

Name: _____

Pid: _____

1. (6 points) Check all of the following that are correct statements

The equation $x_1 + y = z$ is a linear equation in the variables y and z .

The equation $x_1 \cdot x + y = z$ is a linear equation in the variables x , y , and z .

Solution: It is a linear equation since x_1 is a constant, not a variable.

The equation $x_1 \cdot y = z$ is a linear equation in the variables x_1 , y , and z .

Solution: It is not a linear equation because x_1 and y are both variables.

The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is in a reduced echelon form.

Solution: It is not in a reduced echelon form, since the leading element in the third row is 2

The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is in a reduced echelon form.

Solution: It is not in a reduced echelon form, since the second row is a zero row.

The matrix $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ is in a reduced echelon form.

Solution: It is not in a reduced echelon form, since the leading element in the third row is 2

2. (3 points) Is the following system consistent?

$$x_1 + x_2 + 2x_3 = 4$$

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + x_3 = 1$$

$$2x_2 + x_3 = 3$$

Solution: Let us consider an augmented matrix of this system and transform it into reduced echelon form.

$$\begin{aligned} \begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 3 \end{bmatrix} &\sim \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 0 & -1 & -2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 3 \end{bmatrix} \sim \\ &\begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 0 & -1 & -2 \\ 0 & -1 & -1 & 3 \\ 0 & 2 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \\ 0 & 2 & 1 & 3 \end{bmatrix} \sim \\ &\begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & -1 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -1 & -1 & 3 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 11 \end{bmatrix} \end{aligned}$$

Hence, the system is not consistent.

3. (5 points) Write the solution of the vector equation $xa + yb + zc = d$ in the parametric form where

$$a = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, c = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \text{ and } d = \begin{bmatrix} 3 \\ 14 \\ 13 \\ 8 \end{bmatrix}$$

Solution: Let us transform this equation into system of linear equations.

$$\begin{aligned} x + y &= 3 \\ x + 2y + 2z &= 14 \\ x + y + 2z &= 13 \\ x + y + z &= 8 \end{aligned}$$

Let us now consider an augmented matrix of this system and transform it into reduced

echelon form.

$$\begin{aligned} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 1 & 2 & 2 & 14 \\ 1 & 1 & 2 & 13 \\ 1 & 1 & 1 & 8 \end{bmatrix} &\sim \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 11 \\ 1 & 1 & 2 & 13 \\ 1 & 1 & 1 & 8 \end{bmatrix} \sim \\ &\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 11 \\ 0 & 0 & 2 & 10 \\ 1 & 1 & 1 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 11 \\ 0 & 0 & 2 & 10 \\ 0 & 0 & 1 & 5 \end{bmatrix} \sim \\ &\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 11 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \\ &\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

As a result,

$$\begin{aligned} x &= 2 \\ y &= 1 \\ z &= 5 \end{aligned}$$