Name: ____________________

Pid: ____________________

1. (3 points) Are these vectors linearly independent?

$$\begin{bmatrix}1\\2\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\1\end{bmatrix} \quad \begin{bmatrix}1\\0\\2\end{bmatrix}$$

Solution: Let us consider the matrix with columns equal to these vectors.

$$\begin{bmatrix}1&0&1\\2&1&0\\0&1&2\end{bmatrix} \sim \begin{bmatrix}1&0&1\\0&1&-1\\0&1&2\end{bmatrix} \sim \begin{bmatrix}1&0&1\\0&1&-1\\0&0&3\end{bmatrix}$$

Hence, they are linearly independent.

2. (3 points) Write the solution of the following equation in parametric vector form.

$$\begin{bmatrix}1&2&-1&1\\1&1&3&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}4\\3\\2\end{bmatrix}$$

Solution: Let us consider the augmented matrix corresponding to this equation and transform it into the reduced echelon form.

$$\begin{bmatrix}1&2&-1&1&4\\1&1&3&0&3\\0&0&1&0&2\end{bmatrix} \sim \begin{bmatrix}1&2&-1&1&4\\0&-1&4&-1&-1\\0&0&1&0&2\end{bmatrix} \sim$$

$$\begin{bmatrix}1&2&-1&1&4\\0&1&-4&1&1\\0&0&1&0&2\end{bmatrix} \sim \begin{bmatrix}1&2&-1&1&4\\0&1&0&1&9\\0&0&1&0&2\end{bmatrix} \sim$$

$$\begin{bmatrix}1&2&0&1&6\\0&1&0&1&9\\0&0&1&0&2\end{bmatrix} \sim \begin{bmatrix}1&0&0&-1&-12\\0&1&0&1&9\\0&0&1&0&2\end{bmatrix}$$

Hence, the asnwer is

$$\begin{bmatrix}-12\\9\\2\\0\end{bmatrix} + x_4\begin{bmatrix}1\\-1\\0\\1\end{bmatrix}$$

3. (3 points) Does the following system have nontrivial solution?

$$\begin{bmatrix}1&2&3&1\\1&0&2&1\\0&1&1&1\\1&2&3&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

Solution: Let us consider the coefficient matrix corresponding to this equation and transform it into the reduced echelon form.

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -2 & -1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So there are no nontrivial solutions.