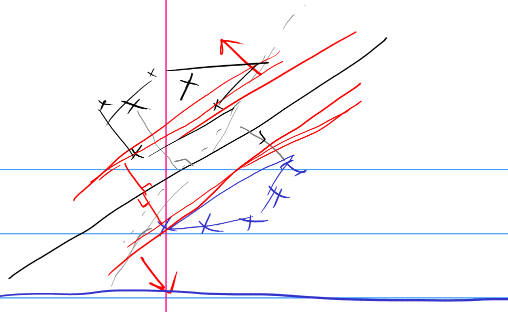
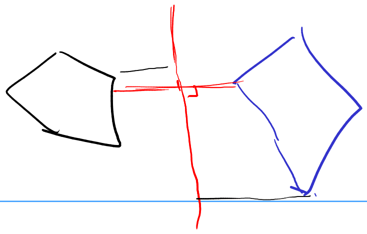




$\in \{\pm 1\}$

$t_n(\bar{w}^T x_n + w_0) \geq 0$

$$\max_{\bar{w}} \left[\min_n \frac{t_n(\bar{w}^T x_n + w_0)}{\|\bar{w}\|} \right] =$$

$$\arg\max_{\bar{w}} \frac{1}{\|\bar{w}\|} = \arg\min_{\bar{w}} \|\bar{w}\|^2 + C\sum z_n$$

$t_n(\bar{w}^T x_n + w_0) \geq 1 + z_n$

$z_n \geq 0$

d_H

$\min \|\bar{w}\|^2$

$t_n(\bar{w}^T x_n + w_0) - 1 \geq 0$

$\bar{w}^T\left(\sum_n \alpha_n t_n \bar{x}_n\right) + \ldots$

$$L(\bar{w}, w_0, \alpha) = \frac{1}{2}\|\bar{w}\|^2 - \sum_n \alpha_n\left(t_n(\bar{w}^T \bar{x}_n + w_0) - 1\right) \to \min \qquad \alpha_n \geq 0$$

$= \bar{w}^T\bar{w}$

$$\frac{\partial L}{\partial w_0} = -\sum_n \alpha_n t_n \qquad \sum_n \alpha_n t_n = 0$$

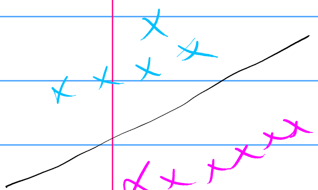
$$\nabla_{\bar{w}} L = \bar{w} - \sum_n \alpha_n t_n \bar{x}_n = 0 \qquad \boxed{\bar{w} = \sum_n \alpha_n t_n \bar{x}_n}$$

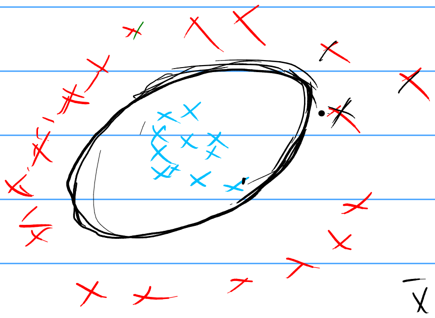
$$L = \frac{1}{2}\left(\sum_n \alpha_n t_n \bar{x}_n\right)^T\left(\sum_m \alpha_m t_m \bar{x}_m\right) - \sum_n\left[\alpha_n t_n \bar{x}_n^T\left(\sum_m \alpha_m t_m \bar{x}_m\right)\right] + \sum_n \alpha_n \to \min$$

$$= \frac{1}{2}\sum_{n,m} \alpha_n \alpha_m t_n t_m \bar{x}_n^T \bar{x}_m - \sum_n \alpha_n \alpha_m t_n t_m \bar{x}_n^T \bar{x}_m + \sum_n \alpha_n$$

$$\boxed{L(\bar{\alpha}) = \sum_n \alpha_n - \frac{1}{2}\sum_{n,m} t_n t_m (\bar{x}_n^T \bar{x}_m)\, \alpha_n \alpha_m} \xrightarrow[\bar{\alpha}]{} \min \qquad \alpha_n \geq 0$$

N





$y \sim w_0 + w_1 x + w_2 x^2$

$\begin{pmatrix} x \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} x^2 \\ x \\ 1 \end{pmatrix}$

$\mathbb{R}^2 \qquad \mathbb{R}^3$

$\mathbb{R}^d \qquad \mathbb{R}^D$

$$\bar{x}_n = \begin{pmatrix} x_{n1} \\ x_{n2} \end{pmatrix} \mapsto \begin{pmatrix} x_{n1}^2 \\ x_{n1}x_{n2} \\ x_{n2}^2 \\ x_{n1} \\ x_{n2} \end{pmatrix}$$

$\mathbb{R}^2 \qquad \mathbb{R}^5$

$\bar{x} \overset{\varphi}{\mapsto} \varphi(\bar{x})$

$y = w_0 + w_1 x_1^2 + w_2 x_1 x_2 + w_3 x_2^2 + w_4 x_1 + w_5 x_2$

SVM

$\bar{w}$

$y = \bar{w}^T \bar{x} + w_0$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_d \end{pmatrix} \overset{\varphi}{\mapsto} \begin{pmatrix} x_1^2 \\ x_1 x_2 \\ \ldots \\ x_d^2 \\ x_1 \\ \vdots \\ x_d \end{pmatrix}$$

