RECOMMENDER SYSTEMS

Sergey Nikolenko

Harbour Space University, Barcelona, Spain
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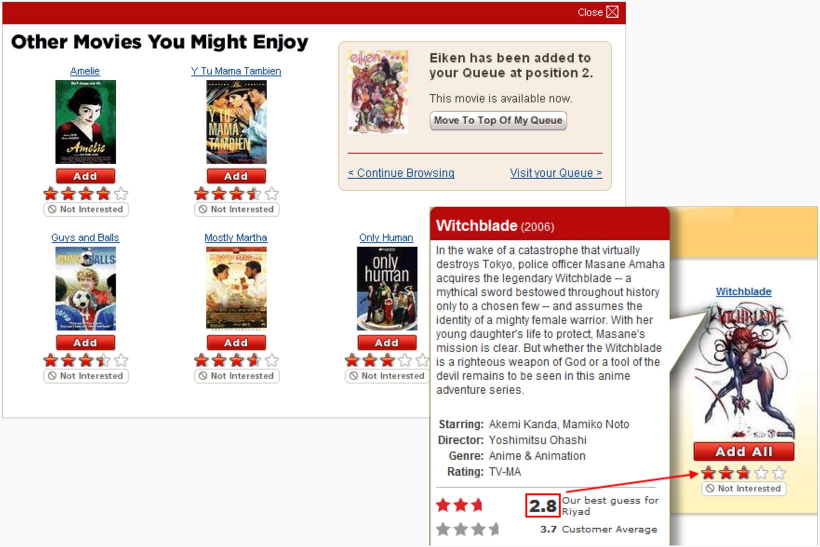
INTRODUCTION

PREVIEW

- Main ideas:
  (1) classical collaborative filtering: nearest neighbors and how to scale them;
  (2) matrix decompositions: why, how, and what else;
  (3) extensions: what can we add to a recommender system; in particular, content-based recommendations;
  (4) non-personalized recommendations with emphasis on speed.

- Recommender systems analyze the users' interests and aim to predict what is most interesting for a specific user at this time.
- Leading recommender systems usually fall into one of two categories:
  (1) we "sell" some goods or services online; the users either explicitly rate the goods or simply buy something; we want to recommend an item that would most interest this user; examples: Netflix, Amazon;
  (2) we are a web portal and make money through advertising; we need to show links that the users will click: Mail.Ru, Yahoo!, Google, Yandex, content providers, news web sites.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Other Movies You Might Enjoy
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Witchblade (2006)
In the wake of a catastrophe that virtually destroys Tokyo, police officer Masane Amaha acquires the legendary Witchblade -- a mythical sword bestowed throughout history only to a chosen few -- and assumes the identity of a mighty female warrior. With her young daughter's life to protect, Masane's mission is clear. But whether the Witchblade is a righteous weapon of God or a tool of the devil remains to be seen in this anime adventure series.
Starring: Akemi Kanda, Mamiko Noto
Director: Yoshimitsu Ohashi
Genre: Anime & Animation
Rating: TV-MA
2.8 Our best guess for Riyad
3.7 Customer Average
Witchblade
Add All
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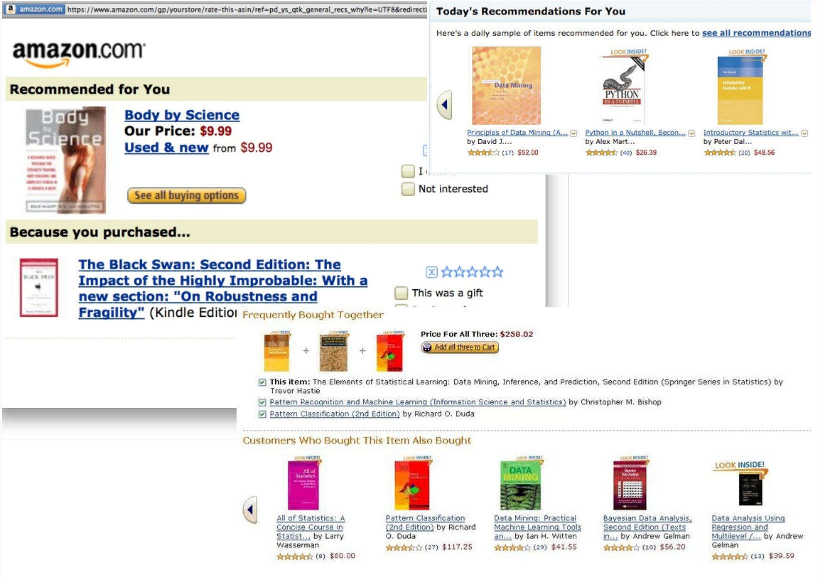
AMAZON



