Name: ______________________

Pid: ______________________

1. (2 points) Check all of the following that are correct statements

☐ The matrix $\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$ is invertible.

> **Solution:** No, this matrix is not invertible since $2 \cdot 3 - 1 \cdot 6 = 0$.

■ The matrix $\begin{bmatrix} 1 & 6 \\ 1 & 3 \end{bmatrix}$ is invertible.

> **Solution:** Yes, this matrix is invertible since $1 \cdot 3 - 1 \cdot 6 = 3$.

2. (3 points) Is the following matrix invertible?

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

If yes, write the inverse of this matrix.

> **Solution:** Let us use the algorithm presented in the class and reduce the following matrix into reduced echelon form.
>
> $$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix} \sim$$
>
> $$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{bmatrix} \sim$$
>
> $$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1/2 & 1/2 & 1/2 \\ 0 & 1 & 0 & -1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{bmatrix}$$
>
> Hence, the matrix is invertible and the inverse is equal to
>
> $$\frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

3. (3 points) Check all of the following that are correct statements

☐ The set of all positive functions (i.e. functions f such that for any $x \in \mathbb{R}$, $f(x) > 0$) with standard operations is a vector space.

> **Solution:** No it is not a vector space since $x^2 + 1$ belongs to this set, but $(-1)(x^2 + 1)$ does not.

■ The set of all functions f such that $f(0) = 0$, with standard operations is a vector space.

> **Solution:** It is easy to see that, the zero function belongs to this set. Additionaly, if $f(0) = 0$ and $g(0) = 0$, then $(f+g)(0) = f(0)+g(0) = 0$ and $(cf)(0) = cf(0) = 0$. Hence this set is closed under addition and multiplication.

□ The set of all pairs of integers with standard operations is a vector space.

> **Solution:** No it is not a vector space since $\begin{bmatrix}1\\1\end{bmatrix}$ belongs to this set, but $\frac{1}{2}\begin{bmatrix}1\\1\end{bmatrix}$ does not.