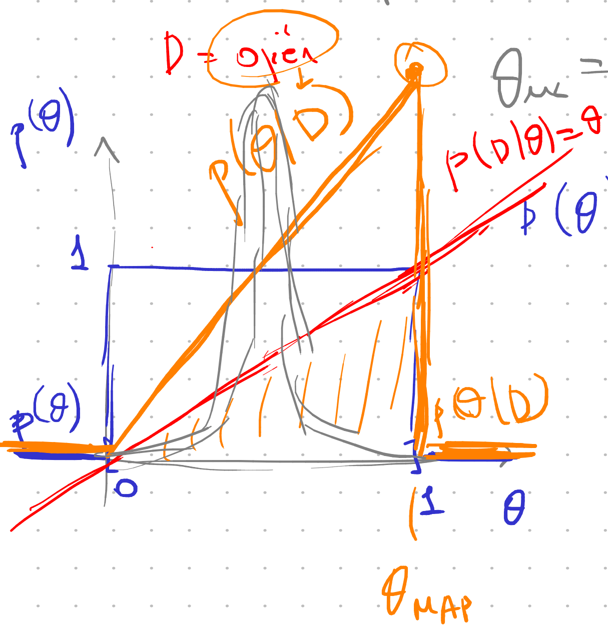
Likelihood

$$p(\bar\theta|D) \propto p(\bar\theta)\, p(D|\bar\theta)$$

$$p(D|\bar\theta) \;\parallel\; \prod_{n=1} p(d_n|\theta), \quad d_n \in \{\text{орёл}, \text{решка}\}$$

$$D = \{n \times \text{орёл},\ m \times \text{решка}\}$$

$$p(D|\theta) = \theta^n (1-\theta)^m$$



$$\theta_{ML} = \frac{n}{n+m}$$

$$p(\theta) = \begin{cases} 1, & [0,1] \\ 0, & -''- \end{cases}$$

$$p(\theta|D) \propto p(\theta)\,p(D|\theta) = \begin{cases} \theta^n(1-\theta)^m, & [0,1] \\ 0, & -''- \end{cases}$$

$$\theta_{MAP} = \frac{n}{n+m}$$

$$p(\text{орёл}|D) = \int p(\text{орёл}, \theta|D)\, d\theta = \int \overbrace{p(\text{орёл}|\theta)}^{=\theta}\ \overbrace{p(\theta|D)}^{=\{-\}/p(D)}\, d\theta =$$

$$p(D) = \int p(\theta)\,p(D|\theta)\, d\theta = \int_0^1 \theta^n (1-\theta)^m\, d\theta = B(n+1, m+1) =$$

$$= \frac{\Gamma(n+1)\,\Gamma(m+1)}{\Gamma(n+m+2)} = \frac{n!\,m!}{(n+m+1)!}$$

Laplace's Rule of Succession

$$= \int_0^1 \theta \cdot \frac{(n+m+1)!}{n!\,m!} \cdot \theta^n (1-\theta)^m\, d\theta = \frac{\cancel{(n+m+1)!}}{\cancel{n!\,m!}} \cdot \frac{(n+1)!\,\cancel{m!}}{(n+m+2)\cancel{!}} = \frac{n+1}{n+m+2}$$

Bayesian smoothing