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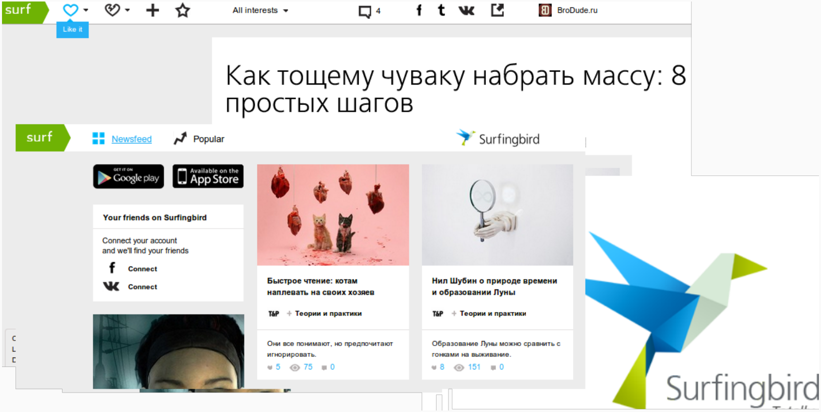


ONLINE VS. OFFLINE

- A recommender system may have two different “levels”:
    - global estimates, slowly changing features and preferences, dependence on permanent user features (geography, demographics) etc.;
    - short-term trends, hotness, fast changes in interest.

- These are different problems with different methods, so there are two classes of models:
  - *offline models* extract global dependencies (this is usually called collaborative filtering). The purpose is to find and recommend something for a specific user, work with “long tails” of the distributions of both users and items;
  - *online models* must react very quickly, they extract short-term trends and recommend whatever is hot right now.

CLASSICAL COLLABORATIVE FILTERING

- Notation:
  - i always denotes users (N in total, $i = 1..N$);
  - a always denotes items (web sites, goods, movies...) that we recommend (M in total, $a = 1..M$);
  - when a user i rates item a, it is captured as a response (rating) $r_{i,a}$; this is a random value, of course.
- The goal is to predict estimates $r_{i,a}$ by features x_i and x_a for all elements in the dataset and some already known $r_{i',a'}$.
- We denote prediction by $\hat{r}_{i,a}$.

- Nearest neighbors: let's introduce a distance (similarity) between users and recommend to you what people similar to you have liked.
- Distance:
  - correlation coefficient (Pearson's coefficient)

$$w_{i,j} = \frac{\sum_a \left(r_{i,a} - \bar{r}_a\right)\left(r_{j,a} - \bar{r}_a\right)}{\sqrt{\sum_a \left(r_{i,a} - \bar{r}_a\right)^2}\sqrt{\sum_a \left(r_{j,a} - \bar{r}_a\right)^2}},$$

    where $\bar{r}_a$ is the average rating of item a among all users;
  - cosine of the angle between rating vectors:

$$w_{i,j} = \frac{\sum_a r_{i,a} r_{j,a}}{\sqrt{\sum_a r_{i,a}^2}\sqrt{\sum_a r_{j,a}^2}}.$$

- The simplest way to construct a prediction for a new rating $\hat{r}_{i,a}$ is the sum of ratings for other users weighted by their similarities to user i:

$$\hat{r}_{i,a} = \bar{r}_a + \frac{\sum_j \left(r_{j,a} - \bar{r}_j\right) w_{i,j}}{\sum_j |w_{i,j}|}.$$

- This is called the GroupLens algorithm, the grandfather of recommender systems.
- We can restrict the sum to nearest neighbors so that we don't have to sum over everybody:

$$\hat{r}_{i,a} = \bar{r}_a + \frac{\sum_{j \in \mathrm{knn}(i)} \left(r_{j,a} - \bar{r}_j\right) w_{i,j}}{\sum_{j \in \mathrm{knn}(i)} |w_{i,j}|}.$$

