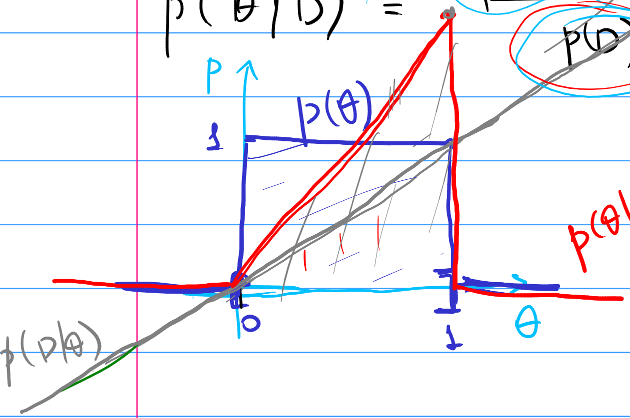
$$p(D)=\int p(\theta,D)d\theta=\int p(\theta)p(D|\theta)d\theta \quad \theta^n(1-\theta)^m$$

$\theta = p(\text{„решка"})$ $\quad D=$ „ n решек, m орлов"

$$p(\theta|D)=\frac{p(\theta)\,p(D|\theta)}{p(D)}$$

$$p(D|\theta)=\theta^n(1-\theta)^m \xrightarrow[\theta]{} \max$$

$$\theta_{ML}=\frac{n}{m+n}$$



$D=$ решка

$$p(D|\theta)=\theta \xrightarrow[\theta]{} \max$$

$$p(\theta)=\begin{cases}1, & \theta\in[0,1]\\ 0, & \theta\notin[0,1]\end{cases}$$

$p(\text{бол.}|\text{кровь})\propto p(\text{кровь})\cdot p(\text{кровь}|\text{бол.})$

$p(\text{кровь})=0.01$

$$p(\theta|D)\propto p(\theta)\cdot p(D|\theta)=\begin{cases}\theta^n(1-\theta)^m, & \theta\in[0,1]\\ 0, & \theta\notin[0,1]\end{cases} \xrightarrow[\theta]{} \max$$

$$p(x,y|z)=p(x|z)\,p(y|x,z)$$

$$\theta_{MAP}=\frac{n}{n+m}$$

$$p(\text{решка}|D)=\int p(\text{решка},\theta|D)\,d\theta=\int \overbrace{p(\theta|D)}^{\text{posterior}}\cdot \overbrace{p(\text{решка}|\theta,D)}^{\theta}\,d\theta=$$

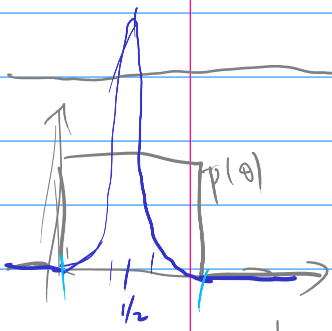
$$p(D)=\int_{-\infty}^{\infty}\boxed{p(\theta)}p(D|\theta)\,d\theta=\int_0^1\theta^n(1-\theta)^m d\theta=B(n+1,m+1)=$$

$$=\frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+2)}=\frac{n!\,m!}{(n+m+1)!}$$

$$=\int_0^1\theta\cdot\frac{\theta^n(1-\theta)^m}{n!\,m!/(n+m+1)!}d\theta=\frac{\cancel{(n+m+1)!}}{\cancel{n!}\,\cancel{m!}}\cdot\frac{(n+1)!\,\cancel{m!}}{(n+m+2)!}$$

$$p(\text{решка}|D)=\frac{n+1}{n+m+2}$$

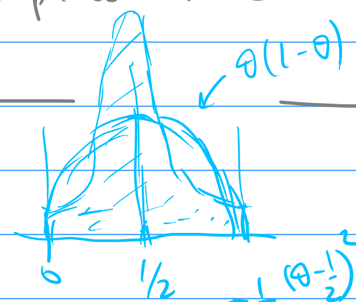
Laplace's rule



$$p(\theta|D)\propto p(\theta)\,p(D|\theta)$$

$$\theta^{\alpha-1}(1-\theta)^{\beta-1}\times\theta^n(1-\theta)^m=$$

$$=\theta^{\alpha+n-1}(1-\theta)^{\beta+m-1}$$



$$p(\theta|\tfrac12,\sigma^2)\propto\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(\theta-\frac12)^2}$$

$p(\theta|\tfrac12,\tfrac12)\propto\theta^{-\frac12}(1-\theta)^{-\frac12}$ Conjugate prior

$$p(\theta|\alpha)\cdot p(D|\theta)\propto p(\theta|\alpha')$$

$$p(\theta|\alpha,\beta)=\begin{cases}\frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}, & \theta\in[0,1]\\ 0, & \theta\notin[0,1]\end{cases}$$

Бета-распр.