Worst-case upper bounds for SAT: automated proof

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February 22, 2003

Abstract

Propositional satisfiability (SAT), an NP-hard problem, can certainly be solved in less than 2^K steps, where K denotes the number of clauses in the input formula, for instance, with the help of DPLL-type SAT solvers. In this paper, describing a work in progress, we deal with automated proofs of upper bounds on the running time of DPLL-type algorithms for SAT.

For a class of algorithms working with the pure literals elimination, we prove a previously unknown upper bound w.r.t. the number of clauses in the input formula. We also note that the employed line of reasoning cannot lead to better results.

1 Introduction

The problem of propositional satisfiability is known to be NP-complete, and, therefore, it is highly unlikely that a polynomial-time algorithm for SAT will be discovered. Therefore, the need in "good" exponential-time algorithms arose. The naive algorithm would in the general case take at least $poly(N)2^N$ steps to find a satisfying assignment, where N is a complexity measure such as number of variables, clauses or length of the input formula. Therefore, it has for quite a long time been a challenging problem to prove better and better worst-case upper bounds for various SAT solvers. The current record holder is due to Edward A. Hirsch. He proved in [6] that his (rather complex) algorithm works for the time $2^{0.30897\ldots K}$, where K is the number of clauses in the input formula. We prove a weaker bound, but for a much simpler algorithm.

One of the two main approaches to obtaining less-than-2^N bounds is the local search approach, when an algorithm starts from a random assignment and modifies it to get closer to a satisfying assignment (for references see, for example, [1]).

Algorithms of the other are called DPLL-type, due to the works of Davis, Putnam, Logemann and Loveland ([3], [2]). The DPLL-type algorithms build a tree, the so-called *branching tree*, whose nodes are partial assignments to the variables of the input formula. On each step, a DPLL-type algorithm first tries to simplify the input formula, employing a number of *simplification rules*. Then, if the result is not obvious yet, it makes a *splitting*, that is, it chooses a literal l in the input formula, constructs two formulae, one corresponding to $l := true$ and the other to $l := false$, and

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recursively calls itself for these two formulas. If any of the calls returns the answer "Satisfiable", the algorithm also returns this answer. Otherwise, it returns "Unsatisfiable".

During the past decade, described in Section 2, the proofs of worst-case upper bounds for DPLL-type SAT solvers have become quite similar. The point of such proofs is to construct the branching tree and prove that in any possible case of the literals' mutual arrangement in the input formula the algorithm in question will be able to pick a literal, the splitting by which would provide the necessary branching vector with respect to the necessary complexity measure.

The proofs have become less elegant and more technical. Simple combinatorial properties of all the possible cases usually comprise the whole proof (probably [6] is the best example of such a proof). The only "creative" element of these proofs are the elimination rules.

In the sequel we present a successful attempt to automate the proofs of worst-case upper bounds for DPLL-type algorithms. Attempts to prove such bounds usually employ the Kullmann-Luckhardt's lemma, to which Section 2 is devoted. Sections 3 describes the ideology of automating these proofs, Section 4 describes a sample output of our program, and Section 5 presents a new bound proven with the help of a computer, the complete output of whose work is presented in the appendix.

2 Basic definitions and Kullmann-Luckhardt's lemma

We denote by X a set of boolean variables. The negation of a variable x is denoted by $\overline{x}$. If $U \subseteq X$, then $\overline{U} = \{\overline{x} \mid x \in U\}$. *Literals* are members of the set $X \cup \overline{X}$. A *clause* is a set of literals that does not contain simultaneously any variable together with its negation. A *formula in CNF* is a finite set of clauses. A clause is called *unit* if it consists of one literal.

We count the number of appearances of a chosen literal in the formula. We call l an (i, j)-*literal* with respect to a formula F, if F contains i appearances of l and j appearances of $\overline{l}$. We call l an $(i, j+)$-*literal* with respect to a formula F, if F contains i appearances of l and at least j appearances of $\overline{l}$. A literal is called *pure* with respect to a formula if the formula contains only the literal, but does not contain its negation.

An *assignment* is a finite subset $I \subseteq X \cup \overline{X}$ that does not contain any variable together with its negation. We denote by $F[I]$ a formula that results from F and an assignment $I = \{x_1, x_2, \ldots, x_n\}$ after removing all clauses containing the literals x_i and deleting all occurrences of the literals $\overline{x_i}$ from the other clauses. An assignment I is said to *satisfy* the formula F, if $F[I]$ is the empty formula (that is, $F[I]$ contains no clauses).

Basically, all one needs to prove worst-case upper bounds on such algorithms is to solve recurrent equations. However, the job was significantly simplified (and made available for automatization) by O. Kullmann and H. Luckhardt. The notion of a branching tree was introduced by them in [7]. The branching tree in our case is not binary, though the number of sons of a node is limited to eight. We attach to each formula (each node of the tree) a non-negative integer $\mu(F)$, which denotes the *complexity* of F. In the sequel we prove an upper bound with respect to the number of clauses in F: $\mu(F) = \mathcal{K}(F)$.

Let us now consider a node of the branching tree (labeled with a formula F) and denote its sons by $F_1, \ldots, F_m$. We call a *branching vector of a node* an m-tuple $(t_1, \ldots, t_m)$, where $t_i = \mu(F) - \mu(F_i)$. Note that the following makes sense only when each of the resulting formulae is "simpler" that its predecessor, that is, when $\mu(F) \geq \mu(F_i)$.

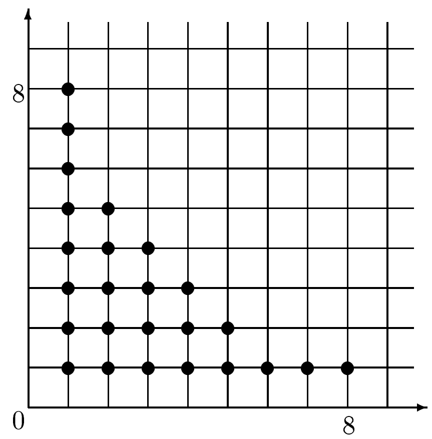

Figure 1: Branching numbers $(i,j) \in \mathbb{N} \times \mathbb{N}$ such that $\tau(i,j) \geq \tau(3,4)$

The *characteristic polynomial* of a branching vector $t = (t_1, \ldots, t_m)$ is defined by

$$h_t(x) = 1 - \sum_{i=1}^{m} x^{-t_i}.$$

This polymonial has exactly one positive root (since it is a monotone function on $(0, +\infty)$). Following [6], we denote this root by $\tau(t) = \tau(t_1, t_2, \ldots, t_m)$ and call it the *branching number.*

The branching numbers have quite a few remarkable properties. For our purposes two of them, presented in [7], are necessary.

Lemma 1 (Kullmann and Luckhardt [7]). *Let T be a branching tree, and let its root be labeled with a formula F. Then the number of leaves in T does not exceed $(\tau_{max,T})^{\mu(F)}$, where $\tau_{max,T}$ is the largest of all the branching numbers of the nodes of T.*

Lemma 2 (Kullmann and Luckhardt [7]). *Let $m, k \in \mathbb{N}$, $x_1, x_2, \ldots, x_m$, $y_1, y_2, \ldots, y_k \in \mathbb{Q}_+$. The problem whether $\tau(x_1, \ldots, x_m)$ is not greater than $\tau(y_1, \ldots, y_k)$ is solvable in time polynomial of* $\max(x_1, \ldots, x_m, y_1, \ldots, y_k)$.

These lemmata allow us to estimate the running time of a DPLL-type algorithm. Indeed, if we know the branching number of its splitting tree ($\tau_{max,T}$) and provided it processes each of its leaves in polynomial time, Lemma 1 gives a worst-case upper bound on its running time as $poly(\mu(F))\tau_{max,T}^{\mu(F)}$, where F is its input formula and μ an arbitrary complexity measure, strictly decreasing with each splitting.

Besides, Lemma 2 allows the algorithm to estimate the branching numbers in polynomial time and choose the next literal according to the suitable branching number.

3 Programming issues

The main idea of the program was to introduce in it the notion of an *unknown literal.* The program actually looks at a rather small piece of formula, just big enough to hold all the occurrences of several literals. The rest of the literals is supposed unknown; they are not subject to elimination rules.

First of all, the program reduces the possibilities for positive and negative occurrences of the literals to a certain finite subset of $\mathbb{Z} \times \mathbb{Z}$, depending on the target branching number. It cuts off the possibilities when splitting on a single literal without any elimination rules would already provide the necessary branching vector (see an example on Fig. 1).

Then the program goes through all the remaining possibilities of i and j for a literal to be an (i, j)-literal and tries to reduce the resulting formula. If it does not succeed, it initializes another literal and tries to split on it, achieving the target branching vector.

4 Lower bounds for this type of reasoning

Of course, this kind of reasoning has its limitations. We present here an example of a formula that is very simple for any solver by itself, but constitutes an unavoidable obstacle for proving for the solver based on pure literals elimination anything better than the bound proven in the following section.

Consider the following formula:

$$G(x_1, x_2, x_3) = (\overline{x_1} \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee x_2 \vee \overline{x_3}).$$

Note that in the terms of Kullmann-Luckhardt's lemma, whichever literal we split on first, we produce a $(1, 3)$-split (that is, one clause is eliminated when x_i is set to $false$ and all three clauses disappear when $x_i := true$). The second splitting (we have to consider it only in one case, since in the other the whole formula disappears) would inevitably reduce the formula to an empty one.

