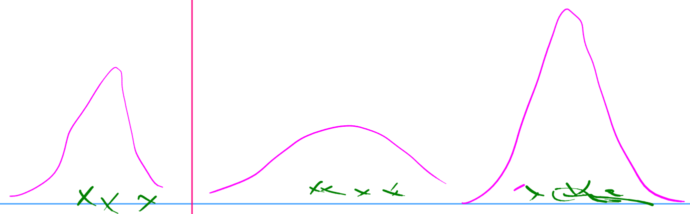


$$p(\bar x|\bar\mu,\Sigma,\bar\pi) = \sum_k \pi_k N(\bar x|\bar\mu_k,\Sigma_k)$$

$$L = \prod_n p(\bar x_n|\bar\pi,\bar\mu,\Sigma) = \prod_n\Big(\sum_k \pi_k N(\ldots)\Big) \xrightarrow[\pi,\mu,\Sigma]{} \max$$

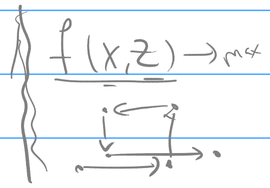
$z_{nk}=1$ iff $\bar x_n \in C_k$

E-шаг: $$\mathbb{E} z_{nk} = \frac{\pi_k N(\bar x_n|\bar\mu_k,\Sigma_k)}{\sum_\ell \pi_\ell N(\bar x_n|\bar\mu_\ell,\Sigma_\ell)}$$
$$\| \\ p(\bar x_n \in C_k|\bar x_n\in C,\bar\mu,\bar\pi,\Sigma)$$

$\mathbb{E}[z_{nk}]$

M-шаг: $$\bar\pi,\bar\mu,\Sigma = \arg\max_{\pi,\mu,\Sigma} p(X,Z|\pi,\mu,\Sigma) = \arg\max_{\pi,\mu,\Sigma} \prod_n\prod_k \pi_k N(\bar x_n|\bar\mu_k,\Sigma_k)$$

$$\sum_{n,k}\Big[\mathbb{E}z_{nk}\ln\pi_k + \mathbb{E}z_{nk}\ln N(\bar x_n|\bar\mu_k,\Sigma_k)\Big]$$



$$p(X|\theta)\xrightarrow[\theta]{}\max$$
$$p(X,Z|\theta)\xrightarrow[\theta]{}\max$$

$$\ln p(X|\theta) - \ln p(X|\theta^{(m)}) =$$

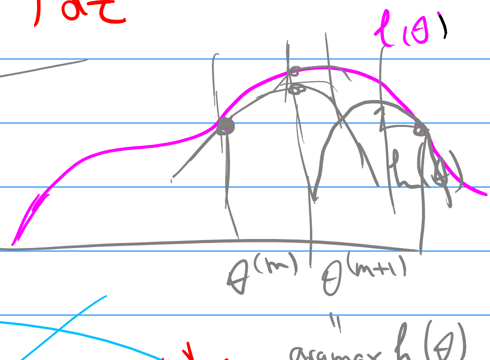
$$= \ln\int p(X,Z|\theta)\,dZ - \ell_m = \qquad // \ \mathbb{E}_{p(Z|X,\theta^{(m)})}\frac{p(X,Z|\theta)}{p(\ldots)}$$

$\ln\mathbb{E} \geq \mathbb{E}\ln$

$$= \ln\int\frac{p(X,Z|\theta)}{p(Z|X,\theta^{(m)})}p(Z|X,\theta^{(m)})\,dZ - \ell_m \geq$$

$$\geq \int\ln\frac{p(X,Z|\theta)}{p(Z|X,\theta^{(m)})}p(Z|X,\theta^{(m)})\,dZ - \ln p(X|\theta^{(m)})$$

$$\ell(\theta) = \ln p(X|\theta) - \ln p(X|\theta^{(m)}) \geq \underbrace{\int\ln\frac{p(X,Z|\theta)}{p(Z|X,\theta^{(m)})\,p(X|\theta^{(m)})}p(Z|X,\theta^{(m)})\,dZ}_{h(\theta,\theta^{(m)})}$$

