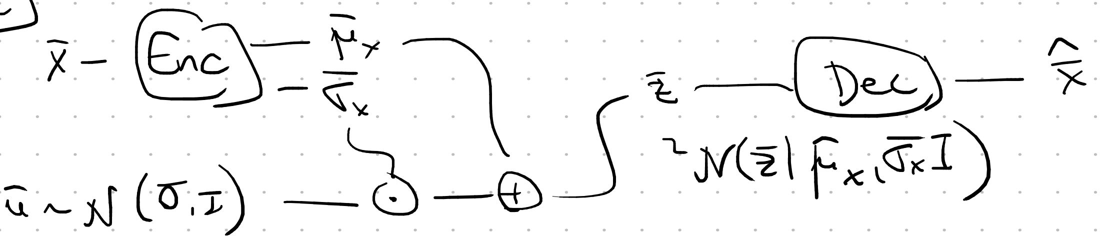
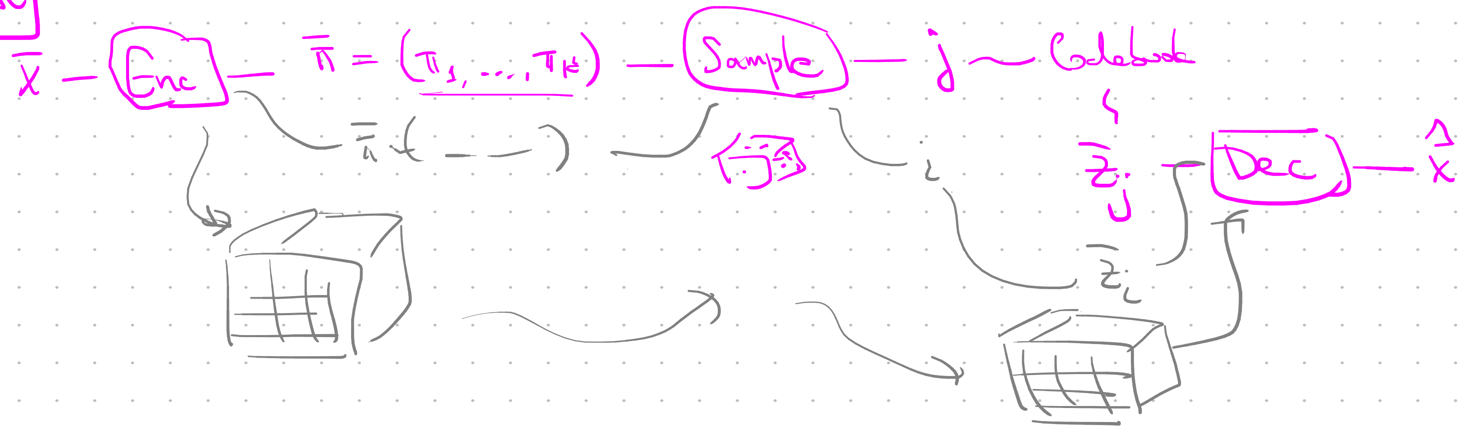
VAE



dVAE



Gumbel-Max trick

$z \sim \text{Mult}(\bar{\pi})$, $\bar{\pi} = (\pi_1 \dots \pi_K)$ $\iff$ $z = \underset{i}{\arg\max}\,(g_i + \log \pi_i)$,

where $g_i \sim$ Gumbel

$p(g_i) = e^{-(g_i + e^{-g_i})}$, $F(g_i) = e^{-e^{-g_i}}$

Proof.

$$p(z=k) = p(\forall_j \; g_k + \log \pi_k \ge g_j + \log \pi_j) =$$

$$= \int_{-\infty}^{\infty} p(\forall_j \; g_j + \log \pi_j \le \overbrace{g_k + \log \pi_k}^{\text{Const}} | g_k) \, p(g_k) \, dg_k$$

$$= \int_{-\infty}^{\infty} \prod_{j \ne k} p(g_j \le g_k + \log \pi_k - \log \pi_j | g_k) \cdot p(g_k) \, dg_k =$$

$$= \int_{-\infty}^{\infty} \prod_{j \ne k} e^{-e^{-g_k - \log \pi_k + \log \pi_j}} \cdot e^{-g_k - e^{-g_k}} \, dg_k =$$

