

Name: _____

Pid: _____

Note that every statement in the homework should be proved.

The only exceptions are statements that were proven in previous homework or midterms and statements proven earlier in the class.

1. Alice and Bob has several apples and bananas and they want to split these fruits. Both of them like both fruits, but value them differently. For Alice, 1 apple is exactly equivalent to 2 bananas. For Bob, 2 apples are exactly equivalent to 1 banana.

Show that the way to split all the fruits is Pareto optimal if and only if

- either Alice has no bananas
- or Bob has no apples.

2. Let us consider a modified game of Nim: on each turn a player may remove some number of pebbles from one pile or split this pile into two piles. Compute the Grundy function for this game for all the initial states with one pile.