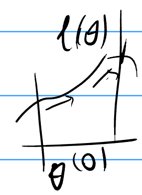


$$h(\theta,\theta^{(m)}) = \int\ln\frac{p(X,Z|\theta)}{p(X,Z|\theta^{(m)})}p(Z|X,\theta^{(m)})\,dZ \xrightarrow[\theta]{}\max$$

$$= \int\ln p(X,Z|\theta)\,p(Z|X,\theta^{(m)})\,dZ - \cancel{\int\ln p(X,Z|\theta^{(m)})\,p(Z|X,\theta^{(m)})\,dZ}$$

$$Q(\theta,\theta^{(m)}) = \int\ln p(X,Z|\theta)\,p(Z|X,\theta^{(m)})\,dZ =$$

$$= \mathbb{E}_{p(Z|X,\theta^{(m)})}\big[\ln p(X,Z|\theta)\big]$$



for $m=0,\ldots$

E-шаг: находим $Q(\theta,\theta^{(m)})$

M-шаг: $\theta^{(m+1)} := \arg\max_\theta Q(\theta,\theta^{(m)})$

$$\theta^{(0)} = \Big(\underbrace{\pi_k^{(0)}}_{\sim 1/K}, \underbrace{\bar\mu_k^{(0)}}_{\sim \text{Data}}, \underbrace{\Sigma_k^{(0)}}_{\sim \alpha \cdot I}\Big), \quad k = 1..K$$

$\bar z_n = (\ldots z_{nk} \ldots)$

$z_{nk} = 1$ iff $\bar x_n \in C_k$

$$p(X,Z|\theta) = \prod_n \prod_k \Big(\pi_k \mathcal{N}(\bar x_n|\bar\mu_k, \Sigma_k)\Big)^{z_{nk}}$$

$$\ln p(X,Z|\theta) = \sum_n \sum_k z_{nk} \cdot \ln\big(\pi_k \mathcal{N}(\bar x_n|\bar\mu_k, \Sigma_k)\big)$$

$$Q(\theta, \theta^{(m)}) = \mathbb{E}_{Z|X,\theta^{(m)}}\big[\ln p(X,Z|\theta)\big] =$$

$$= \mathbb{E}_{Z|X,\theta^{(m)}}\Big[\sum_n \sum_k z_{nk} \ln\big(\pi_k \mathcal{N}(\bar x_n|\bar\mu_k, \Sigma_k)\big)\Big] =$$

$$= \sum_n \sum_k \big(\ln \pi_k \mathcal{N}(\bar x_n|\bar\mu_k, \Sigma_k)\big) \cdot \mathbb{E}_{Z|X,\theta^{(m)}}[z_{nk}]$$

$$\theta^{(m+1)} = \arg\max_\theta Q(\theta, \theta^{(m)})$$

$$\mathbb{E}_{Z|X,\theta^{(m)}}[z_{nk}] = \frac{p(\bar x_n, z_{nk}|\theta^{(m)})}{\sum_\ell p(\bar x_n, z_{n\ell}|\theta^{(m)})} = \frac{\pi_k^{(m)} \mathcal{N}(\bar x_n|\mu_k^{(m)}, \Sigma_k^{(m)})}{\sum_\ell \pi_\ell^{(m)} \mathcal{N}(\bar x_n|\bar\mu_\ell^{(m)}, \Sigma_k^{(m)})}$$

$y = (X, \underline{Z})$ $\qquad p(x,z|\theta) = \prod_n \theta_i \theta_j$

$$p(X|\theta) = \prod_n \Big(\sum_k \theta_{ik}\theta_{jk}\Big) \to \max$$

$\bar x$ $\quad$ ACGTGTAC / ACTTGTTC $\qquad C_k$

$p_{isk} = p(x_i = s | \bar x \in C_k)$ $\qquad \sum_s p_{isk} = 1 \quad \forall i,k$

$\pi_k = p(\bar x \in C_k)$

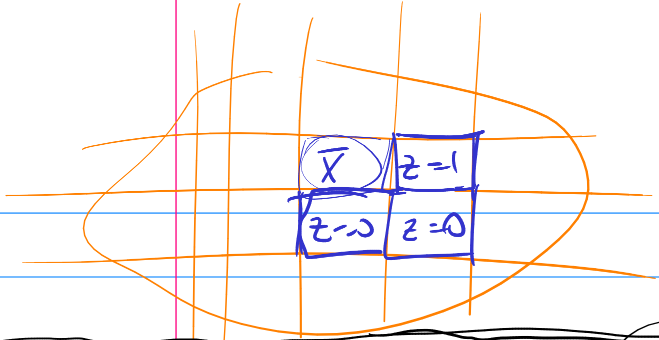
E-war: $$\mathbb{E} z_{nk} = p(\bar x_n \in C_k) = \frac{\pi_k p(\bar x_n|C_k)}{\sum_\ell \pi_\ell p(\bar x_n|C_\ell)} = \frac{\pi_k \cdot \prod_i p_{i,x_{in},k}}{\sum_\ell \pi_\ell \prod_i p_{i,x_{in},\ell}}$$

M-war: $$\theta := \arg\max_{\pi, p} \prod_n \prod_k \Big(\pi_k \prod_i p_{i,x_{in},k}\Big)^{\mathbb{E} z_{nk}} =$$

$$= \arg\max_{\pi, p} \sum_{n,k} \mathbb{E} z_{nk} \Big(\ln \pi_k + \sum_i \ln p_{i,x_{in},k}\Big)$$

$$\begin{pmatrix} \vdots \\ \ell_i \\ \vdots \end{pmatrix} \qquad \ell_i - \underbrace{\log\big(\sum e^{\ell_i}\big)}_{}$$

np.log(np.sum(np.exp(...)))



