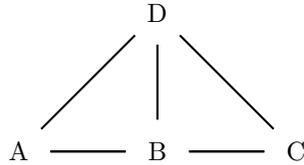Name: _____

Pid: _____

1. (80 points) Check all the correct statements.

☐ There are 6 permutations of [4] with 3 cycles.

☐ The number of different strings you can get by reordering letters in the word aabbc is 30.

☐ The following graph is connected.

D

A ———— B ———— C

☐ There are 125 different strings of length 5 over the alphabet with 3 letters.

☐ If you have 26 balls in 5 boxes, then there is a box with at least 6 balls.

☐ There are 6 different surjective functions from [3] to [3].

☐ The following graph has an Euler path.

D

A ———— B ———— C

E
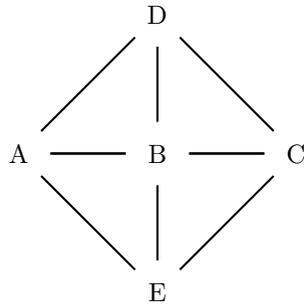
☐ There are 15 variants to put 4 identical balls into 3 different boxes.

2. (10 points) Let $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$, $a_1 = 2$, and $a_0 = 1$. Find a closed formula (no summation signs) for the recurrent sequence a_n.

3. (10 points) Let $i_1, \ldots, i_k \in [n]$ be some different integers. Find a closed formula (no summation signs) for number of permutations π such that

$$\pi^{-1}(i_1) \le \pi^{-1}(i_2) \le \cdots \le \pi^{-1}(i_k).$$