$D = \frac{d(d-1)}{2} + d$

SVM $\quad \mathbb{R}^D$

$$L(\bar\alpha) = \sum_n \alpha_n - \frac{1}{2}\sum_n\sum_m t_n t_m (\bar{x}_n^T \bar{x}_m)\,\alpha_n \alpha_m \to \min$$

$$\boxed{\varphi(\bar{x}_n)^T \varphi(\bar{x}_m)} = k(\bar{x}_n, \bar{x}_m)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \overset{\varphi}{\mapsto} \begin{pmatrix} x_1^2 \\ x_1 x_2 \\ x_2^2 \end{pmatrix} \quad \mathbb{R}^D$$

$$\varphi\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}^T \varphi\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1^2 \\ \sqrt{2}x_1x_2 \\ x_2^2 \end{pmatrix}^T \begin{pmatrix} y_1^2 \\ \sqrt{2}y_1y_2 \\ y_2^2 \end{pmatrix} = x_1^2y_1^2 + 2x_1x_2y_1y_2 + x_2^2y_2^2$$

$$(x_1y_1 + x_2y_2)^2 = x_1^2y_1^2 + 2x_1y_1x_2y_2 + x_2^2y_2^2$$

kernel trick

$$\varphi(\bar{x})^T\varphi(\bar{y}) = \underbrace{(\bar{x}^T\bar{y})^2}_{\in\mathbb{R}^d} = k(\bar{x},\bar{y})$$

$$\begin{pmatrix} x_1^2 \\ | \\ x_1 \\ x_2 \end{pmatrix}^T \begin{pmatrix} y_1^2 \\ | \\ y_1 \\ y_2 \end{pmatrix}$$

$$(\bar{x}^T\bar{y})^2 + \bar{x}^T\bar{y}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \overset{\varphi}{\mapsto} \begin{pmatrix} x_1^{10} \\ \dots \\ x_2^{10} \\ \vdots \end{pmatrix} \in \mathbb{R}^{45}$$

$$\begin{pmatrix} x_1^k \\ | \\ x_2^k \end{pmatrix}^T \begin{pmatrix} y_1^k \\ | \\ y_2^k \end{pmatrix}$$

$$(\bar{x}^T\bar{y})^k = x_1^k y_1^k + k\,x_1^{k-1}x_2 \dots + x_2^k y_2^k$$

$$(\bar{x}^T\bar{y} + 1)^k - 1$$

$$(x_1y_1 + \dots + x_dy_d + 1)^k - 1$$

$$\bar\alpha = (\alpha_1, \dots, \alpha_N)$$

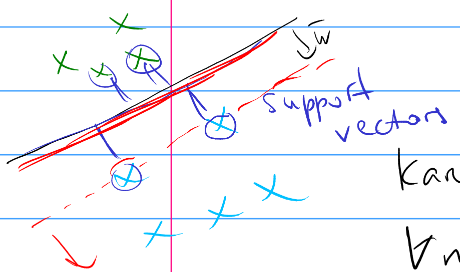
$$\bar{w} = \sum_n \alpha_n t_n \bar{x}_n$$

$$y(\bar{x}) = w_0 + \bar{w}^T\bar{x} = w_0 + \sum_n \alpha_n t_n \bar{x}_n^T\bar{x}$$

$$\boxed{y(\bar{x}) = w_0 + \sum_n \alpha_n k(\bar{x}_n, \bar{x})\, t_n}$$

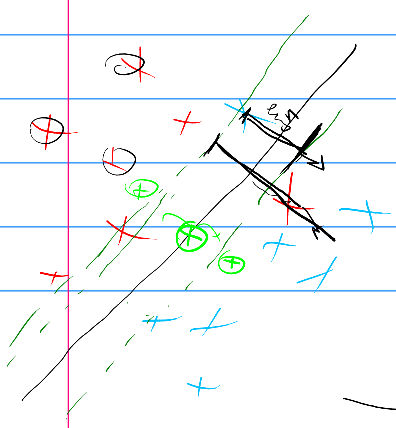
RBF

$$k(\bar{x}_n, \bar{x}) = e^{-\frac{1}{\sigma^2}\|\bar{x}_n - \bar{x}\|^2}$$



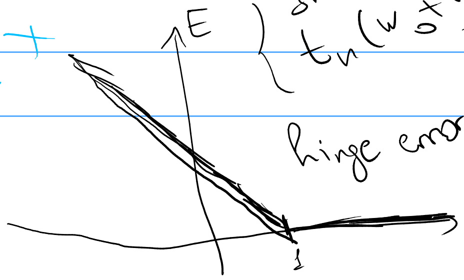
Karush-Kuhn-Tucker

$$\forall n \quad \alpha_n\left(t_n(w_0 + \bar{w}^T\bar{x}_n) - 1\right) = 0$$



$$C\sum \xi_n + \frac{1}{2}\|\bar{w}\|^2$$

$$\left\{\begin{array}{l} \xi_n \geq 0 \\ t_n(w_0 + \bar{w}^T\bar{x}_n) + \xi_n \geq 1 \end{array}\right.$$



hinge error function