Now let us consider the formula

$$F(x_1, \ldots, x_n) = G(x_1, x_2, x_3) \wedge G(x_4, x_5, x_6) \wedge \ldots \wedge G(x_{n-2}, x_{n-1}, x_n).$$

Due to the symmetry of the formula and the independence of subformulas with three variables, we can rearrange the splittings made by the pure literals elimination algorithm in any suitable order (provided we preserve the order of every pair of splittings made in any independent subformula). Let us rearrange them in such a way that splittings made by variables occurring in each subformula appear one after another. Then it is easy to see that every sequence of two splittings result in a branching vector with a branching number of $\tau(3, 3, 3) = 1.442250\ldots$.

Note that the formula itself is remerkably simple, because every assignment is a satisfiable assignment, and it may not be considered a hard instance for any solver. However, the line of reasoning along which the automated search is going would inevitably fail at this point.

5 Proof of the $1.442250\ldots^K = 2^{0.528321\ldots K}$ upper bound

We present here the formal proof of a previously unknown upper bound for the same solver, working with pure literals elimination only. We begin with several lemmata. Throughout this section we denote by F the formula in the current node of the branching tree and fix the number of clauses as the complexity measure. The bound in question is obtained as $\tau(3, 3, 3) = 1.442250\ldots$.

Lemma 3. *If F contains a $(1+,4+)$-literal or a $(2+,2+)$-literal, then its branching number is greater than the necessary bound.*

Proof. Straightforward calculations. $\square$

Lemma 4. *For any input formula F, one of the following alternatives hold:*

- *After applying to F the pure literals elimination rule, F will be either empty or contain an empty clause;*
- *There exists a clause in F such that it has more than one literal in it (a non-unit clause).*

Proof. If the second alternative does not hold, F consists of unit clauses. Then, if there is an $(1+,1+)$-literal in F, F will contain an empty clause after reduction. If not, F will be empty. $\square$

Lemma 5. *If F contains a non-unit clause, does not contain pure literals and does not contain $(1+,4+)$-literals or $(2+,2+)$-literals, then it is possible to pick two literals l_1, l_2 in F such that $\tau(\mathcal{K}(F)-\mathcal{K}(F[\overline{l_1}]_{red}), \mathcal{K}(F)-\mathcal{K}(F[l_1,\overline{l_2}])_{red}, \mathcal{K}(F)-\mathcal{K}(F[l_1,l_2])_{red}) \geq \tau(3,3,3)$ or a single literal l such that $\tau(\mathcal{K}(F)-\mathcal{K}(F[\overline{l}]_{red}), \mathcal{K}(F)-\mathcal{K}(F[l]_{red}) \geq \tau(3,3,3)$, where F_{red} denotes the result of applying the pure literals rule to F.*

Proof. The automated proof of this statement is supplied in the appendix. We denote the first literal in the existing non-unit clause by 1 and the, by 2, and consider all possibilities of their mutual arrangement (provided, by Lemma 3, that each of them is either a $(1,1)$-literal or a $(1,2)$-literal). $\square$

All of the above, after applying Lemmata 1 and 2, proves the following

Theorem 1. *The described DPLL-type algorithm with pure literals elimination as the only simplification rule gives the correct answer in the time not exceeding $poly(K)2^{0.64743\ldots K}$, where K is the number of clauses in the input formula.*

6 Further work

One direction for further work in this field is obvious: we will implement other elimination rules and obtain better bounds. Other known elimination rules include elimination by resolution, unit clause elimination, blocked clauses elimination and the black-and-white literals rule (for an example where all of them are used see [6]). It would be also interesting to try to prove bounds w.r.t. other complexity measures, the most interesting being the number of variables. We are currently working on it, and on this way we expect some new results and, possibly, records.

Another direction is to extend these methods to other NP-complete problems. There exist algorithms for other problems that employ similar splitting methods and bounds on which also have been achieved with the help of Kullmann-Luckhardt's lemma. An example of such algorithms for MAX-2-SAT and MAX-CUT can be found in [5], and the current record holder for MAX-CUT ([4]) also uses such techniques.

And the last but not least. It has been an equally challenging problem to obtain exponential lower bounds for such algorithms. Previous attempts usually employed graph-theoretical techniques ([9],[10],[11],[8]). The automated search for upper bounds may yield explicit formulae that are hard for DPLL-type algorithms: after all, it does not succeed every time, and analysis of its output when a bound cannot be proven might turn out to be very useful. For example, this output may provide the so-called "bottleneck cases", i.e. explicit formulae where the algorithm performs with complexity close to its proven worst-case upper bound.

Acknowledgements

The authors would like to express gratitude to Edward A. Hirsch, who posed the problem and supervised our work.

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Appendix

We quote here the actual proof of Lemma 5 supplied by the program. Whenever the algorithm deals with a formula on which it cannot prove the necessary bound, it introduces a new literal.

We work here in 5-CNF to make sure that the algorithm never eliminates a clause just because it had three literals. Since no line of the proof makes use of more than three literals (and, therefore, at least one unknown literal remains in every clause), no clauses are eliminated in this way.

`Proving bound:` $\tau(3,3,3) = 1.442250\ldots$.

Case 1: $1 * * * *$ | $-1 * * * *$ |
`Heuristic codenumber is 1 for literal 2`

Case 1.1: $1\ 2 * * *$ | $-1\ -2 * * *$ |
`Bound for this case is (2,2)=1.414214:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 1.2: $1\ 2 * * *$ | $-1 * * * *$ | $-2 * * * *$ |
`Can't prove bound.  Try subcases:`

`Heuristic codenumber is 2 for literal 3`

Case 1.2.1: $1\ 2\ -3 * *$ | $-1\ 3 * * *$ | $-2 * * * *$ |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 1.2.2: $1\ 2 * * *$ | $-1\ 3 * * *$ | $-2\ -3 * * *$ |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 1.2.3: $1\ 2 * * *$ | $-1\ 3 * * *$ | $-2 * * * *$ | $-3 * * * *$ |
`Bound for this case is (2,2)=1.414214:`
$F_1 = F[-1]:$ $2 * * *$ | $-2 * * * *$ |
$F_2 = F[1]:$ $3 * * *$ | $-3 * * * *$ |
`Heuristics no.  2 bound for literal 3 is 1.442250`
`Heuristics no.  1 bound for literal 2 is 1.442250`

Case 2: $1 * * * *$ | $-1 * * * *$ | $-1 * * * *$ |
`Heuristic codenumber is 1 for literal 2`

Case 2.1: $1\ 2 * * *$ | $-1\ -2 * * *$ | $-1 * * * *$ |
`Bound for this case is (3,2)=1.324718:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ $* * * *$ |

Case 2.2: $1\ 2 * * *$ | $-1 * * * *$ | $-1 * * * *$ | $-2 * * * *$ |
`Bound for this case is (2,2)=1.414214:`

$F_1 = F[-1]:$ 2 ∗ ∗ ∗ | −2 ∗ ∗ ∗ ∗ |
$F_2 = F[1]:$ ∗ ∗ ∗ ∗ | ∗ ∗ ∗ ∗ |

Case 2.3: 1 2 ∗ ∗ ∗ | −1 −2 ∗ ∗ ∗ | −1 −2 ∗ ∗ ∗ |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4: 1 −2 ∗ ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 ∗ ∗ ∗ |
`Can't prove bound. Try subcases:`

`Heuristic codenumber is 0 for literal 3`

Case 2.4.1: 1 −2 | −1 2 | −1 −2 |
`Bound for this case is (3,3,3)=1.442250:`
$F_1 = F[-1]:$
$F_2 = F[1,-2]:$
$F_3 = F[1,2]:$ `Maximum number of literals achieved. Current upper bound is 1.442250`

Case 2.4.2: 1 −2 3 ∗ ∗ | −1 2 −3 ∗ ∗ | −1 −2 ∗ ∗ ∗ |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.3: 1 −2 3 ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 ∗ ∗ |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.4: 1 −2 −3 ∗ ∗ | −1 2 3 ∗ ∗ | −1 −2 ∗ ∗ ∗ |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.5: 1 −2 ∗ ∗ ∗ | −1 2 3 ∗ ∗ | −1 −2 −3 ∗ ∗ |
`Bound for this case is (3,3,3)=1.442250:`
$F_1 = F[-1]:$
$F_2 = F[1,-2]:$
$F_3 = F[1,2]:$

Case 2.4.6: 1 −2 −3 ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 3 ∗ ∗ |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.7: 1 −2 ∗ ∗ ∗ | −1 2 −3 ∗ ∗ | −1 −2 3 ∗ ∗ |
`Bound for this case is (3,3,3)=1.442250:`
$F_1 = F[-1]:$
$F_2 = F[1,-2]:$

$F_3 = F[1,2]:$

Case 2.4.8: $1\ -2\ 3\ *\ *\ \mid\ -1\ 2\ *\ *\ *\ \mid\ -1\ -2\ *\ *\ *\ \mid\ -3\ *\ *\ *\ *\ \mid$
Bound for this case is (4,2)=1.272020:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $2\ *\ *\ *\ \mid\ -2\ *\ *\ *\ \mid$

Case 2.4.9: $1\ -2\ *\ *\ *\ \mid\ -1\ 2\ 3\ *\ *\ \mid\ -1\ -2\ *\ *\ *\ \mid\ -3\ *\ *\ *\ *\ \mid$
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $2\ 3\ *\ *\ \mid\ -2\ *\ *\ *\ \mid\ -3\ *\ *\ *\ *\ \mid$

