Name: ____________________

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1. (3 points) Find eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & 19 & 17 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

> **Solution:** Note that the characteristic polynomial of this matrix is $(1-\lambda)\left((2-\lambda)^2 - 1\right) = (1-\lambda)(3-4\lambda+\lambda^2) = (1-\lambda)^2(3-\lambda)$. Hence, the eigenvalues of this matrix are 1 and 3.

2. (3 points) Compute the area of the parallelogram with vertices $(2, 1)$, $(1, 4)$, $(3, 5)$, $(2, 8)$.

> **Solution:** The vectors corresponding to this parallelogram are $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.
>
> Hence, the area of this parallelogram is equal to $\left| \det \begin{bmatrix} -1 & 1 \\ 3 & 4 \end{bmatrix} \right| = 7$.

3. (3 points) Find a basis of the eigenspace corresponding to 1 of the matrix $A = \begin{bmatrix} -9 & -4 & -12 \\ 10 & 5 & 12 \\ 5 & 2 & 7 \end{bmatrix}$.

> **Solution:** In order to find the basis, let us solve the system $\begin{bmatrix} -9 & -4 & -12 \\ 10 & 5 & 12 \\ 5 & 2 & 7 \end{bmatrix} x = x$. It is equivalent to $\begin{bmatrix} -10 & -4 & -12 \\ 10 & 4 & 12 \\ 5 & 2 & 6 \end{bmatrix} x = 0$. Hence, the solution in the parametric vector form is $x = x_2 \begin{bmatrix} -0.4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -0.6 \\ 0 \\ 1 \end{bmatrix}$. As a result the basis of the space is $\left\{ \begin{bmatrix} -0.4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1.2 \\ 0 \\ 1 \end{bmatrix} \right\}$.