- Natural extension: let's re-weigh the items according to how often they have been rated; if something is liked by everybody it's not very useful.
- Inverse user frequency: $f_a = \log \frac{N}{N_a}$, where N is the total number of users, N_a – number of users who rated a. We get

$$w_{i,j}^{\mathrm{iuf}} = \frac{\sum_a f_a \sum_a f_a r_{i,a} r_{j,a} - \left(\sum_a f_a r_{i,a}\right)\left(\sum_a f_a r_{j,a}\right)}{\sqrt{\sum_a f_a \left(\sum_a f_a r_{i,a}^2 - \left(\sum_a f_a r_{i,a}\right)^2\right)}\sqrt{\sum_a f_a \left(\sum_a f_a r_{j,a}^2 - \left(\sum_a f_a r_{j,a}\right)^2\right.}}$$

and for the cosine:

$$w_{i,j}^{\mathrm{iuf}} = \frac{\sum_a f_a^2 r_{i,a} r_{j,a}}{\sqrt{\sum_a (f_a r_{i,a})^2}\sqrt{\sum_a (f_a r_{j,a})^2}}.$$

- Symmetrical approach – item-based collaborative filtering. Compute similarity between items, choose similar items.
- Amazon: customers who bought this item also bought...
- Can be more efficient since we can always compute item similarity offline and get new predictions for a user online.

- It's hard to find nearest neighbors algorithmically (k-d-trees don't work in large dimensions).
- Large-scale recommender systems use approximations.
- E.g., LSH (locality sensitive hashing) with min-hashing:
    - take several hash functions, compute them for every item;
    - for every user compute the minimal value of hash functions for its items;
    - look for neighbors only among those users that have identical values in at least one hash.

WHAT ABOUT LIKES?

- We have only considered explicit ratings.
- But often we only have the sets of “consumed” or “liked” items I and J for users i and j:
    - likes (usually very few dislikes);
    - bought goods without explicit ratings.
- This is called *implicit feedback*. What do we do?

- We need to define distance between sets; Jaccard similarity

$$w_{i,j} = \mathrm{Jaccard}(I, J) = \frac{|I \cap J|}{|I \cup J|}.$$

- We can introduce user weights and then use GroupLens.

- Jaccard similarity is even more popular for item-based CF.
- We define similarity between a and b via the sets of users A and B who consumed it:

$$w_{a,b} = \mathrm{Jaccard}(A, B) = \frac{|A \cap B|}{|A \cup B|}.$$

- Often works well, but there are problems.
- E.g., what if item a is rare, b is more popular, and almost all $i \in A$ have also consumed b? (very common situation)

WHAT ABOUT LIKES?

- Jaccard similarity does not suit imbalanced cases because it is symmetric.
- Let's make it asymmetric:

$$w_{a,b} = \frac{|A \cap B|}{|A|}, \quad w_{b,a} = \frac{|A \cap B|}{|B|}.$$

- Now the previous example is fine, but there still are problems.
- What if one of the items is very popular, and everybody has seen it? *Banana trap*.

- One more variation – the method of associations:

$$w_{a,b} = \frac{|A \cap B| \,/\, |A|}{|\bar{A} \cap B| \,/\, |\bar{A}|},$$

where $\bar{A}$ is the complement of A.
- In practice it is usually easy to simply try all of this and choose what works best.

MATRIX DECOMPOSITIONS

- Let's now try to construct a model for a rating.
- What does a rating that user i gives to item a consist of?
- There are kind and harsh users, good and bad items.
- *Baseline predictors* $b_{i,a}$:

$$b_{i,a} = \mu + b_i + b_a.$$

- To find the predictors, let's make it into a probabilistic model.
- We add normally distributed noise and get the model

$$r_{i,a} \sim \mathcal{N}\left(\mu + b_i + b_a, \sigma^2\right).$$

- We can now add prior distributions and optimize

$$b_* = \arg\min_b \sum_{(i,a)} \left(r_{i,a} - \mu - b_i - b_a\right)^2 + \lambda_1 \left(\sum_i b_i^2 + \sum_a b_a^2\right).$$

- How do we train this model?

