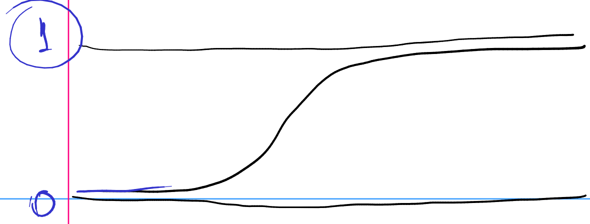


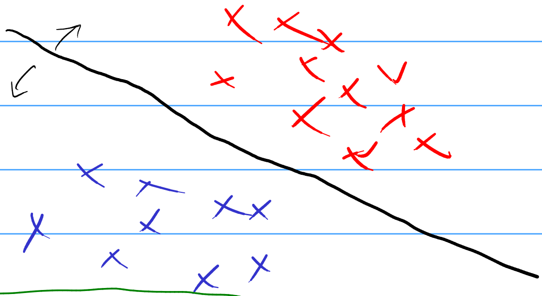
probit

$$\Phi(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx$$

$$\int_{-\infty}^{a} e^{-w_0 - w_1 x - w_2 x^2}\,dx$$

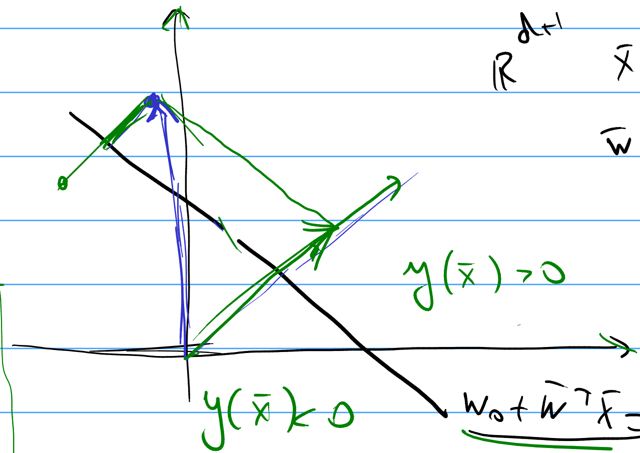
$$y \sim e^{-w_0 - w_1 x - w_2 x^2}$$

$$-\ln y \sim w_0 + w_1 x + w_2 x^2$$

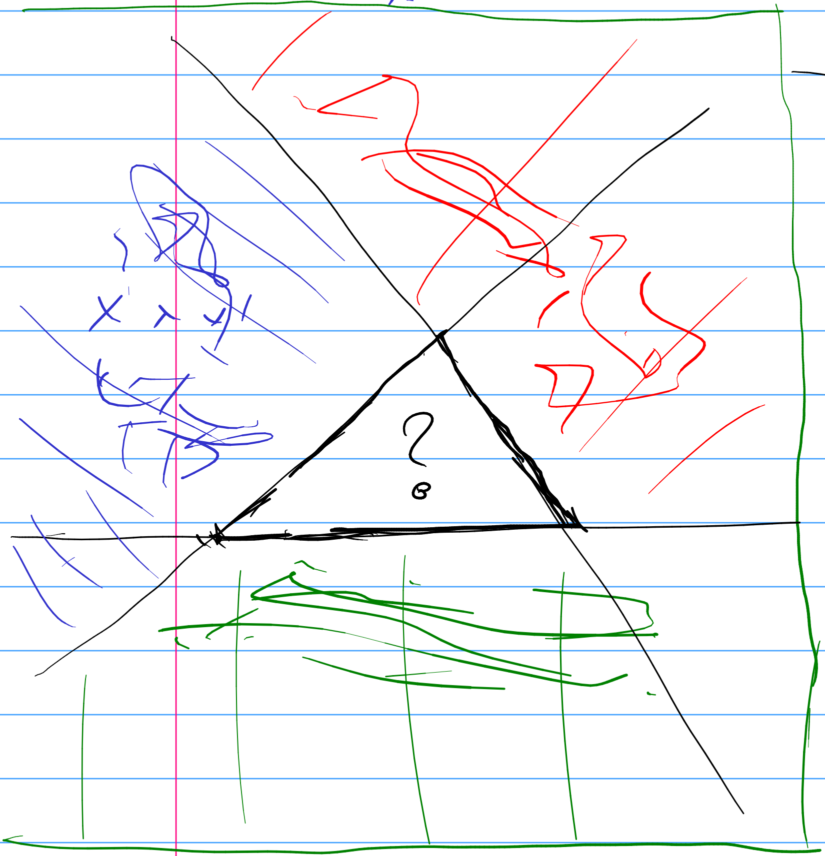


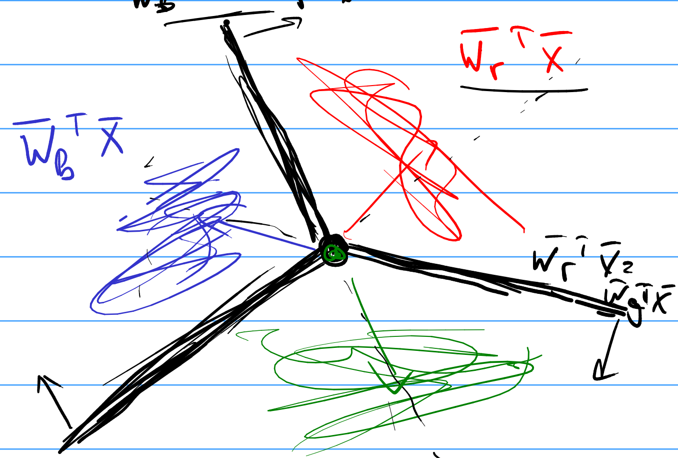
$\mathbb{R}^{d+1}$ $\quad \bar{x} \mapsto \begin{pmatrix} \bar{x} \\ 1 \end{pmatrix}$

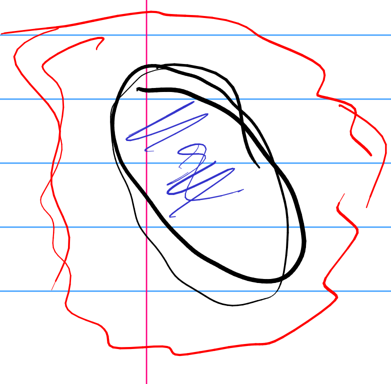
$\bar{w} \mapsto \begin{pmatrix} \bar{w} \\ w_0 \end{pmatrix}$

