Catch up Review

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- **Your seats are already assigned to you;** you can find your seat on TritonEd.

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But remember that problems in the midterm will be different!

If I am not Prepared?

Visit me or your TA during the office hours and ask questions!

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$$\begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ -2 & 0 & 2 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 4 & 0 & 2 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0.5 & 0.25 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0.5 & 0.25 & 0 \\ 0 & 0 & 2 & 0.5 & 0.25 & 1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0.5 & 0.25 & 0 \\ 0 & 0 & 1 & 0.25 & 0.125 & 0.5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 1.25 & 0.125 & 0.5 \\ 0 & 1 & 0 & 0.5 & 0.25 & 0 \\ 0 & 0 & 1 & 0.25 & 0.125 & 0.5 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 0.25 & -0.375 & 0.5 \\ 0 & 1 & 0 & 0.5 & 0.25 & 0 \\ 0 & 0 & 1 & 0.25 & 0.125 & 0.5 \end{bmatrix}$$

Vector spaces

1. Is the set of all functions $f\colon \mathbb{R} \to \mathbb{R}$ such that $f(-x) = -f(x)$ for all $x \in \mathbb{R}$ is a subspace of the space of all functions from $\mathbb{R} \to \mathbb{R}$?

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    - $\mathbf{0}(-x) = -\mathbf{0}(x)$,
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    - $\mathbf{0}(-x) = -\mathbf{0}(x)$,
    - if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$, then $(cf)(-x) = xf(-x) = -cf(x)$, and
    - if $f(-x) = -f(x)$ and $g(-x) = -g(x)$ for all $x \in \mathbb{R}$, then $(f+g)(-x) = f(-x)+g(-x) = -f(x)-g(x) = -(f+g)(x)$

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2. Is the set of all vectors $\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$ such that $xy = 0$ is a subspace of $\mathbb{R}^2$.

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2. Is the set of all vectors $\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$ such that $xy = 0$ is a subspace of $\mathbb{R}^2$. Not it is not. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ belong to this set, however their sum not. Hence this set is not closed under addition.

Rank and Column Spaces

Let A and B be $n \times n$ matrices such that $Ax = 0$ iff for some $y \in \mathbb{R}^n$, $x = By$ and let $\dim \text{Row } A = n - 4$.

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We know that $\dim \text{Col } A = \dim \text{Row } A = n - 4$. We also know that $\dim \text{Col } A + \dim \text{Nul } A = n$. Since $Ax = 0$ iff for some $yin\mathbb{R}^n$, $x = By$, $\text{Nul } A = \text{Col } B$. Hence, $4 = \dim \text{Nul } A = \dim \text{Col } B = \text{rank } B$.

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Dimension of a Subspace

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It is easy to see that $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$. Hence $\dim H \leq 2$.

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