- That's just linear regression!
- Note that often ratings are binary (like/dislike).
- Then it makes more sense to use the logistic sigmoid:

$$b_{i,a} = \sigma(\mu + b_i + b_a), \quad \sigma(x) = \frac{1}{1 + e^{-x}}.$$

- And we now have logistic regression instead of linear.
- This is often a good idea even for several ratings.

- How do we personalize and train the rest of a rating?
- We have a huge $N \times M$ matrix where only some small fraction of elements are known.
- So we make assumptions on the structure of the matrix and predict the rest.

- SVD (singular value decomposition) – assume that matrix X has low rank and decompose it.
- But we can also get to the same model from the other side.
- Fix some number f of *latent factors* that define each item and the preferences of each user.
- A user is now a vector $p_i \in \mathbb{R}^f$; an item, a vector $q_a \in \mathbb{R}^f$.

- And we model the preference as a scalar product $q_a^\top p_i = \sum_{j=1}^{f} q_{a,j} p_{i,j}$.
- Adding baseline predictors, we get the following model for a rating:

$$\hat{r}_{i,a} \sim \mu + b_i + b_a + q_a^\top p_i.$$

- How do we train it?

- SGD – stochastic gradient descent.
- Compute the gradient of the likelihood function, iterate over training samples, update on every step:

$$
\begin{aligned}
b_i &:= b_i + \gamma \left(e_{i,a} - \lambda_2 b_i \right), \\
b_a &:= b_a + \gamma \left(e_{i,a} - \lambda_2 b_a \right), \\
q_{a,j} &:= q_{a,j} + \gamma \left(e_{i,a} p_{i,j} - \lambda_2 q_{i,j} \right) \text{ for all } j, \\
p_{i,j} &:= p_{i,j} + \gamma \left(e_{i,a} q_{a,j} - \lambda_2 p_{i,j} \right) \text{ for all } j,
\end{aligned}
$$

where γ is the learning rate

- ALS – alternating least squares.
- Note that if in $\hat{r}_{i,a} \sim \mu + b_i + b_a + q_a^\top p_i$ we fix p_i, we will get linear regression w.r.t. q_a, and vice versa.
- ALS is similar to EM; repear until convergence:
    - fix p_i, train q_a;
    - fix q_a, train p_i.
- Usually faster and more robust than SGD.

- The same remark about the logistic variation: for binary ratings we can consider

$$\hat{r}_{i,a} \sim \sigma(\mu + b_i + b_a + q_a^\top p_i).$$

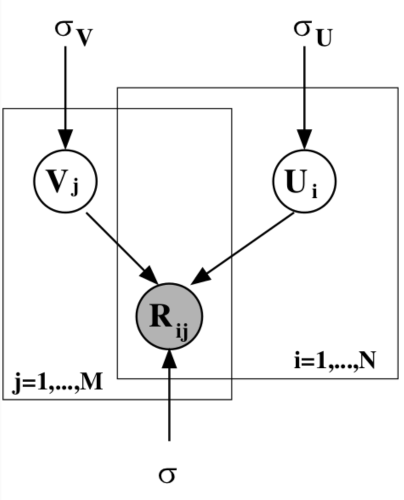
- Then the SGD will simply get the sigmoid's probabilities $\sigma'(x) = \sigma(x)(1 - \sigma(x))$.
- And in ALS instead of a linear regression we will have to train logistic regression on every iteration.

- We can also add external information to this model.
- Suppose there are extra factors y_a for the items that characterize a user based on what he has seen but not rated.
- The model is now

$$\hat{r}_{i,a} = \mu + b_i + b_a + q_a^\top \left(p_i + \frac{1}{\sqrt{|V(i)|}} \sum_{b \in V(i)} y_b \right),$$

  where $V(i)$ is the set of items that this user has seen ($\frac{1}{\sqrt{|V(i)|}}$ controls the variance).
- This is called SVD++.

