Catch up Review

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The Midterm Rules

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- You cannot use calculators, books, computers, or phones. But **you can use a cheatsheet**: one handwritten side of a sheet of paper.
- **Your seats are already assigned to you;** you can find your seat on TritonEd.

Preparations to the Midterm

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- Make sure that your know how to solve **every** problem from your quizzes.
- Make sure that you understand statements of **all** theorems from the course.
- Solve few problems from MyMathLab or from the book.

If I am not Prepared

Visit me or your TA during the office hours and ask questions!

Problems

PROBLEM

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix}$ and $T(x) = Ax$.

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SOLUTION

T is a onto transformation iff columns of A span $\mathbb{R}^3$.

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Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix}$ and $T(x) = Ax$. Is T a onto transformation?

SOLUTION

T is a onto transformation iff columns of A span $\mathbb{R}^3$. In order to check this we have to chek that $Ax = b$ has a solution for every $b \in \mathbb{R}^3$.

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Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 2 \\ 2 & 4 & 3 \end{bmatrix}$ and $T(x) = Ax$. Is T a onto transformation?

SOLUTION

Let us transform $\begin{bmatrix} 1 & 2 & 3 & b_1 \\ -1 & -1 & 2 & b_2 \\ 2 & 4 & 3 & b_3 \end{bmatrix}$ into reduced echelon form.

$$\begin{bmatrix} 1 & 2 & 3 & b_1 \\ 0 & 1 & 5 & b_2 + b_1 \\ 0 & 0 & -3 & b_3 - 2b_1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 & 3 & b_1 \\ 0 & 1 & 5 & b_2 + b_1 \\ 0 & 0 & -3 & b_3 - 2b_1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & b_1 \\ 0 & 1 & 5 & b_2 + b_1 \\ 0 & 0 & 1 & \frac{2b_1 - b_3}{3} \end{bmatrix}$$

Problems

PROBLEM

Let us consider the following system

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ 2x_1 - x_3 &= 4 \\ -x_1 + 2x_2 - x_3 &= 6 \end{aligned}$$

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Find solutions in parametric vector form.

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$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & 0 & -1 & 4 \\ -1 & 2 & -1 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -2 \\ 0 & 3 & 0 & 9 \end{bmatrix}$$

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Let us consider the following system

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ 2x_1 - x_3 &= 4 \\ -x_1 + 2x_2 - x_3 &= 6 \end{aligned}$$

Find solutions in parametric vector form.

SOLUTION

Hence, $x_3 = -\frac{4}{3}$, $x_2 = 3$, and $x_1 = \frac{4}{3}$. In parametric vector form solution is $\begin{bmatrix} 4/3 \\ 3 \\ -4/3 \end{bmatrix}$.

Problems

PROBLEM

Let us consider the homogeneous version of the previous system

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ 2x_1 - x_3 &= 0 \\ -x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

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Any solution of this system plus some solution of the pervious system is a solution of the previous system.

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Find solutions in parametric vector form.

SOLUTION

Any solution of this system plus some solution of the pervious system is a solution of the previous system. Hence this system has only trivial solution, since there is only one solution of the previous system.

Problems

PROBLEM

Let $T_1(x) = \begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & 1 \\ 4 & 3 & -2 \end{bmatrix} x$ and $T_2(x) = \begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix} x$

Find the standard matrix for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined as follows $T(x) = T_2(T_1(x))$.

Problems

SOLUTION

The standard matrix of T_1 is $\begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & 1 \\ 4 & 3 & -2 \end{bmatrix}$ and standard matrix of T_2 is $\begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix}$.

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The standard matrix of T_1 is $\begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & 1 \\ 4 & 3 & -2 \end{bmatrix}$ and standard matrix of T_2 is $\begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix}$.

Hence the standart matrix of $T_2(T_1(x))$ is

$$\begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & 1 \\ 4 & 3 & -2 \end{bmatrix}$$

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The standard matrix of T_1 is $\begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & 1 \\ 4 & 3 & -2 \end{bmatrix}$ and standard matrix of T_2 is $\begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix}$.

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$\begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & 1 \\ 4 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -6 & -5 & 0 \\ 39 & 30 & -15 \end{bmatrix}$ Since it is equivalent to

$$\left[\begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -5 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \right].$$

Problems

PROBLEM

Let $S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \end{bmatrix} \right\}$.

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Let $S = \left\{ \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ k \end{bmatrix} \right\}$. Is S linearly dependent?

SOLUTION

Let us transform the following matrix into reduced echelon form

$$\begin{bmatrix} 1 & 2 & -3 \\ -2 & -3 & 4 \\ 5 & 1 & k \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 & -3 \\ -2 & -3 & 4 \\ 5 & 1 & k \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -2 \\ 0 & -9 & k+15 \end{bmatrix}$$

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Hence S is linearly dependant iff $k = 3$.

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SOLUTION

Note that the following sustem has solution for all b iff $k \neq 3$

$$\begin{bmatrix} 1 & 2 & -3 & b_1 \\ 0 & 1 & -2 & b_2 \\ 0 & 0 & k-3 & b_3 \end{bmatrix}$$

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Hence S spans $\mathbb{R}^3$ iff $k = 3$.