.

Show that there are exactly three lines.

3. (10 points) Let  $a_0 = 2$ ,  $a_1 = 5$ , and  $a_n = 5a_{n-1} - 6a_{n-2}$  for all integers  $n \geq 2$ . Show that  $a_n = 3^n + 2^n$  for all integers  $n \geq 0$ .

4. (10 points) Show that  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$  for all integers  $n \geq 1$ .