- Suppose we want to decompose the rating matrix into low rank matrices
$$\hat{R} = U^\top V.$$
- The likelihood is
$$p(R \mid U, V, \sigma^2) = \prod_i \prod_a \left(\mathcal{N}(r_{i,a} \mid u_i^\top v_j, \sigma^2)\right)^{[i \text{ rated } a]}.$$
- Adding Gaussian priors on U and V, we get
$$p(U \mid \sigma_U^2) = \prod_i \mathcal{N}(U_i \mid 0, \sigma_U^2 I), \quad p(V \mid \sigma_V^2) = \prod_a \mathcal{N}(V_a \mid 0, \sigma_V^2 I).$$


σ_V
σ_U
V_j
U_i
R_{ij}
i=1,...,N
j=1,...,M
σ

- If we simply fix σ^2, σ_V^2, and σ_U^2, they will serve as regularizers, as in regular SVD.
- The difference is that we can now find optimal $\sigma = (\sigma^2, \sigma_V^2, \sigma_U^2)$ by maximizing the total likelihood of the model

$$\sigma^* = \arg\max_\sigma p(R \mid \sigma) = \arg\max_\sigma \int p(R, U, V \mid \sigma) dU dV$$

with EM:

  - first fix σ and find

$$f(\sigma) = \mathrm{E}_{U,V|R,\sigma} \left[\log p(R, U, V \mid \sigma) \right];$$

  - then maximize

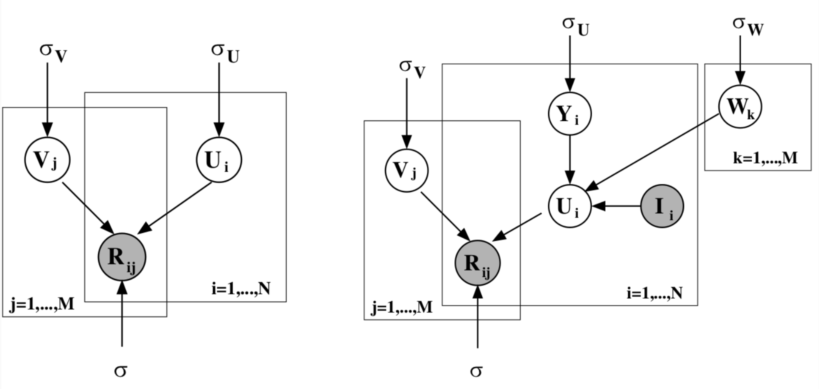
$$\sigma := \arg\max_\sigma f(\sigma).$$

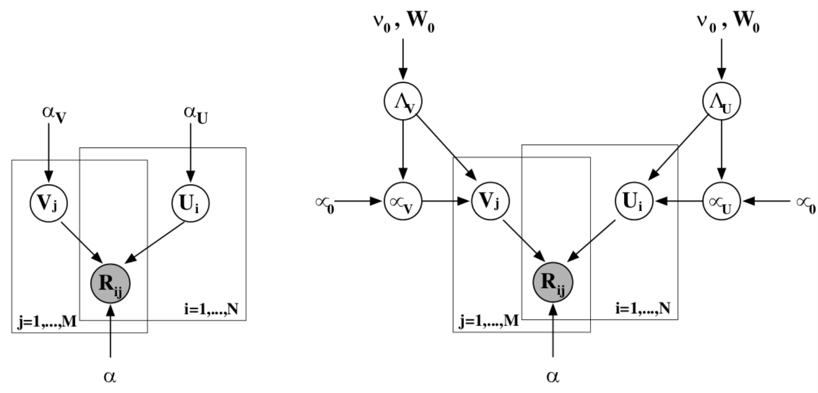
- Modification: users with few ratings in PMF will get posteriors very close to the "average user".
- To generalize better to this case, we can add factors that change the priors depending on how many and what items a user has rated:

$$U_i = Y_i + \frac{\sum_a [i \text{ rated } a] W_a}{\sum_a [i \text{ rated } a]}.$$

- And we get $p(W \mid \sigma_W^2) = \prod_i \mathcal{N}(W_i \mid 0, \sigma_W^2 I)$.