Case 2.4.10: $1\ -2\ *\ *\ *\ \mid\ -1\ 2\ *\ *\ *\ \mid\ -1\ -2\ 3\ *\ *\ \mid\ -3\ *\ *\ *\ *\ \mid$
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $2\ *\ *\ *\ \mid\ -2\ 3\ *\ *\ \mid\ -3\ *\ *\ *\ *\ \mid$

Case 2.4.11: $1\ -2\ -3\ *\ *\ \mid\ -1\ 2\ *\ *\ *\ \mid\ -1\ -2\ *\ *\ *\ \mid\ 3\ *\ *\ *\ *\ \mid$
Bound for this case is (4,2)=1.272020:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $2\ *\ *\ *\ \mid\ -2\ *\ *\ *\ \mid$

Case 2.4.12: $1\ -2\ *\ *\ *\ \mid\ -1\ 2\ -3\ *\ *\ \mid\ -1\ -2\ *\ *\ *\ \mid\ 3\ *\ *\ *\ *\ \mid$
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $2\ -3\ *\ *\ \mid\ -2\ *\ *\ *\ \mid\ 3\ *\ *\ *\ *\ \mid$

Case 2.4.13: $1\ -2\ *\ *\ *\ \mid\ -1\ 2\ *\ *\ *\ \mid\ -1\ -2\ -3\ *\ *\ \mid\ 3\ *\ *\ *\ *\ \mid$
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $2\ *\ *\ *\ \mid\ -2\ -3\ *\ *\ \mid\ 3\ *\ *\ *\ *\ \mid$

Case 2.4.14: $1\ -2\ 3\ *\ *\ \mid\ -1\ 2\ -3\ *\ *\ \mid\ -1\ -2\ -3\ *\ *\ \mid$
Bound for this case is (3,3)=1.259921:
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.15: $1\ -2\ -3\ *\ *\ \mid\ -1\ 2\ 3\ *\ *\ \mid\ -1\ -2\ -3\ *\ *\ \mid$
Bound for this case is (3,3,3)=1.442250:
$F_1 = F[-1]:$
$F_2 = F[1,-2]:$
$F_3 = F[1,2]:$

Case 2.4.16: $1\ -2\ -3\ *\ *\ \mid\ -1\ 2\ -3\ *\ *\ \mid\ -1\ -2\ 3\ *\ *\ \mid$
Bound for this case is (3,3,3)=1.442250:
$F_1 = F[-1]:$
$F_2 = F[1,-2]:$
$F_3 = F[1,2]:$

Case 2.4.17: $1\ -2\ 3\ *\ *\ \mid\ -1\ 2\ -3\ *\ *\ \mid\ -1\ -2\ *\ *\ *\ \mid\ -3\ *\ *\ *\ *\ \mid$
Bound for this case is (4,4)=1.189207:

$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.18: 1 −2 3 * * | −1 2 * * * | −1 −2 −3 * * | −3 * * * * |
`Bound for this case is (4,4)=1.189207:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.19: 1 −2 −3 * * | −1 2 3 * * | −1 −2 * * * | −3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 3 * * | −2 * * * | −3 * * * * |

Case 2.4.20: 1 −2 * * * | −1 2 3 * * | −1 −2 −3 * * | −3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 3 * * | −2 −3 * * | −3 * * * * |

Case 2.4.21: 1 −2 −3 * * | −1 2 * * * | −1 −2 3 * * | −3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 * * * | −2 3 * * | −3 * * * * |

Case 2.4.22: 1 −2 * * * | −1 2 −3 * * | −1 −2 3 * * | −3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 −3 * * | −2 3 * * | −3 * * * * |

Case 2.4.23: 1 −2 −3 * * | −1 2 −3 * * | −1 −2 * * * | 3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 −3 * * | −2 * * * | 3 * * * * |

Case 2.4.24: 1 −2 −3 * * | −1 2 * * * | −1 −2 −3 * * | 3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 * * * | −2 −3 * * | 3 * * * * |

Case 2.4.25: 1 −2 * * * | −1 2 −3 * * | −1 −2 −3 * * | 3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 −3 * * | −2 −3 * * | 3 * * * * |

Case 2.4.26: 1 −2 3 * * | −1 2 * * * | −1 −2 * * * | −3 * * * * | −3 * * * * |
`Bound for this case is (5,3)=1.193859:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 * * * | −2 * * * |

Case 2.4.27: 1 −2 * * * | −1 2 3 * * | −1 −2 * * * | −3 * * * * | −3 * * * * |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$

$F_2 = F[1]:$ 2 3 ∗ ∗ | −2 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

Case 2.4.28: 1 −2 ∗ ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 3 ∗ ∗ | −3 ∗ ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

`Bound for this case is (5,1)=1.324718:`

$F_1 = F[-1]:$

$F_2 = F[1]:$ 2 ∗ ∗ ∗ | −2 3 ∗ ∗ | −3 ∗ ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

Case 2.4.29: 1 −2 −3 ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 ∗ ∗ ∗ | 3 ∗ ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

`Bound for this case is (4,1)=1.380278:`

$F_1 = F[-3]:$ ∗ ∗ ∗ ∗ |

$F_2 = F[3]:$ 1 −2 ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 ∗ ∗ ∗ | ∗ ∗ ∗ ∗ |

Case 2.4.30: 1 −2 ∗ ∗ ∗ | −1 2 −3 ∗ ∗ | −1 −2 ∗ ∗ ∗ | 3 ∗ ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

`Bound for this case is (4,1)=1.380278:`

$F_1 = F[-3]:$ ∗ ∗ ∗ ∗ |

$F_2 = F[3]:$ 1 −2 ∗ ∗ ∗ | −1 2 ∗ ∗ | −1 −2 ∗ ∗ ∗ | ∗ ∗ ∗ ∗ |

Case 2.4.31: 1 −2 ∗ ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 ∗ ∗ | 3 ∗ ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

`Can't prove bound. Try subcases:`

`Heuristic codenumber is 4 for literal 4`

Case 2.4.31.1: 1 −2 −4 ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 ∗ ∗ | 3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

`Bound for this case is (5,5)=1.148698:`

$F_1 = F[-1]:$

$F_2 = F[1]:$

Case 2.4.31.2: 1 −2 ∗ ∗ ∗ | −1 2 −4 ∗ ∗ | −1 −2 −3 ∗ ∗ | 3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

`Bound for this case is (5,1)=1.324718:`

$F_1 = F[-1]:$

$F_2 = F[1]:$ 2 −4 ∗ ∗ | −2 −3 ∗ ∗ | 3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

Case 2.4.31.3: 1 −2 ∗ ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 −4 ∗ | 3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

`Bound for this case is (5,1)=1.324718:`

$F_1 = F[-1]:$

$F_2 = F[1]:$ 2 ∗ ∗ ∗ | −2 −3 −4 ∗ | 3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

Case 2.4.31.4: 1 −2 ∗ ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 ∗ ∗ | 3 4 ∗ ∗ ∗ | −3 −4 ∗ ∗ ∗ |

`Bound for this case is (3,5,3)=1.363965:`

$F_1 = F[-1]:$ 3 4 ∗ ∗ ∗ | −3 −4 ∗ ∗ ∗ |

$F_2 = F[1,-3]:$

$F_3 = F[1,3]:$ 2 ∗ ∗ ∗ | −2 ∗ ∗ |

Case 2.4.31.5: 1 −2 ∗ ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 ∗ ∗ | 3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ | −4 ∗ ∗ ∗ ∗ |

`Bound for this case is (3,4,3)=1.395337:`

$F_1 = F[-1]:$ 3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ | −4 ∗ ∗ ∗ ∗ |

$F_2 = F[1,-3]:$ 4 ∗ ∗ ∗ | −4 ∗ ∗ ∗ ∗ |

$F_3 = F[1,3]:$ 2 ∗ ∗ ∗ | −2 ∗ ∗ | ∗ ∗ ∗ ∗ |

Case 2.4.31.6: 1 −2 4 ∗ ∗ | −1 2 −4 ∗ ∗ | −1 −2 −3 ∗ ∗ | 3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

`Bound for this case is (5,5)=1.148698:`

$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.31.7: 1 −2 4 ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 −4| ∗3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |
`Bound for this case is (5,5)=1.148698:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.31.8: 1 −2 4 ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 ∗ ∗| 3 −4 ∗ ∗ ∗ | −3 −4 ∗ ∗ ∗ |
`Bound for this case is (5,5)=1.148698:`
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.4.31.9: 1 −2 −4 ∗ ∗ | −1 2 4 ∗ ∗ | −1 −2 −3 ∗ ∗| 3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 4 ∗ ∗ | −2 −3 ∗ ∗ | 3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

Case 2.4.31.10: 1 −2 ∗ ∗ ∗ | −1 2 4 ∗ ∗ | −1 −2 −3 −4| ∗3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 4 ∗ ∗ | −2 −3 −4 ∗ | 3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

Case 2.4.31.11: 1 −2 ∗ ∗ ∗ | −1 2 4 ∗ ∗ | −1 −2 −3 ∗ ∗| 3 −4 ∗ ∗ ∗ | −3 −4 ∗ ∗ ∗ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 4 ∗ ∗ | −2 −3 ∗ ∗ | 3 −4 ∗ ∗ ∗ | −3 −4 ∗ ∗ ∗ |

Case 2.4.31.12: 1 −2 −4 ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 4 ∗| 3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 ∗ ∗ ∗ | −2 −3 4 ∗ | 3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

Case 2.4.31.13: 1 −2 ∗ ∗ ∗ | −1 2 −4 ∗ ∗ | −1 −2 −3 4 ∗| 3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 −4 ∗ ∗ | −2 −3 4 ∗ | 3 −4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

