$p(x)$ $\quad \sum_x p(x) = 1, \quad p(x) \geq 0$

$p(x,y)$ $\quad p(x) = \sum_y p(x,y)$

$p(x|y) = \frac{p(x,y)}{p(y)}$ $\quad p(x,y) = p(x|y)p(y) =$

likelihood $= p(y|x)p(x)$

posterior

prior

$\theta^* = \arg\max p(\theta|D) = \frac{p(\theta)p(D|\theta)}{p(D)}$

w

PPOOOPP

$\theta = p(\text{„решка"})$

$1-\theta$

$p(D|\theta) = \theta^4(1-\theta)^3 \xrightarrow[\theta]{} \max$

$= \prod_{d \in D} p(d|\theta)$

$\frac{\partial p(D|\theta)}{\partial \theta} = 4\theta^3(1-\theta)^3 - 3(1-\theta)^2\theta^4 = 0$

$\theta^3(1-\theta)^2(4(1-\theta) - 3\theta) = 0$

$\theta^3(1-\theta)^2(\cancel{4\theta - 3(1-\theta)}) = 0$

$\theta = 0 \quad \theta = 1 \quad \text{"} 0 \quad \theta = 4/7$ $\quad \theta^n(1-\theta)^m$

$$\boxed{\theta_{ML} = \frac{n}{n+m}}$$

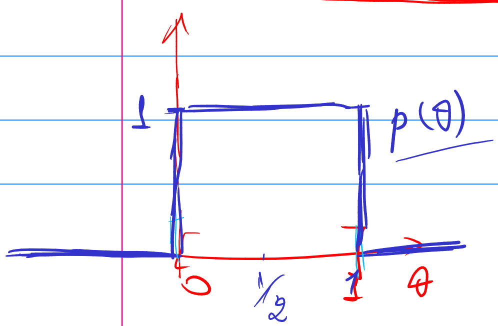
$p(\theta|D) = \frac{p(\theta)p(D|\theta)}{p(D)}$

$p(D|\theta) = \theta \to \max$

$\max_\theta$

$p(D) = \int p(\theta)p(D|\theta)\,d\theta$

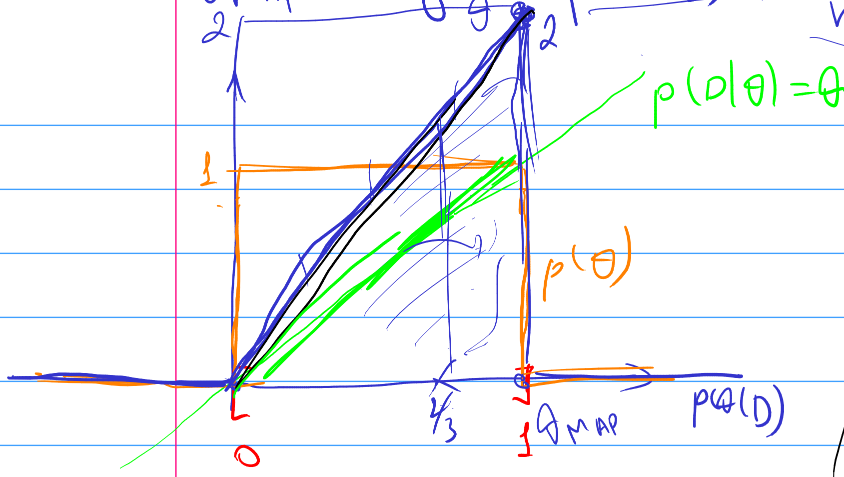
$p(\theta|D) \propto p(\theta)p(D|\theta)$



$p(\theta) = \begin{cases} 1, & \theta \in [0,1] \\ 0, & \theta \notin [0,1] \end{cases}$ $\quad p(D|\theta) = \theta^n(1-\theta)^m$

$p(\theta|D) = \begin{cases} \frac{\theta^n(1-\theta)^m}{p(D)}, & \theta \in [0,1] \\ 0, & \theta \in [0,1] \end{cases}$

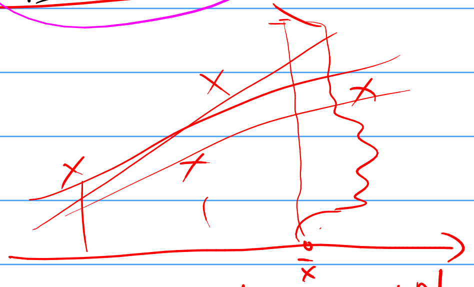
$$\theta_{MAP} = \underset{\theta}{\operatorname{argmax}}\; p(\theta|D) = \frac{n}{n+m}$$



$\theta \sim p(\theta|D)$ $\quad$ $p(x) = \sum_y p(x,y)$

D $\quad$ $p(\theta)$, $p(D|\theta)$

$p(\theta|D)$



$D = \{(x_n, y_n)\}_{n=1}^{N}$

$D)$

$, |D)$

$p(\text{„решка"}|D)$

$$p(x|D) = \int p(x,\theta|D)\,d\theta =$$

$$= \int p(x|\theta, D)\, p(\theta|D)\,d\theta$$

$$p(x|\theta, D) = p(x|\theta)$$

$$= \int p(x|\theta)\,p(\theta|D)\,d\theta = \mathbb{E}_{p(\theta|D)}\left[p(x|\theta)\right]$$

$$p(\text{решка}|D) = \int_{-\infty}^{\infty} p(\text{решка}|\theta)\cdot p(\theta|D)\,d\theta = \int_0^1 \theta \cdot \frac{\theta^n(1-\theta)^m}{p(D)}\,d\theta = (*)$$

$$p(\text{решка}|\theta) = \theta$$

$$p(D) = \int_{-\infty}^{\infty} p(\theta)\,\underbrace{p(D|\theta)}_{\theta^n(1-\theta)^m}\,d\theta = \int_0^1 \theta^n(1-\theta)^m\,d\theta = \frac{n!\,m!}{(n+m+1)!}$$