GRAPHICAL MODEL





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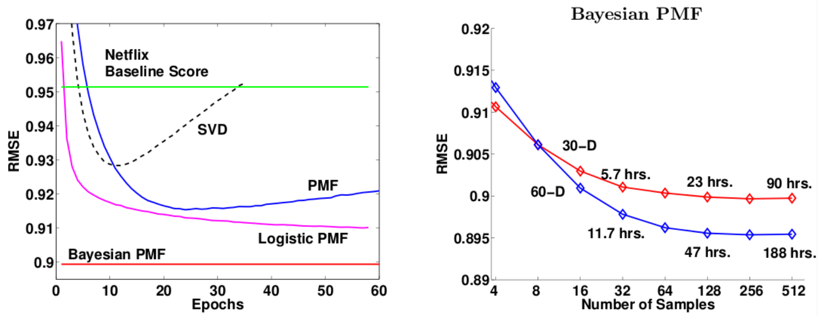
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RMSE
Netflix
Baseline Score
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PMF
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47 hrs.
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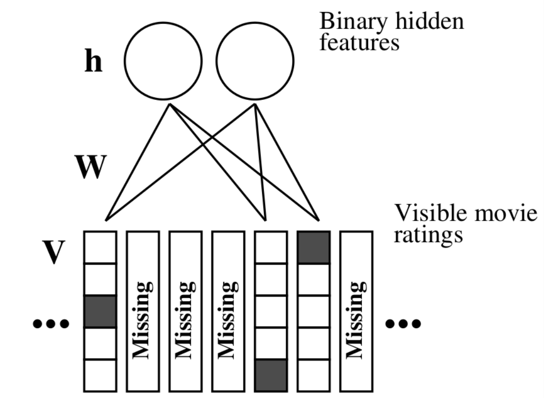
Number of Samples

- One more kind of probabilistic modeling – *restricted Boltzmann machines*.
- Undirected graphical model with two levels, visible and hidden.
- In collaborative filtering we model user preferences with RBM:



- As a result, on the hidden layer we train the user model.
- Training by contrastive divergence (approximation to max likelihood).
- RBM is not *better* than SVD, but often makes *different errors*, so a combination of these models is a big improvement.

- We have considered SVD (singular value decomposition) – decomposing matrix X into a product of low rank matrices.
- This is not the only matrix decomposition, and they are all interesting in their own way.
- PCA (principal components analysis) tries to explain as much variance in the original dataset as possible.
- But often directions to clusters in the data are not orthogonal, and PCA features are hard to interpret.

- SVD (singular decomposiion) does exactly what we need when ratings are available:
    - maximizes the likelihood for known ratings (minimizes rating prediction error);
    - works with sparse matrices (minimizes only over known ratings).
- But what do we do if there are no ratings, only the fact of use? SVD won't work...

- NMF (nonnegative matrix factorization): we still decompose as

$$X \approx UV^\top,$$

  where U is $n \times f$, V is $m \times f$, and f is much less than n and m.
- But we now require that elements U and V are nonnegative.
- The features, by the way, are often better interpretable – this is a common theme.
- NMF can be implemented with ALS, but with additional complications due to constraints.

QUALITY METRICS AND EXTENSIONS

QUALITY METRICS

- One more important topic: how do we evaluate the quality of recommendations? What is the quality metric?
- When we train SVD (maximize likelihood), we optimize the mean squared error.
- Netflix Prize, for instance, asked for the same: optimize RMSE.
- But what do we need in the real application? What do we have in the test set?

- The test set has ratings of certain items evaluated by the users.
- But the problem is to give a user new recommendations!
- We don’t have to predict all ratings, we need to find items with the largest rating.
- So in reality this is a *ranking* problem! And it’s best to take quality metrics from information retrieval, where search results are evaluated not by the RMSE of the relevance function.
- In what follows we consider the binary case (like/dislike) for simplicity.

- Classical quality metrics:
  (1) precision – number of “good” (relevant, positively ranked) items in the results divided by the total number of items in the results;
  (2) recall – number of “good” items in the results divided by the total number of “good” items in the database.
- Same problems: these parameters do not depend on the ranking, we need to know in advance how many recommendations will be needed.

- Ranking quality metrics:
  - NDCG, Normalized Discounted Commulative Gain; choose top-k recommendations (k can be larger than the necessary number) and compute

$$\mathrm{DCG}_k = \sum_{i=1}^{k} \frac{2^{\hat{r}_i} - 1}{\log_2(1+i)},$$
$$\mathrm{NDCG}_k = \frac{\mathrm{DCG}_k}{\mathrm{IDCG}_k},$$

    where $\hat{r}_i$ is our estimate of the rating of item on position i, and IDCG_k is the value of DCG_k in the ranking by true values (from the test set);
  - NDCG ranges from 0 to 1 but it's hard to interpret as probability.