Case 2.4.31.14: 1 −2 ∗ ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 4 ∗| 3 −4 ∗ ∗ ∗ | −3 −4 ∗ ∗ ∗ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 ∗ ∗ ∗ | −2 −3 4 ∗ | 3 −4 ∗ ∗ ∗ | −3 −4 ∗ ∗ ∗ |

Case 2.4.31.15: 1 −2 −4 ∗ ∗ | −1 2 −4 ∗ ∗ | −1 −2 −3 ∗ ∗| 3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 −4 ∗ ∗ | −2 −3 ∗ ∗ | 3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |

Case 2.4.31.16: 1 −2 −4 ∗ ∗ | −1 2 ∗ ∗ ∗ | −1 −2 −3 −4| ∗3 4 ∗ ∗ ∗ | −3 ∗ ∗ ∗ ∗ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$

$F_2 = F[1]$: 2 * * * | −2 −3 −4 * | 3 4 * * * | −3 * * * * |

Case 2.4.31.17: 1 −2 −4 * * | −1 2 * * * | −1 −2 −3 * * | 3 4 * * * | −3 −4 * * * |
`Bound for this case is (3,5,3)=1.363965:`
$F_1 = F[-1]$: 3 4 * * * | −3 −4 * * * |
$F_2 = F[1, -3]$:
$F_3 = F[1, 3]$: 2 * * * | −2 * * |

Case 2.4.31.18: 1 −2 * * * | −1 2 −4 * * | −1 −2 −3 −4 | *3 4 * * * | −3 * * * * |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]$:
$F_2 = F[1]$: 2 −4 * * | −2 −3 −4 * | 3 4 * * * | −3 * * * * |

Case 2.4.31.19: 1 −2 * * * | −1 2 −4 * * | −1 −2 −3 * * | 3 4 * * * | −3 −4 * * * |
`Bound for this case is (3,5,5)=1.298030:`
$F_1 = F[-1]$: 3 4 * * * | −3 −4 * * * |
$F_2 = F[1, -3]$:
$F_3 = F[1, 3]$:

Case 2.4.31.20: 1 −2 * * * | −1 2 * * * | −1 −2 −3 −4 | *3 4 * * * | −3 −4 * * * |
`Bound for this case is (3,5,5)=1.298030:`
$F_1 = F[-1]$: 3 4 * * * | −3 −4 * * * |
$F_2 = F[1, -3]$:
$F_3 = F[1, 3]$:

Case 2.4.31.21: 1 −2 −4 * * | −1 2 * * * | −1 −2 −3 * * | 3 −4 * * * | −3 4 * * * |
`Bound for this case is (3,5,3)=1.363965:`
$F_1 = F[-1]$: 3 −4 * * * | −3 4 * * * |
$F_2 = F[1, -3]$:
$F_3 = F[1, 3]$: 2 * * * | −2 * * |

Case 2.4.31.22: 1 −2 * * * | −1 2 −4 * * | −1 −2 −3 * * | 3 −4 * * * | −3 4 * * * |
`Bound for this case is (3,5,2)=1.429108:`
$F_1 = F[-1]$: 3 −4 * * * | −3 4 * * * |
$F_2 = F[1, -3]$:
$F_3 = F[1, 3]$: 2 −4 * * | −2 * * | 4 * * * |

Case 2.4.31.23: 1 −2 * * * | −1 2 * * * | −1 −2 −3 −4 | *3 −4 * * * | −3 4 * * * |
`Bound for this case is (3,5,2)=1.429108:`
$F_1 = F[-1]$: 3 −4 * * * | −3 4 * * * |
$F_2 = F[1, -3]$:
$F_3 = F[1, 3]$: 2 * * * | −2 −4 * | 4 * * * |

Case 2.4.31.24: 1 −2 4 * * | −1 2 * * * | −1 −2 −3 * * | 3 −4 * * * | −3 * * * * | −4 * * * * |
`Bound for this case is (6,6)=1.122462:`
$F_1 = F[-1]$:
$F_2 = F[1]$:

Case 2.4.31.25: 1 −2 * * * | −1 2 4 * * | −1 −2 −3 * * | 3 −4 * * * | −3 * * * * | −4 * * * * |
`Bound for this case is (6,1)=1.285199:`

$F_1 = F[-1]:$
$F_2 = F[1]:$ $2\ 4\ *\ * \mid -2\ -3\ *\ * \mid 3\ -4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$

Case 2.4.31.26: $1\ -2\ *\ *\ * \mid -1\ 2\ *\ *\ * \mid -1\ -2\ -3\ 4\ * \mid 3\ -4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
`Bound for this case is (6,1)=1.285199:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ $2\ *\ *\ * \mid -2\ -3\ 4\ * \mid 3\ -4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$

Case 2.4.31.27: $1\ -2\ -4\ *\ * \mid -1\ 2\ *\ *\ * \mid -1\ -2\ -3\ *\ * \mid 3\ 4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
`Bound for this case is (3,4,3)=1.395337:`
$F_1 = F[-1]:$ $3\ 4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
$F_2 = F[1,-3]:$ $4\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
$F_3 = F[1,3]:$ $2\ *\ *\ * \mid -2\ *\ * \mid *\ *\ *\ * \mid$

Case 2.4.31.28: $1\ -2\ *\ *\ * \mid -1\ 2\ -4\ *\ * \mid -1\ -2\ -3\ *\ * \mid 3\ 4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
`Bound for this case is (3,4,5)=1.324718:`
$F_1 = F[-1]:$ $3\ 4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
$F_2 = F[1,-3]:$ $4\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
$F_3 = F[1,3]:$ $*\ *\ *\ * \mid$

Case 2.4.31.29: $1\ -2\ *\ *\ * \mid -1\ 2\ *\ *\ * \mid -1\ -2\ -3\ -4 \mid *3\ 4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
`Bound for this case is (3,4,5)=1.324718:`
$F_1 = F[-1]:$ $3\ 4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
$F_2 = F[1,-3]:$ $4\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
$F_3 = F[1,3]:$ $*\ *\ *\ * \mid$

Case 2.4.31.30: $1\ -2\ *\ *\ * \mid -1\ 2\ *\ *\ * \mid -1\ -2\ -3\ *\ * \mid 3\ 4\ *\ *\ * \mid -3\ -4\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
`Bound for this case is (3,4,4)=1.353210:`
$F_1 = F[-1]:$ $3\ 4\ *\ *\ * \mid -3\ -4\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
$F_2 = F[1,-3]:$ $4\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
$F_3 = F[1,3]:$ $2\ *\ *\ * \mid -2\ *\ * \mid$

Case 2.4.31.31: $1\ -2\ *\ *\ * \mid -1\ 2\ *\ *\ * \mid -1\ -2\ -3\ *\ * \mid 3\ -4\ *\ *\ * \mid -3\ 4\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
`Bound for this case is (3,6,2)=1.403602:`
$F_1 = F[-1]:$ $3\ -4\ *\ *\ * \mid -3\ 4\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$
$F_2 = F[1,-3]:$
$F_3 = F[1,3]:$ $2\ *\ *\ * \mid -2\ *\ * \mid 4\ *\ *\ * \mid -4\ *\ *\ *\ * \mid$

Case 2.4.31.32: $1\ -2\ -4\ *\ * \mid -1\ 2\ *\ *\ * \mid -1\ -2\ -3\ *\ * \mid 3\ -4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid 4\ *\ *\ *\ * \mid$
`Bound for this case is (3,4,3)=1.395337:`
$F_1 = F[-1]:$ $3\ -4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid 4\ *\ *\ *\ * \mid$
$F_2 = F[1,-3]:$ $-4\ *\ *\ * \mid 4\ *\ *\ *\ * \mid$
$F_3 = F[1,3]:$ $2\ *\ *\ * \mid -2\ *\ * \mid *\ *\ *\ * \mid$

Case 2.4.31.33: $1\ -2\ *\ *\ * \mid -1\ 2\ -4\ *\ * \mid -1\ -2\ -3\ *\ * \mid 3\ -4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid 4\ *\ *\ *\ * \mid$
`Bound for this case is (3,5,2)=1.429108:`
$F_1 = F[-1]:$ $3\ -4\ *\ *\ * \mid -3\ *\ *\ *\ * \mid 4\ *\ *\ *\ * \mid$
$F_2 = F[1,-4]:$ $*\ *\ *\ * \mid$
$F_3 = F[1,4]:$ $2\ *\ * \mid -2\ -3\ *\ * \mid 3\ *\ *\ * \mid -3\ *\ *\ *\ * \mid$

Case 2.4.31.34: 1 −2 * * * | −1 2 * * * | −1 −2 −3 −4 * | 3 −4 * * * | −3 * * * * | 4 * * * * |
Bound for this case is (3,5,2)=1.429108:
$F_1 = F[-1]:$ 3 −4 * * * | −3 * * * * | 4 * * * * |
$F_2 = F[1,-4]:$ * * * * |
$F_3 = F[1,4]:$ 2 * * * | −2 −3 * | 3 * * * | −3 * * * * |

Case 2.4.31.35: 1 −2 * * * | −1 2 * * * | −1 −2 −3 * * | 3 −4 * * * | −3 −4 * * * | 4 * * * * |
Bound for this case is (3,5,2)=1.429108:
$F_1 = F[-1]:$ 3 −4 * * * | −3 −4 * * * | 4 * * * * |
$F_2 = F[1,-4]:$ * * * * |
$F_3 = F[1,4]:$ 2 * * * | −2 −3 * * | 3 * * * | −3 * * * |

