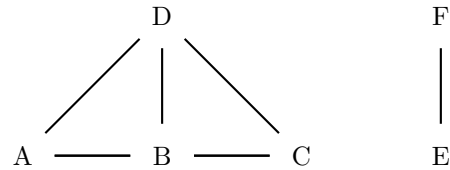


Name: _____

Pid: _____

1. (80 points) Check all the correct statements.

- The number of different strings you can get by reordering letters in the word aabbc is 30.
- The following graph is connected.



- There are 32 different strings of length 5 over the alphabet with 2 letters.
- There are 4 different surjective functions from $[4]$ to $[4]$.
- There are 3 ways to put 4 identical balls into 3 different boxes such that all the boxes are not empty.
- A graph on 4 vertices has at most 6 edges.
- A disconnected graph on 5 vertices has at most 6 edges.
- If a graph on 5 vertices has 3 edges it should be disconnected.

2. (10 points) Let A_1, \dots, A_ℓ be different subsets of $[n]$ such that $A_i \cap A_j \neq \emptyset$ for all $i \neq j \in [\ell]$. Prove that $\ell \leq 2^{n-1}$.

3. (10 points) In Durmstrang Institute every semester consists of two parts with two midterms in each of them. Find a closed formula for number of ways to organize semester if there are n days in this semester.

4. (10 points) Prove the following equality:

$$S(n, k) = \sum_{i=1}^n S(n-i, k-1) \cdot k^{i-1}.$$

5. (10 points) Show that the following equality is always true:

$$\binom{n+m}{k} = \sum_{i=0}^k \binom{n}{i} \cdot \binom{m}{k-i}.$$

6. (10 points) Let $i_1, \dots, i_k \in [n]$. Find number of permutations π such that i_1, \dots, i_k are in the same cycle in π .