Name: ______________________

Pid: ______________________

1. (3 points) Find a basis of the column space of the matrix

$$\begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & 1 \\ -1 & -3 & -1 \end{bmatrix}$$

Solution: Let us transform the matrix into reduced echelon form.

$$\begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & 1 \\ -1 & -3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 5 & -5 \\ 0 & -1 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & -1 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & -5 \end{bmatrix}$$

Hence, all columns are pivot columns and the basis of the column space is the following.

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix} \right\}$$

2. (3 points) Is the set $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 8 \\ -2 \\ 1 \\ 9 \end{bmatrix} \right\}$ is a basis of its span? If not, find a basis of the space Span S.

Solution: It is easy to see that

$$2\begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix} + 3\begin{bmatrix} 2 \\ -2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 1 \\ 9 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 2 \\ -2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 1 \\ 5 \end{bmatrix},$$

Hence, the basis of the space is

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \\ 1 \end{bmatrix} \right\}$$

3. (3 points) Let $\mathfrak{B} = \left\{ \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 4 \\ -2 \end{bmatrix} \right\}$ be a basis of $\mathbb{R}^3$ find $[x]_{\mathfrak{B}}$ where $x = \begin{bmatrix} 12 \\ 2 \\ 13 \end{bmatrix}$.

Solution: Let us trusform the following matrix into reduced echelon form

$$\begin{bmatrix} 1 & 2 & 8 & 12 \\ -3 & 1 & 4 & 2 \\ 2 & 1 & -2 & 13 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 8 & 12 \\ 0 & 7 & 28 & 38 \\ 0 & -3 & -18 & -11 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 2 & 8 & 12 \\ 0 & 1 & 4 & \frac{38}{7} \\ 0 & 1 & 6 & \frac{11}{3} \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 8 & 12 \\ 0 & 1 & 4 & \frac{38}{7} \\ 0 & 0 & 2 & \frac{11}{3} - \frac{38}{7} \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 2 & 0 & 12 - \frac{44}{3} + 4\frac{38}{7} \\ 0 & 1 & 0 & 3\frac{38}{7} - \frac{22}{3} \\ 0 & 0 & 2 & \frac{11}{3} - \frac{38}{7} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 12 - \frac{22}{3} + 3\frac{38}{7} \\ 0 & 1 & 0 & 3\frac{38}{7} - \frac{22}{3} \\ 0 & 0 & 2 & \frac{11}{3} - \frac{38}{7} \end{bmatrix}$$

Hence,

$$[x]_{\mathfrak{B}} = \begin{bmatrix} \frac{8}{7} \\ \frac{188}{21} \\ -\frac{37}{42} \end{bmatrix}$$