Case 2.4.31.36: 1 −2 * * * | −1 2 * * * | −1 −2 −3 * * | 3 4 * * * | −3 * * * * | −4 * * * * | −4 * * * * |
Bound for this case is (3,4,4)=1.353210:
$F_1 = F[-1]:$ 3 4 * * * | −3 * * * * | −4 * * * * | −4 * * * * |
$F_2 = F[1,-3]:$ 4 * * * | −4 * * * * | −4 * * * * |
$F_3 = F[1,3]:$ 2 * * * | −2 * * | * * * * |

Case 2.4.31.37: 1 −2 * * * | −1 2 * * * | −1 −2 −3 * * | 3 −4 * * * | −3 * * * * | 4 * * * * | −4 * * * * |
Bound for this case is (3,6,2)=1.403602:
$F_1 = F[-1]:$ 3 −4 * * * | −3 * * * * | 4 * * * * | −4 * * * * |
$F_2 = F[1,-4]:$ * * * * |
$F_3 = F[1,4]:$ 2 * * * | −2 −3 * * | 3 * * * | −3 * * * * | * * * * |
Heuristics no. 4 bound for literal 4 is 1.442250
Heuristics no. 0 bound for literal 3 is 1.442250

Case 2.5: 1 2 * * * | −1 −2 * * * | −1 * * * * | −2 * * * * |
Bound for this case is (2,3)=1.324718:
$F_1 = F[-1]:$ 2 * * * | −2 * * * * |
$F_2 = F[1]:$ * * * * |

Case 2.6: 1 −2 * * * | −1 2 * * * | −1 * * * * | −2 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ 2 * * * | * * * * | −2 * * * * |

Case 2.7: 1 −2 * * * | −1 −2 * * * | −1 * * * * | 2 * * * * |
Can't prove bound. Try subcases:

Heuristic codenumber is 3 for literal 3

Case 2.7.1: 1 −2 −3 * * | −1 −2 * * * | −1 3 * * * | 2 * * * * |
Bound for this case is (4,2)=1.272020:
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 * * * | 2 * * * * |

Case 2.7.2: 1 −2 * * * | −1 −2 −3 * * | −1 3 * * * | 2 * * * * |
Can't prove bound. Try subcases:

Heuristic codenumber is 4 for literal 4

Case 2.7.2.1: 1 −2 −4 * * | −1 −2 −3 * * | −1 3 * * * | 2 4 * * * |
`Bound for this case is (2,4)=1.272020:`
$F_1 = F[-1]:$ −2 −4 * * | 2 4 * * * |
$F_2 = F[1]:$

Case 2.7.2.2: 1 −2 * * * | −1 −2 −3 −4 * | −1 3 * * * | 2 4 * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 −3 −4 * | 3 * * * | 2 4 * * * |

Case 2.7.2.3: 1 −2 * * * | −1 −2 −3 * * | −1 3 −4 * * | 2 4 * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 −3 * * | 3 −4 * * | 2 4 * * * |

Case 2.7.2.4: 1 −2 * * * | −1 −2 −3 * * | −1 3 * * * | 2 4 * * * | −4 * * * * |
`Bound for this case is (3,2)=1.324718:`
$F_1 = F[-2]:$ 4 * * * | −4 * * * * |
$F_2 = F[2]:$ 1 * * * | −1 −3 * * | −1 3 * * * |

Case 2.7.2.5: 1 −2 4 * * | −1 −2 −3 −4 * | −1 3 * * * | 2 −4 * * * |
`Bound for this case is (2,4)=1.272020:`
$F_1 = F[-1]:$ −2 4 * * | 2 −4 * * * |
$F_2 = F[1]:$

Case 2.7.2.6: 1 −2 4 * * | −1 −2 −3 * * | −1 3 −4 * * | 2 −4 * * * |
`Bound for this case is (2,4)=1.272020:`
$F_1 = F[-1]:$ −2 4 * * | 2 −4 * * * |
$F_2 = F[1]:$

Case 2.7.2.7: 1 −2 −4 * * | −1 −2 −3 4 * | −1 3 * * * | 2 −4 * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 −3 4 * | 3 * * * | 2 −4 * * * |

Case 2.7.2.8: 1 −2 * * * | −1 −2 −3 4 * | −1 3 −4 * * | 2 −4 * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 −3 4 * | 3 −4 * * | 2 −4 * * * |

Case 2.7.2.9: 1 −2 −4 * * | −1 −2 −3 * * | −1 3 4 * * | 2 −4 * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 −3 * * | 3 4 * * | 2 −4 * * * |

Case 2.7.2.10: 1 −2 * * * | −1 −2 −3 −4 * | −1 3 4 * * | 2 −4 * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 −3 −4 * | 3 4 * * | 2 −4 * * * |

Case 2.7.2.11: 1 −2 −4 * * | −1 −2 −3 −4 * | −1 3 * * * | 2 4 * * * |

Bound for this case is (2,4,4)=1.414214:
$F_1 = F[-1]:$ −2 −4 * * | 2 4 * * * |
$F_2 = F[1,-2]:$
$F_3 = F[1,2]:$

Case 2.7.2.12: 1 −2 −4 * * | −1 −2 −3 * * | −1 3 −4 * * | 2 4 * * * |
Bound for this case is (2,4,4)=1.414214:
$F_1 = F[-1]:$ −2 −4 * * | 2 4 * * * |
$F_2 = F[1,-2]:$
$F_3 = F[1,2]:$

Case 2.7.2.13: 1 −2 * * * | −1 −2 −3 −4 * | −1 3 −4 * * | 2 4 * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 −3 −4 * | 3 −4 * * | 2 4 * * * |

Case 2.7.2.14: 1 −2 4 * * | −1 −2 −3 * * | −1 3 * * * | 2 −4 * * * | −4 * * * * |
Bound for this case is (2,5)=1.236506:
$F_1 = F[-1]:$ −2 4 * * | 2 −4 * * * | −4 * * * * |
$F_2 = F[1]:$

Case 2.7.2.15: 1 −2 * * * | −1 −2 −3 4 * | −1 3 * * * | 2 −4 * * * | −4 * * * * |
Bound for this case is (5,1)=1.324718:
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 −3 4 * | 3 * * * | 2 −4 * * * | −4 * * * * |

Case 2.7.2.16: 1 −2 * * * | −1 −2 −3 * * | −1 3 4 * * | 2 −4 * * * | −4 * * * * |
Bound for this case is (5,1)=1.324718:
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 −3 * * | 3 4 * * | 2 −4 * * * | −4 * * * * |

Case 2.7.2.17: 1 −2 −4 * * | −1 −2 −3 * * | −1 3 * * * | 2 4 * * * | −4 * * * * |
Bound for this case is (3,5)=1.193859:
$F_1 = F[-2]:$ 4 * * * | −4 * * * * |
$F_2 = F[2]:$

Case 2.7.2.18: 1 −2 * * * | −1 −2 −3 −4 * | −1 3 * * * | 2 4 * * * | −4 * * * * |
Bound for this case is (2,3,5)=1.429108:
$F_1 = F[-1]:$ −2 * * * | 2 4 * * * | −4 * * * * |
$F_2 = F[1,-2]:$ 4 * * * | −4 * * * * |
$F_3 = F[1,2]:$

Case 2.7.2.19: 1 −2 * * * | −1 −2 −3 * * | −1 3 −4 * * | 2 4 * * * | −4 * * * * |
Bound for this case is (2,3,5)=1.429108:
$F_1 = F[-1]:$ −2 * * * | 2 4 * * * | −4 * * * * |
$F_2 = F[1,-2]:$ 4 * * * | −4 * * * * |
$F_3 = F[1,2]:$

Case 2.7.2.20: 1 −2 −4 * * | −1 −2 −3 * * | −1 3 * * * | 2 −4 * * * | 4 * * * * |
Bound for this case is (3,3,3)=1.442250:

$F_1 = F[-2]:$ $-4\ *\ *\ *$ | $4\ *\ *\ *\ *$ |
$F_2 = F[2,-1]:$ $-4\ *\ *$ | $4\ *\ *\ *\ *$ |
$F_3 = F[2,1]:$ $-3\ *\ *$ | $3\ *\ *\ *$ |

Case 2.7.2.21: $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ -4$ | $*\ -1\ 3\ *\ *\ *$ | $2\ -4\ *\ *\ *$ | $4\ *\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-4]:$ $*\ *\ *\ *$ |
$F_2 = F[4]:$ $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ *$ | $-1\ 3\ *\ *\ *$ | $2\ *\ *\ *$ |

Case 2.7.2.22: $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ 3\ -4\ *\ *$ | $2\ -4\ *\ *\ *$ | $4\ *\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-4]:$ $*\ *\ *\ *$ |
$F_2 = F[4]:$ $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ 3\ *\ *$ | $2\ *\ *\ *$ |

Case 2.7.2.23: $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ 4\ *\ *\ *$ | $-4\ *\ *\ *\ *$ | $-4\ *\ *\ *\ *$ |
Bound for this case is (3,3)=1.259921:
$F_1 = F[-2]:$ $4\ *\ *\ *$ | $-4\ *\ *\ *\ *$ | $-4\ *\ *\ *\ *$ |
$F_2 = F[2]:$ $1\ *\ *\ *$ | $-1\ -3\ *\ *$ | $-1\ 3\ *\ *\ *$ |