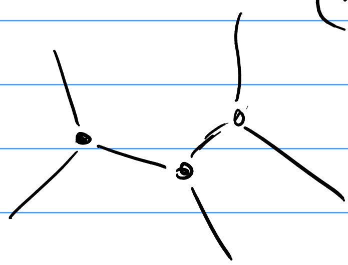


$$d(\bar{x}, \dots) = \frac{\bar{w}^T \bar{x} + w_0}{\|\bar{w}\|}$$







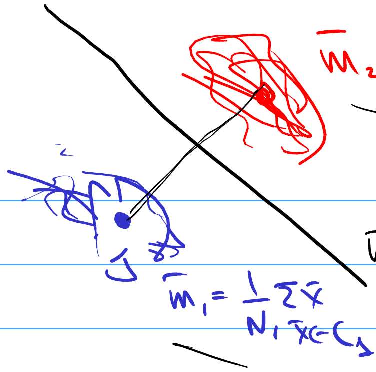


$\max(\bar{w}_b^T \bar{x}, \bar{w}_g^T \bar{x}, \bar{w}_r^T \bar{x})$

не единственный

tropical variety

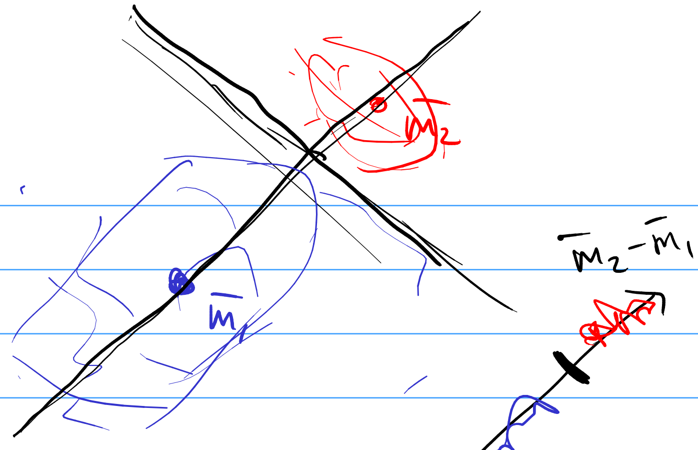
(min, +)-algebra

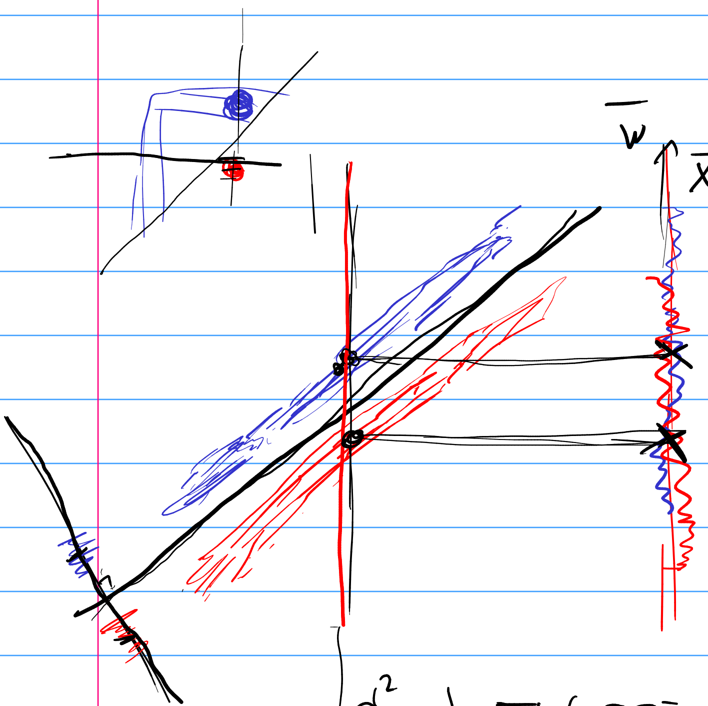


$$\bar m_2 = \frac{1}{N_2}\sum_{\bar x \in C_2} \bar x$$

$$\bar w^T \bar x + w_0 = 0$$

$$\bar m_1 = \frac{1}{N_1}\sum_{\bar x \in C_1} \bar x$$

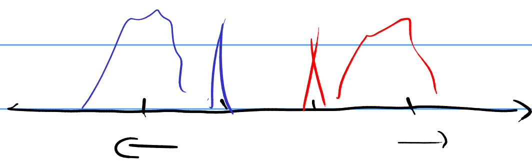




$$\bar x^T(\bar m_2 - \bar m_1)$$

$$\left(\bar w^T(\bar m_2 - \bar m_1)\right)^2 \xrightarrow[\bar w]{} \max$$

$$\|$$



$$\nabla_{\bar x}(\bar x^T A \bar x) = (A + A^T)\bar x$$

$$s_1^2 = \frac{1}{N_1}\sum_{\bar x \in C_1} \underbrace{(\bar w^T \bar x - \bar w^T \bar m_1)^2}_{(\bar w^T(\bar x - \bar m_1))^2} = \frac{1}{N_1}\bar w^T S_1 \bar w$$

$$S_1 = \sum (\bar x - \bar m_1)(\bar x - \bar m_1)^T$$

$$S_W = s_1^2 + s_2^2 \qquad \bar w^T(S_1 + S_2)\bar w \xrightarrow[\bar w]{} \min$$

Fischer algor.