$y=1$ — „суслик водится“ $\qquad \eta(\bar x) = \bar w^T \bar x + w_0$

$$p(y=1\mid\bar x) = \sigma(\eta(\bar x)) = \frac{1}{1+e^{-\eta(\bar x)}}$$

$$\pi = p(y=1)$$

	$d=0$	$d=1$
$x=0$	π_{00}	π_{01}
$x=1$	π_{10}	π_{11}

$p(d\mid x) = \frac{n_{d,x}}{n_x}$ — prospective

$\pi_0 = p(s=1\mid d=0)$
$\pi_1 = p(s=1\mid d=1)$ — retrospective

$$\sigma(\eta(x)) = \frac{e^{\eta(x)}}{1+e^{\eta(x)}}$$

$$\pi_{00} = p(d=0, x=0 \mid s=1) \quad \leftarrow \sigma(\eta(x)) \quad „\pi_1“$$

$$\boxed{\frac{\pi_{xd}}{\pi_{xd}+\pi_{x\bar d}}} = p(d\mid s=1, x) = \frac{p(d\mid x)\,p(s=1\mid d,\cancel{x})}{p(d\mid x)\,\underbrace{p(s=1\mid d,\cancel{x})}_{\pi_1} + p(\bar d\mid x)\,\underbrace{p(s=1\mid \bar d,\cancel{x})}_{\pi_0}} =$$

$$= \frac{\pi_1 \sigma}{\pi_1\sigma + \pi_0(1-\sigma)} = \frac{\pi_1 e^{\eta(x)}}{\pi_1 e^{\eta(x)} + \pi_0} = \frac{1}{1+\left(\frac{\pi_0}{\pi_1}\right)e^{-\eta(x)}} =$$

$$= \frac{1}{1+e^{-\eta(x)+\log\frac{\pi_0}{\pi_1}}} = \sigma(\eta^*(x))$$

$$\eta^*(x) = \eta(x) - \log\frac{\pi_0}{\pi_1}, \qquad \eta(x) = wx + w_0 \quad + \log\frac{\pi_1}{\pi_0}$$

$$\sigma(\eta^*(\bar x)) = \frac{\pi_{xd}}{\pi_{xd}+\pi_{x\bar d}}$$

$$\eta^*(\bar x) = w^T x + w^*, \qquad w^* = w_0 - \log\frac{\pi_0}{\pi_1}$$

$$p(z=1\mid s=1,\bar x) = p(z=1\mid y=1, s=1, \cancel{\bar x})\, p(y=1\mid s=1,\bar x) + \cancel{p(z=1\mid y=0, s=1,\bar x)\, p(y=0\mid s=1,\bar x)}$$

$$= \frac{p(z=1, y=1\mid s=1)}{p(y=1\mid s=1)} = \frac{\frac{n_p}{n_p+n_u}}{\frac{n_p+\pi n_u}{n_p+n_u}} = \frac{n_p}{n_p+\pi n_u}$$

$n_p = \#\{z=1\}$
$n_u = \#\{z=0\}$
πn_u

$$p(z=1\mid s=1,\bar x) = \frac{n_p}{n_p+\pi n_u}\,\sigma(\eta^*(\bar x))$$

$$\eta^*(\bar x) = \eta(\bar x) + \log\frac{n_p+\pi n_u}{(1-\pi)n_u}$$

$$\frac{\pi_1}{\pi_0} = \frac{p(s=1, y=1)}{p(s=1, y=0)} = \frac{n_p+\pi n_u}{(1-\pi)n_u}$$

$$\prod_n p(z_n\mid s=1,\bar x_n) \longrightarrow \max$$

E-шаг: $$y^{(m)} = E\, y^{(m)} = \frac{e^{\eta(\bar x)}+1}{1+e^{\eta(\bar x)}+2}$$

$$C \cdot \frac{1}{1+e^{-\eta^*}} =$$

$\pi = p(y=1)$

M-шаг: логист. регр. $\sigma(\eta^*(\bar{x})) \sim y^{(m)}$

E-шаг: $\eta(\bar{x}) = \eta^*(\bar{x}) - \log \dfrac{n_p + \pi n_u}{\pi n_u}$

$$y^{(m)} = \frac{e^{\eta(\bar{x})} + 1}{1 + e^{\eta(\bar{x})} + 2}$$