Case 2.7.2.24: $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ -4\ *\ *\ *$ | $4\ *\ *\ *\ *$ | $-4\ *\ *\ *\ *$ |
Bound for this case is (5,1)=1.324718:
$F_1 = F[-4]:$ $*\ *\ *\ *$ |
$F_2 = F[4]:$ $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ *\ *\ *$ | $*\ *\ *\ *$ |
Heuristics no. 4 bound for literal 4 is 1.442250

Case 2.7.3: $1\ -2\ *\ *\ *$ | $-1\ -2\ *\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $-2\ *\ *\ *$ | $3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ |

Case 2.7.4: $1\ -2\ *\ *\ *$ | $-1\ -2\ *\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ *\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-2]:$ $*\ *\ *\ *$ |
$F_2 = F[2]:$ $1\ *\ *\ *$ | $-1\ *\ *\ *$ | $-1\ 3\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |

Case 2.7.5: $1\ -2\ 3\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ -3\ *\ *\ *$ | $2\ *\ *\ *\ *$ |
Bound for this case is (4,4)=1.189207:
$F_1 = F[-1]:$
$F_2 = F[1]:$

Case 2.7.6: $1\ -2\ 3\ *\ *$ | $-1\ -2\ *\ *\ *$ | $-1\ -3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ |
Bound for this case is (2,4)=1.272020:
$F_1 = F[-1]:$ $-2\ 3\ *\ *$ | $2\ -3\ *\ *\ *$ |
$F_2 = F[1]:$

Case 2.7.7: $1\ -2\ -3\ *\ *$ | $-1\ -2\ 3\ *\ *$ | $-1\ -3\ *\ *\ *$ | $2\ *\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $-2\ 3\ *\ *$ | $-3\ *\ *\ *$ | $2\ *\ *\ *\ *$ |

Case 2.7.8: $1\ -2\ *\ *\ *$ | $-1\ -2\ 3\ *\ *$ | $-1\ -3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ |

Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $-2\ 3\ *\ *$ | $-3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ |

Case 2.7.9: $1\ -2\ -3\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ *\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $-2\ -3\ *\ *$ | $3\ *\ *\ *$ | $2\ *\ *\ *\ *$ |

Case 2.7.10: $1\ -2\ -3\ *\ *$ | $-1\ -2\ *\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $-2\ *\ *\ *$ | $3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ |

Case 2.7.11: $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $-2\ -3\ *\ *$ | $3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ |

Case 2.7.12: $1\ -2\ -3\ *\ *$ | $-1\ -2\ *\ *\ *$ | $-1\ -3\ *\ *\ *$ | $2\ 3\ *\ *\ *$ |
Bound for this case is (4,4)=1.189207:
$F_1 = F[-2]:$
$F_2 = F[2]:$

Case 2.7.13: $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ -3\ *\ *\ *$ | $2\ 3\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $-2\ -3\ *\ *$ | $-3\ *\ *\ *$ | $2\ 3\ *\ *\ *$ |

Case 2.7.14: $1\ -2\ 3\ *\ *$ | $-1\ -2\ *\ *\ *$ | $-1\ -3\ *\ *\ *$ | $2\ *\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |
Bound for this case is (2,3)=1.324718:
$F_1 = F[-1]:$ $-2\ 3\ *\ *$ | $2\ *\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |
$F_2 = F[1]:$ $-2\ *\ *\ *$ | $2\ *\ *\ *\ *$ |

Case 2.7.15: $1\ -2\ *\ *\ *$ | $-1\ -2\ 3\ *\ *$ | $-1\ -3\ *\ *\ *$ | $2\ *\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |
Bound for this case is (3,3,3)=1.442250:
$F_1 = F[-1]:$ $-2\ *\ *\ *$ | $2\ *\ *\ *\ *$ |
$F_2 = F[1,-3]:$ $-2\ *\ *$ | $2\ *\ *\ *\ *$ |
$F_3 = F[1,3]:$ $*\ *\ *$ | $*\ *\ *\ *$ |

Case 2.7.16: $1\ -2\ -3\ *\ *$ | $-1\ -2\ *\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ *\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |
Bound for this case is (5,1)=1.324718:
$F_1 = F[-1]:$
$F_2 = F[1]:$ $-2\ *\ *\ *$ | $3\ *\ *\ *$ | $2\ *\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |

Case 2.7.17: $1\ -2\ *\ *\ *$ | $-1\ -2\ -3\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ *\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-2]:$ $*\ *\ *\ *$ |
$F_2 = F[2]:$ $1\ *\ *\ *$ | $-1\ -3\ *\ *$ | $-1\ 3\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |

Case 2.7.18: $1\ -2\ *\ *\ *$ | $-1\ -2\ *\ *\ *$ | $-1\ 3\ *\ *\ *$ | $2\ -3\ *\ *\ *$ | $-3\ *\ *\ *\ *$ |

Bound for this case is (5,1)=1.324718:
$F_1 = F[-1]$:
$F_2 = F[1]$: -2 * * * | 3 * * * | 2 -3 * * * | -3 * * * * |

Case 2.7.19: 1 -2 * * * | -1 -2 * * * | -1 -3 * * * | 2 3 * * * | -3 * * * * |
Bound for this case is (3,3)=1.259921:
$F_1 = F[-2]$: 3 * * * | -3 * * * * |
$F_2 = F[2]$: 1 * * * | -1 * * * |

Case 2.7.20: 1 -2 -3 * * | -1 -2 * * * | -1 -3 * * * | 2 * * * * | 3 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-2]$: * * * * |
$F_2 = F[2]$: 1 -3 * * | -1 * * * | -1 -3 * * * | 3 * * * * |

Case 2.7.21: 1 -2 * * * | -1 -2 -3 * * | -1 -3 * * * | 2 * * * * | 3 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-2]$: * * * * |
$F_2 = F[2]$: 1 * * * | -1 -3 * * | -1 -3 * * * | 3 * * * * |

Case 2.7.22: 1 -2 * * * | -1 -2 * * * | -1 -3 * * * | 2 -3 * * * | 3 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-3]$: * * * * |
$F_2 = F[3]$: 1 -2 * * * | -1 -2 * * * | -1 * * * | 2 * * * |

Case 2.7.23: 1 -2 * * * | -1 -2 * * * | -1 3 * * * | 2 * * * * | -3 * * * * | -3 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]$: -2 * * * | 2 * * * * |
$F_2 = F[1]$: -2 * * * | 3 * * * | 2 * * * * | -3 * * * * | -3 * * * * |

Case 2.7.24: 1 -2 * * * | -1 -2 * * * | -1 -3 * * * | 2 * * * * | 3 * * * * | -3 * * * * |
Can't prove bound. Try subcases:

Heuristic codenumber is 4 for literal 4

Case 2.7.24.1: 1 -2 -4 * * | -1 -2 * * * | -1 -3 * * * | 2 4 * * * | 3 * * * * | -3 * * * * |
Bound for this case is (2,3)=1.324718:
$F_1 = F[-1]$: -2 -4 * * | 2 4 * * * | 3 * * * * | -3 * * * * |
$F_2 = F[1]$: -3 * * * | 3 * * * * | -3 * * * * |

Case 2.7.24.2: 1 -2 * * * | -1 -2 -4 * * | -1 -3 * * * | 2 4 * * * | 3 * * * * | -3 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]$: 3 * * * * | -3 * * * * |
$F_2 = F[1]$: -2 -4 * * | -3 * * * | 2 4 * * * | 3 * * * * | -3 * * * * |

Case 2.7.24.3: 1 -2 * * * | -1 -2 * * * | -1 -3 -4 * * | 2 4 * * * | 3 * * * * | -3 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]$: 3 * * * * | -3 * * * * |
$F_2 = F[1]$: -2 * * * | -3 -4 * * | 2 4 * * * | 3 * * * * | -3 * * * * |

Case 2.7.24.4: 1 -2 * * * | -1 -2 * * * | -1 -3 * * * | 2 4 * * * | 3 -4 * * * | -3 * * * * |
Bound for this case is (3,4)=1.220744:

$F_1 = F[-2]:$ 4 * * * | 3 −4 * * * | −3 * * * * |
$F_2 = F[2]:$ 1 * * * | −1 * * * |

Case 2.7.24.5: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 4 * * * | 3 * * * * | −3 −4 * * * |
Bound for this case is (3,2)=1.324718:
$F_1 = F[-2]:$ 4 * * * | 3 * * * * | −3 −4 * * * |
$F_2 = F[2]:$ 1 * * * | −1 * * * | −1 −3 * * * | 3 * * * * |

Case 2.7.24.6: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 4 * * * | 3 * * * * | −3 * * * * | −4 * * * * |
Bound for this case is (3,2)=1.324718:
$F_1 = F[-2]:$ 4 * * * | 3 * * * * | −3 * * * * | −4 * * * * |
$F_2 = F[2]:$ 1 * * * | −1 * * * | −1 −3 * * * | 3 * * * * | −3 * * * * |

Case 2.7.24.7: 1 −2 4 * * | −1 −2 −4 * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 * * * * |
Bound for this case is (2,3)=1.324718:
$F_1 = F[-1]:$ −2 4 * * | 2 −4 * * * | 3 * * * * | −3 * * * * |
$F_2 = F[1]:$ −3 * * * | 3 * * * * | −3 * * * * |

Case 2.7.24.8: 1 −2 4 * * | −1 −2 * * * | −1 −3 −4 * * | 2 −4 * * * | 3 * * * * | −3 * * * * |
Bound for this case is (2,4)=1.272020:
$F_1 = F[-1]:$ −2 4 * * | 2 −4 * * * | 3 * * * * | −3 * * * * |
$F_2 = F[1]:$ 3 * * * * | −3 * * * * |