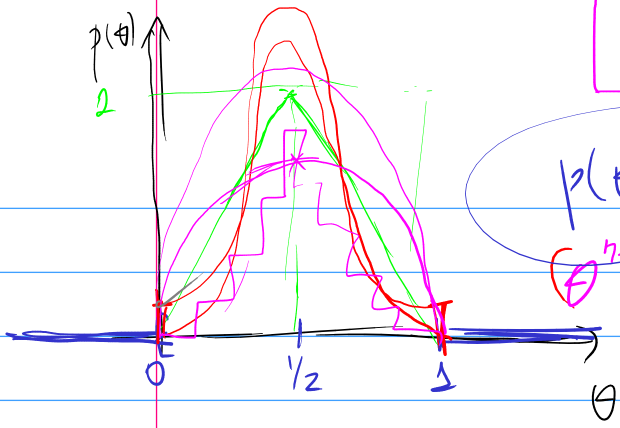
$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx = \frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{(\alpha-1)!\,(\beta-1)!}{(\alpha+\beta-1)!}$$

$$(*) = \frac{(n+m+1)!}{n!\,m!}\int_0^1 \theta^{n+1}(1-\theta)^m\,d\theta =$$

$$= \frac{\cancel{(n+m+1)!}}{\cancel{n!}\,\cancel{m!}}\cdot\frac{(n+1)!\,\cancel{m!}}{(n+m+2)!}$$

$$\boxed{p(\text{решка}|D) = \frac{n+1}{n+m+2}}$$

правило Лапласа



$$p(\theta) = \delta(\theta - 1/2) \cdot \theta^n$$

$$p(\theta|D) \propto p(\theta) \times p(D|\theta)$$

$$\left(\theta^{n+\alpha-1}(1-\theta)^{m+\beta-1}\right) \propto \left(\theta^{\alpha-1}(1-\theta)^{\beta-1}\right) \times \theta^n(1-\theta)^m$$

$$\boxed{p(\theta) = Beta(\theta|\alpha,\beta) = \frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}}$$

$\times$

$$p(D|\theta) = \theta^n(1-\theta)^m \propto \theta^{n+\alpha-1}(1-\theta)^{m+\beta-1} \propto Beta(\theta|n+\alpha, m+\beta)$$

$$Beta(\theta|\alpha,\beta) \times p\left(\begin{smallmatrix} n\times P \\ m\times O \end{smallmatrix}\middle|\theta\right) \propto Beta(\theta|n+\alpha, m+\beta)$$

Conjugate prior

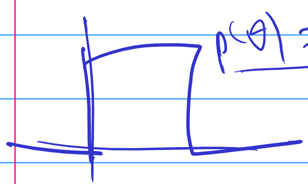
Bernoulli trials

$$p(\theta|\alpha) \cdot p(D|\theta) \propto p(\theta|\alpha')$$

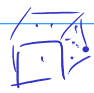
$$p(\theta|\tfrac{1}{2},\sigma) = \begin{cases} \dfrac{\frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2\sigma^2}(\theta-\frac{1}{2})^2}}{\int_0^1 N(\ldots)\,d\theta}, & \theta \in [0,1] \\ 0 & \theta \notin [0,1] \end{cases}$$

$$p(\theta) \propto e^{-\frac{1}{2\sigma^2}(\theta-\frac{1}{2})^2} \times p(D|\theta) = \theta^n(1-\theta)^m$$

$$= \theta^n(1-\theta)^m \cdot e^{-(\ldots\theta)^2}$$



$$p(\theta) = 1 = Beta(\theta|1,1)$$



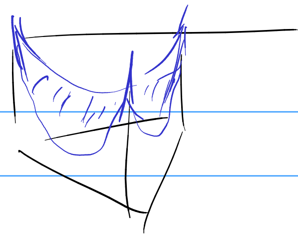
$$\bar\theta = (\theta_1,\theta_2,\ldots\theta_k) \qquad \sum_{i=1}^{k}\theta_i = 1$$

$$p(D|\bar\theta) = \theta_1^{n_1}\theta_2^{n_2}\ldots\theta_k^{n_k}$$

$$p(\bar{\theta} \mid \bar{\alpha}) = \frac{1}{\mathrm{Dir}(\alpha)} \theta_1^{\alpha_1 - 1} \theta_2^{\alpha_2 - 1} \cdots \theta_k^{\alpha_k - 1}$$

$\sum \theta_i = 1$

$\theta_i \geq 0$



$$\mathrm{Dir}\left(\bar{\theta} \,\middle|\, \left(\tfrac{1}{10}, \tfrac{1}{10}, \ldots \tfrac{1}{10}\right)\right)$$