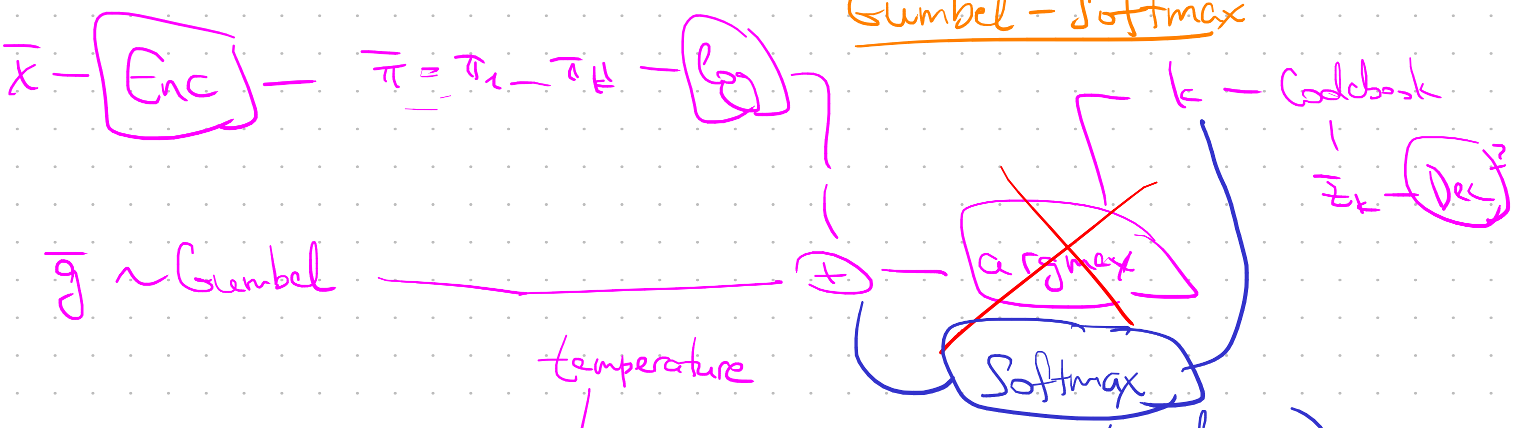
$$= \int e^{-\sum_{j\neq k}\pi_j \cdot e^{-g_k-\log\pi_k}} \cdot e^{-g_k-\log\pi_k+\log\pi_k-e^{-g_k\pm\log\pi_k}}\, dg_k$$

$$-e^{-g_k-\log\pi_k+\log\pi_k} = -\pi_k \cdot e^{-}$$

$$= \int e^{-e^{-g_k-\log\pi_k}\left(\sum_{j\neq k}\pi_j+\pi_k\right)} \cdot e^{-g_k-\log\pi_k} \cdot \pi_k \, dg_k =$$

$$= \pi_k \cdot \int_{-\infty}^{\infty} e^{-(g_k+\log\pi_k)} \cdot e^{-e^{-(g_k+\log\pi_k)}}\, dg_k = \pi_k$$

Gumbel - Softmax



temperature

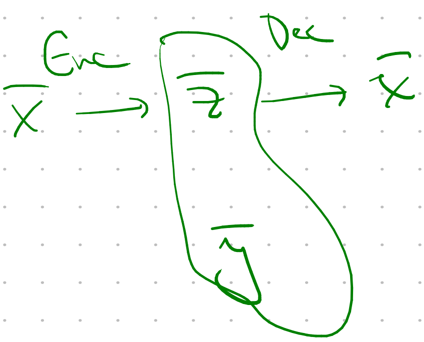
$$z_i = \text{softmax}\left(\frac{1}{\tau}(g_i+\log\pi_i)\right) = \frac{e^{\frac{1}{\tau}(g_i+\log\pi_i)}}{\sum_j e^{\frac{1}{\tau}(g_j+\log\pi_j)}}$$

$\tau \to 0$

$\Rightarrow$ argmax

image, text

$$D = \{(\bar{x}, \bar{y})\}$$

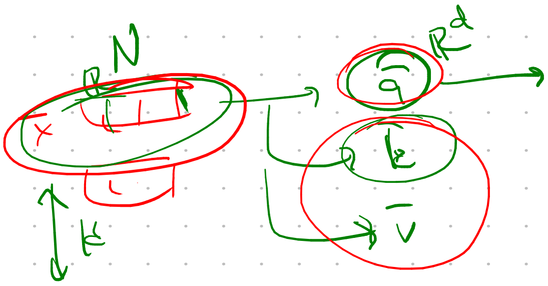


Enc: $p(\bar{z}|\bar{x})$

Dec: $p(\bar{x}|\bar{z})$

Transf: $p(\bar{z}, \bar{y})$

$$\log p(\bar{x}, \bar{y}) \geq E_{q(\bar{z}|\bar{x})}\left[\log p(\bar{x}|\bar{y}, \bar{z}) - \beta \cdot KL\left(q(\bar{z}|\bar{x}) \,\|\, p(\bar{y}, \bar{z})\right)\right]$$



α_{ij} $O(K^2)$

$K \to Q \to \mathrm{softmax}\left(\frac{1}{\sqrt{d}} QK^T\right)V$

$\bar{x} \to K, V$