Case 2.7.24.9: 1 −2 4 * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 −4 * * * | −3 * * * * |
Bound for this case is (2,6)=1.210608:
$F_1 = F[-1]:$ −2 4 * * | 2 −4 * * * | 3 −4 * * * | −3 * * * * |
$F_2 = F[1]:$

Case 2.7.24.10: 1 −2 4 * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 −4 * * * |
Bound for this case is (2,4)=1.272020:
$F_1 = F[-1]:$ −2 4 * * | 2 −4 * * * | 3 * * * * | −3 −4 * * * |
$F_2 = F[1]:$ −3 * * * | 3 * * * * |

Case 2.7.24.11: 1 −2 −4 * * | −1 −2 4 * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$ 3 * * * * | −3 * * * * |
$F_2 = F[1]:$ −2 4 * * | −3 * * * | 2 −4 * * * | 3 * * * * | −3 * * * * |

Case 2.7.24.12: 1 −2 * * * | −1 −2 4 * * | −1 −3 −4 * * | 2 −4 * * * | 3 * * * * | −3 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-1]:$ 3 * * * * | −3 * * * * |
$F_2 = F[1]:$ −2 4 * * | −3 −4 * * | 2 −4 * * * | 3 * * * * | −3 * * * * |

Case 2.7.24.13: 1 −2 * * * | −1 −2 4 * * | −1 −3 * * * | 2 −4 * * * | 3 −4 * * * | −3 * * * * |
Bound for this case is (6,1)=1.285199:
$F_1 = F[-1]:$
$F_2 = F[1]:$ −2 4 * * | −3 * * * | 2 −4 * * * | 3 −4 * * * | −3 * * * * |

Case 2.7.24.14: 1 −2 * * * | −1 −2 4 * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 −4 * * * |
Bound for this case is (6,1)=1.285199:
$F_1 = F[-1]:$

$F_2 = F[1]:$ -2 4 * * | -3 * * * | 2 -4 * * * | 3 * * * * | -3 -4 * * * |

Case 2.7.24.15: 1 -2 -4 * * | -1 -2 * * * | -1 -3 4 * * | 2 -4 * * * | 3 * * * * | -3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$ 3 * * * * | -3 * * * * |
$F_2 = F[1]:$ -2 * * * | -3 4 * * | 2 -4 * * * | 3 * * * * | -3 * * * * |

Case 2.7.24.16: 1 -2 * * * | -1 -2 -4 * * | -1 -3 4 * * | 2 -4 * * * | 3 * * * * | -3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$ 3 * * * * | -3 * * * * |
$F_2 = F[1]:$ -2 -4 * * | -3 4 * * | 2 -4 * * * | 3 * * * * | -3 * * * * |

Case 2.7.24.17: 1 -2 * * * | -1 -2 * * * | -1 -3 4 * * | 2 -4 * * * | 3 -4 * * * | -3 * * * * |
`Bound for this case is (6,1)=1.285199:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ -2 * * * | -3 4 * * | 2 -4 * * * | 3 -4 * * * | -3 * * * * |

Case 2.7.24.18: 1 -2 * * * | -1 -2 * * * | -1 -3 4 * * | 2 -4 * * * | 3 * * * * | -3 -4 * * * |
`Bound for this case is (6,1)=1.285199:`
$F_1 = F[-1]:$
$F_2 = F[1]:$ -2 * * * | -3 4 * * | 2 -4 * * * | 3 * * * * | -3 -4 * * * |

Case 2.7.24.19: 1 -2 -4 * * | -1 -2 -4 * * | -1 -3 * * * | 2 4 * * * | 3 * * * * | -3 * * * * |
`Bound for this case is (4,4)=1.189207:`
$F_1 = F[-2]:$ 3 * * * * | -3 * * * * |
$F_2 = F[2]:$ 3 * * * * | -3 * * * * |

Case 2.7.24.20: 1 -2 -4 * * | -1 -2 * * * | -1 -3 -4 * * | 2 4 * * * | 3 * * * * | -3 * * * * |
`Bound for this case is (4,4)=1.189207:`
$F_1 = F[-2]:$ 3 * * * * | -3 * * * * |
$F_2 = F[2]:$ 3 * * * * | -3 * * * * |

Case 2.7.24.21: 1 -2 -4 * * | -1 -2 * * * | -1 -3 * * * | 2 4 * * * | 3 -4 * * * | -3 * * * * |
`Bound for this case is (3,6)=1.173985:`
$F_1 = F[-2]:$ 4 * * * | 3 -4 * * * | -3 * * * * |
$F_2 = F[2]:$

Case 2.7.24.22: 1 -2 -4 * * | -1 -2 * * * | -1 -3 * * * | 2 4 * * * | 3 * * * * | -3 -4 * * * |
`Bound for this case is (3,6)=1.173985:`
$F_1 = F[-2]:$ 4 * * * | 3 * * * * | -3 -4 * * * |
$F_2 = F[2]:$

Case 2.7.24.23: 1 -2 * * * | -1 -2 -4 * * | -1 -3 -4 * * | 2 4 * * * | 3 * * * * | -3 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-1]:$ 3 * * * * | -3 * * * * |
$F_2 = F[1]:$ -2 -4 * * | -3 -4 * * | 2 4 * * * | 3 * * * * | -3 * * * * |

Case 2.7.24.24: 1 -2 * * * | -1 -2 -4 * * | -1 -3 * * * | 2 4 * * * | 3 -4 * * * | -3 * * * * |
`Bound for this case is (2,6,3)=1.403602:`
$F_1 = F[-1]:$ -2 * * * | 2 4 * * * | 3 -4 * * * | -3 * * * * |
$F_2 = F[1, -4]:$

$F_3 = F[1,4]$: −3 * * * | 3 * * * | −3 * * * * |

Case 2.7.24.25: 1 −2 * * * | −1 −2 −4 * * | −1 −3 * * * | 2 4 * * * | 3 * * * * | −3 −4 * * * |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-2]$: 4 * * * | 3 * * * * | −3 −4 * * * |
$F_2 = F[2]$: 1 * * * | −1 −3 * * * | 3 * * * * |

Case 2.7.24.26: 1 −2 * * * | −1 −2 * * * | −1 −3 −4 * * | 2 4 * * * | 3 −4 * * * | −3 * * * * |
`Bound for this case is (3,4)=1.220744:`
$F_1 = F[-2]$: 4 * * * | 3 −4 * * * | −3 * * * * |
$F_2 = F[2]$: 1 * * * | −1 * * * |

Case 2.7.24.27: 1 −2 * * * | −1 −2 * * * | −1 −3 −4 * * | 2 4 * * * | 3 * * * * | −3 −4 * * * |
`Bound for this case is (3,4)=1.220744:`
$F_1 = F[-2]$: 4 * * * | 3 * * * * | −3 −4 * * * |
$F_2 = F[2]$: 1 * * * | −1 * * * |

Case 2.7.24.28: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 4 * * * | 3 −4 * * * | −3 −4 * * * |
`Bound for this case is (3,4)=1.220744:`
$F_1 = F[-2]$: 4 * * * | 3 −4 * * * | −3 −4 * * * |
$F_2 = F[2]$: 1 * * * | −1 * * * |

Case 2.7.24.29: 1 −2 −4 * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 4 * * * | −3 * * * * |
`Bound for this case is (3,3,5)=1.363965:`
$F_1 = F[-2]$: −4 * * * | 3 4 * * * | −3 * * * * |
$F_2 = F[2,-1]$: −4 * * | 3 4 * * * | −3 * * * * |
$F_3 = F[2,1]$: * * * |

Case 2.7.24.30: 1 −2 * * * | −1 −2 −4 * * | −1 −3 * * * | 2 −4 * * * | 3 4 * * * | −3 * * * * |
`Bound for this case is (3,5,2)=1.429108:`
$F_1 = F[-2]$: −4 * * * | 3 4 * * * | −3 * * * * |
$F_2 = F[2,-1]$: * * * |
$F_3 = F[2,1]$: −4 * * | −3 * * * | 3 4 * * * | −3 * * * * |

Case 2.7.24.31: 1 −2 * * * | −1 −2 * * * | −1 −3 −4 * * | 2 −4 * * * | 3 4 * * * | −3 * * * * |
`Bound for this case is (2,3,5)=1.429108:`
$F_1 = F[-1]$: −2 * * * | 2 −4 * * * | 3 4 * * * | −3 * * * * |
$F_2 = F[1,-3]$: −2 * * * | 2 −4 * * * | 4 * * * |
$F_3 = F[1,3]$: * * * * |

Case 2.7.24.32: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 4 * * * | −3 −4 * * * |
`Bound for this case is (2,3,5)=1.429108:`
$F_1 = F[-1]$: −2 * * * | 2 −4 * * * | 3 4 * * * | −3 −4 * * * |
$F_2 = F[1,-3]$: −2 * * * | 2 −4 * * * | 4 * * * |
$F_3 = F[1,3]$: * * * |

Case 2.7.24.33: 1 −2 −4 * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 4 * * * |
`Bound for this case is (3,3,3)=1.442250:`
$F_1 = F[-2]$: −4 * * * | 3 * * * * | −3 4 * * * |
$F_2 = F[2,-1]$: −4 * * | 3 * * * * | −3 4 * * * |