- Ranking quality metrics:
  - AUC, Area Under (ROC) Curve – the probability of the event that a randomly chosen pair of items with different ratings will be ranked correctly (the higher rating will be higher in the results);
  - in the binary case there is a closed form:

$$\hat{A} = \frac{S_0 - n_0(n_0 + 1)/2}{n_0 n_1},$$

    where n_0, n_1 is the number of items that the user liked and disliked, $S_0 = \sum p_i$ is the sum of positions for the liked items in the results.

- Ranking quality metrics:
  - but simple metrics are also important because a user often looks only at the very top recommendations;
  - WTA (winner takes all) – 1 if the top recommendation is a “like” and 0 otherwise;
  - Top$k$ – share of positive ratings among top-k recommendations (Top10 is sometimes called MAP – mean average precision).

- Problem: cold start.
- If we don't know anything, there's nothing we can do.
- But usually there is some set of external features, and we can try to predict the SVD features:
    - with a simple regression over the features;
    - (usually for items) with topic modeling!

- For user features x_i and item features x_a we consider the model

$$r_{i,a} \sim \mu + b_{\text{user}}(x_i) + b_{\text{item}}(x_a) + q_a^\top p_i(t),$$

where

$$b_{\text{user}}(x_i) \sim \mathcal{N}(u(x_i), \sigma_u^2),$$
$$b_{\text{item}}(x_i) \sim \mathcal{N}(v(x_i), \sigma_v^2),$$

and as u and v we can take any kind of regression [Agarwal, Chen, 2009].

- Or with content:
  - extract topics from the items (LDA); or other features if it's not text;
  - we get a distribution $z_{a,k}$ for every a;
  - and now we train the factors $s_{i,k}$ for how much a user "likes" these topics;
  - then for a new item we estimate the topics $\hat{z}_{a,k}$ with their content and then add to the model

$$r_{i,a} \sim \ldots + \sum_k s_{i,k} \hat{z}_{a,k},$$

    which helps for cold start w.r.t. items.
- We can also train topics that specifically reflect preferences (fLDA).

- Example: let's add time, i.e., we consider user features and baseline predictors as functions of time,

$$\hat{r}_{i,a} = \mu + b_i(t) + b_a(t) + q_a^\top p_i(t),$$

where

$$\begin{aligned} b_a(t) =& b_a + b_{a,\mathrm{Bin}(t)}, \\ b_i(t) =& b_i + \alpha_i \mathrm{dev}_i(t) + b_{i,t}, \\ p_{i,f}(t) =& p_{i,f} + \alpha_{i,f} \mathrm{dev}_i(t) + p_{i,f,t} + \frac{1}{\sqrt{|V(i)|}} \sum_{b \in V(i)} y_b, \\ \mathrm{dev}_i(t) =& \mathrm{sign}(t - t_i) \left| t - t_i \right|^\beta . \end{aligned}$$

- This is called timeSVD++, one of the main components of the Netflix Prize winner.

- Suppose that users come from a social network.
- I.e., we know their friends, a part of the social graph etc.
- We can add this to the recommender model:
    - filtering/reweighting in nearest neighbors;
    - additional terms in an SVD-like decomposition;
    - decomposing the trust matrix (from the social graph) together with the matrix of ratings, change prior distribution for PMF and so on.

- Filter bubble: how do we take a user outside the usual bubble?
- Metrics that value “interesting” results:
    - diversity – make items in the list less similar;
    - novelty – choose less common items (with few ratings);
    - serendipity – choose items that are not like the user’s history.
- We only need to be able to define the similarity of items (preferably without the ratings, by content).

CARS

- CARS (context-aware recommender systems) – we recommend in a context:
  - temporal;
  - situation;
  - geographical;
  - user behaviour, and so on.

- Formally this adds new dimensions to the preference matrix.
- We get a “hypercube” of data, there are tensor decomposition methods similar to SVD.
- But simple approaches like slicing and filtering often work as well as complicated tensor decompositions...

THANK YOU!

Thank you for your attention!