$$J(\bar w) = \frac{\bar w^T(\bar m_2 - \bar m_1)(\bar m_2 - \bar m_1)^T \bar w}{\bar w^T \cdot S_W \bar w} = \frac{\bar w^T S_B \bar w}{\bar w^T S_W \bar w} \to \max$$

S_B – between-class covariance

S_W – within-class

$$\nabla_{\bar w} J = \frac{2(\bar w^T S_W \bar w)\cdot S_B \bar w - 2(\bar w^T S_B \bar w)\cdot S_W \bar w}{(\bar w^T S_W \bar w)^2} = 0$$

$$\cancel{(\bar w^T S_W \bar w)}\, S_B \bar w \propto \cancel{(\bar w^T S_B \bar w)}\, S_W \bar w$$

$$S_W + S_B = S = \frac{1}{N}\sum_{\bar x \in C_1 \cup C_2} (\bar x - \bar m)(\bar x - \bar m)^T$$

$$S_B \bar w \propto S_W \bar w$$

$$\| $$

$$(\bar m_2 - \bar m_1)(\bar m_2 - \bar m_1)^T \bar w$$

$$\boxed{\bar w \propto S_W^{-1}(\bar m_2 - \bar m_1)}$$

$$S_W \bar w \propto \bar m_2 - \bar m_1$$

$p(y|\bar{x})$

$p(\bar{x})$

$p_1(\bar{x}) = p(\bar{x}|C_1)$

$$\bullet \; p(C_1|\bar{x}) = \frac{p(C_1)\,p(\bar{x}|C_1)}{p(C_1)\,p(\bar{x}|C_1) + p(C_2)\,p(\bar{x}|C_2)}$$

Optimal Bayes classifier

$p_2(\bar{x}) = p(\bar{x}|C_2)$

Generative models

Discriminative models

$p(C_1|\bar{x}) = ?$

$$p(\bar{x}) = p(C_1)\,p(\bar{x}|C_1) + p(C_2)\,p(\bar{x}|C_2)$$