$F_3 = F[2,1]:$ $* * *$ | $-3 * * *$ | $3 * * * *$ |

Case 2.7.24.34: $1\ -2\ * * *$ | $-1\ -2\ -4\ * *$ | $-1\ -3\ * * *$ | $2\ -4\ * * *$ | $3\ * * * *$ | $-3\ 4\ * * *$ |
`Bound for this case is (3,5,2)=1.429108:`
$F_1 = F[-2]:$ $-4\ * * *$ | $3\ * * * *$ | $-3\ 4\ * * *$ |
$F_2 = F[2,-1]:$ $* * *$ |
$F_3 = F[2,1]:$ $-4\ * *$ | $-3\ * * *$ | $3\ * * * *$ | $-3\ 4\ * * *$ |

Case 2.7.24.35: $1\ -2\ * * *$ | $-1\ -2\ * * *$ | $-1\ -3\ -4\ * *$ | $2\ -4\ * * *$ | $3\ * * * *$ | $-3\ 4\ * * *$ |
`Bound for this case is (3,5,2)=1.429108:`
$F_1 = F[-2]:$ $-4\ * * *$ | $3\ * * * *$ | $-3\ 4\ * * *$ |
$F_2 = F[2,-1]:$ $* * *$ |
$F_3 = F[2,1]:$ $* * *$ | $-3\ -4\ * *$ | $3\ * * * *$ | $-3\ 4\ * * *$ |

Case 2.7.24.36: $1\ -2\ * * *$ | $-1\ -2\ * * *$ | $-1\ -3\ * * *$ | $2\ -4\ * * *$ | $3\ -4\ * * *$ | $-3\ 4\ * * *$ |
`Bound for this case is (3,4,3)=1.395337:`
$F_1 = F[-2]:$ $-4\ * * *$ | $3\ -4\ * * *$ | $-3\ 4\ * * *$ |
$F_2 = F[2,-3]:$ $1\ * * *$ | $-1\ * * *$ |
$F_3 = F[2,3]:$ $1\ * * *$ | $-1\ * * *$ | $-1\ * * *$ |

Case 2.7.24.37: $1\ -2\ 4\ * *$ | $-1\ -2\ * * *$ | $-1\ -3\ * * *$ | $2\ -4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
`Bound for this case is (2,4)=1.272020:`
$F_1 = F[-1]:$ $-2\ 4\ * *$ | $2\ -4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
$F_2 = F[1]:$ $-3\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ |

Case 2.7.24.38: $1\ -2\ * * *$ | $-1\ -2\ 4\ * *$ | $-1\ -3\ * * *$ | $2\ -4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$ $3\ * * * *$ | $-3\ * * * *$ |
$F_2 = F[1]:$ $-2\ 4\ * *$ | $-3\ * * *$ | $2\ -4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |

Case 2.7.24.39: $1\ -2\ * * *$ | $-1\ -2\ * * *$ | $-1\ -3\ 4\ * *$ | $2\ -4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-1]:$ $3\ * * * *$ | $-3\ * * * *$ |
$F_2 = F[1]:$ $-2\ * * *$ | $-3\ 4\ * *$ | $2\ -4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |

Case 2.7.24.40: $1\ -2\ -4\ * *$ | $-1\ -2\ * * *$ | $-1\ -3\ * * *$ | $2\ 4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
`Bound for this case is (3,5)=1.193859:`
$F_1 = F[-2]:$ $4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
$F_2 = F[2]:$ $3\ * * * *$ | $-3\ * * * *$ |

Case 2.7.24.41: $1\ -2\ * * *$ | $-1\ -2\ -4\ * *$ | $-1\ -3\ * * *$ | $2\ 4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-2]:$ $4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
$F_2 = F[2]:$ $1\ * * *$ | $-1\ -3\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ |

Case 2.7.24.42: $1\ -2\ * * *$ | $-1\ -2\ * * *$ | $-1\ -3\ -4\ * *$ | $2\ 4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
`Bound for this case is (3,3)=1.259921:`
$F_1 = F[-2]:$ $4\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |
$F_2 = F[2]:$ $1\ * * *$ | $-1\ * * *$ | $3\ * * * *$ | $-3\ * * * *$ |

Case 2.7.24.43: $1\ -2\ * * *$ | $-1\ -2\ * * *$ | $-1\ -3\ * * *$ | $2\ 4\ * * *$ | $3\ -4\ * * *$ | $-3\ * * * *$ | $-4\ * * * *$ |

Bound for this case is (2,5,3)=1.429108:
$F_1 = F[-1]$: −2 * * * | 2 4 * * * | 3 −4 * * * | −3 * * * * | −4 * * * * |
$F_2 = F[1,-4]$: −2 * * * | 2 * * * |
$F_3 = F[1,4]$: −3 * * * | 3 * * * | −3 * * * * | * * * * |

Case 2.7.24.44: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 4 * * * | 3 * * * * | −3 −4 * * * | −4 * * * * |
Bound for this case is (3,3)=1.259921:
$F_1 = F[-2]$: 4 * * * | 3 * * * * | −3 −4 * * * | −4 * * * * |
$F_2 = F[2]$: 1 * * * | −1 * * * | −1 −3 * * * | 3 * * * * |

Case 2.7.24.45: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 4 * * * | −3 * * * * | −4 * * * * |
Bound for this case is (2,3,5)=1.429108:
$F_1 = F[-1]$: −2 * * * | 2 −4 * * * | 3 4 * * * | −3 * * * * | −4 * * * * |
$F_2 = F[1,-3]$: −2 * * * | 2 −4 * * * | 4 * * * | −4 * * * * |
$F_3 = F[1,3]$: * * * | * * * * |

Case 2.7.24.46: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 4 * * * | −4 * * * * |
Bound for this case is (6,1)=1.285199:
$F_1 = F[-3]$: * * * * |
$F_2 = F[3]$: 1 −2 * * * | −1 −2 * * * | −1 * * * | 2 −4 * * * | 4 * * * | −4 * * * * |

Case 2.7.24.47: 1 −2 −4 * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 * * * * | 4 * * * * |
Bound for this case is (3,3,3)=1.442250:
$F_1 = F[-2]$: −4 * * * | 3 * * * * | −3 * * * * | 4 * * * * |
$F_2 = F[2,-1]$: −4 * * | 3 * * * * | −3 * * * * | 4 * * * * |
$F_3 = F[2,1]$: * * * | −3 * * * | 3 * * * * | −3 * * * * |

Case 2.7.24.48: 1 −2 * * * | −1 −2 −4 * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 * * * * | 4 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-4]$: 3 * * * * | −3 * * * * | * * * * |
$F_2 = F[4]$: 1 −2 * * * | −1 −2 * * | −1 −3 * * * | 2 * * * | 3 * * * * | −3 * * * * |

Case 2.7.24.49: 1 −2 * * * | −1 −2 * * * | −1 −3 −4 * * | 2 −4 * * * | 3 * * * * | −3 * * * * | 4 * * * * |
Bound for this case is (4,1)=1.380278:
$F_1 = F[-4]$: 3 * * * * | −3 * * * * | * * * * |
$F_2 = F[4]$: 1 −2 * * * | −1 −2 * * * | −1 −3 * * | 2 * * * | 3 * * * * | −3 * * * * |

Case 2.7.24.50: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 −4 * * * | −3 * * * * | 4 * * * * |
Bound for this case is (3,3,3)=1.442250:
$F_1 = F[-2]$: −4 * * * | 3 −4 * * * | −3 * * * * | 4 * * * * |
$F_2 = F[2,-3]$: 1 * * * | −1 * * * | −4 * * * | 4 * * * * |
$F_3 = F[2,3]$: 1 * * * | −1 * * * | −1 * * * | * * * * |

Case 2.7.24.51: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 −4 * * * | 4 * * * * |
Bound for this case is (6,1)=1.285199:
$F_1 = F[-4]$: * * * * |
$F_2 = F[4]$: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 * * * | 3 * * * * | −3 * * * |

Case 2.7.24.52: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 4 * * * | 3 * * * * | −3 * * * * | −4 * * * * | −4 * * * * |
Bound for this case is (3,3)=1.259921:

$F_1 = F[-2]:$ 4 * * * | 3 * * * * | −3 * * * * | −4 * * * * | −4 * * * * |
$F_2 = F[2]:$ 1 * * * | −1 * * * | −1 −3 * * * | 3 * * * * | −3 * * * * |

Case 2.7.24.53: 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 −4 * * * | 3 * * * * | −3 * * * * | 4 * * * * | −4 * * * * |
`Bound for this case is (5,1)=1.324718:`
$F_1 = F[-4]:$ 3 * * * * | −3 * * * * | * * * * |
$F_2 = F[4]:$ 1 −2 * * * | −1 −2 * * * | −1 −3 * * * | 2 * * * | 3 * * * * | −3 * * * * | * * * * |
`Heuristics no.  4 bound for literal 4 is 1.442250`
`Heuristics no.  3 bound for literal 3 is 1.442250`

Case 2.8: 1 2 * * * | −1 * * * * | −1 * * * * | −2 * * * * | −2 * * * * |
`Bound for this case is (2,3)=1.324718:`
$F_1 = F[-1]:$ 2 * * * | −2 * * * * | −2 * * * * |
$F_2 = F[1]:$ * * * * | * * * * |

Case 2.9: 1 −2 * * * | −1 * * * * | −1 * * * * | 2 * * * * | −2 * * * * |
`Bound for this case is (4,1)=1.380278:`
$F_1 = F[-2]:$ * * * * |
$F_2 = F[2]:$ 1 * * * | −1 * * * * | −1 * * * * | * * * * |
`Heuristics no.  1 bound for literal 2 is 1.442250`