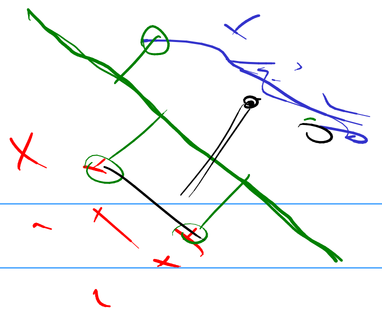


$\bar{x}_n \qquad C_1, C_2$

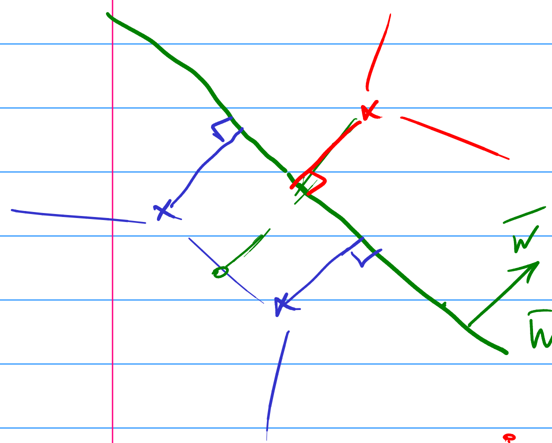
$\bar{a} = \sum_{\bar{x}_n \in C_1} \alpha_n \bar{x}_n, \quad \sum \alpha_n = 1, \alpha_n \geq 0$

$\bar{b} = \sum_{\bar{x}_n \in C_2} \alpha_n \bar{x}_n,$

(1)

$$\left\| \sum_{n: \bar{x}_n \in C_1} \alpha_n \bar{x}_n - \sum_{n: \bar{x}_n \in C_2} \alpha_n \bar{x}_n \right\|^2 \to \min$$

$$\sum_{C_1} \alpha_n = \sum_{C_2} \alpha_n = 1, \quad \alpha_n \geq 0$$



$$d(\bar{x}, \dots) = \frac{|\bar{w}^T \bar{x} + w_0|}{\|\bar{w}\|}$$

$$D = \{(\bar{x}_n, t_n)\}_{n=1}^{N} \qquad t_n = \begin{cases} 1, & C_1 \\ -1, & C_2 \end{cases}$$

$$\max_{\bar{w}} \min_{n} \frac{|\bar{w}^T \bar{x}_n + w_0|}{\|\bar{w}\|}$$

$\forall n \quad \bar{w}^T \bar{x}_n + w_0 > 0, \ x_n \in C_1$

$< 0, \ x_n \in C_2$

$$\max_{\bar{w}} \min_{n} \left[\frac{1}{\|\bar{w}\|} \cdot t_n (\bar{w}^T \bar{x}_n + w_0) \right]$$

$\forall n \quad t_n(\bar{w}^T \bar{x}_n + w_0) \geq 0$

$\|\bar{w}\| = 1$

? $\max_{\bar{w}} \left(\min_{n} t_n (\bar{w}^T \bar{x}_n + w_0) \right)$

$\forall n \ t_n(\bar{w}^T \bar{x}_n + w_0) \geq 0$

$\sum w_i^2 = 1$

Выберем $\|\bar{w}\|$: $\quad \min_{n} t_n(\bar{w}^T \bar{x}_n + w_0) = 1$

$$\max_{\bar{w}} \frac{1}{\|\bar{w}\|} \Leftrightarrow \min_{\bar{w}} \|\bar{w}\|^2 \qquad \forall n \quad t_n(\bar{w}^T \bar{x}_n + w_0) \geq 1$$

(2) $\min_{\bar{w}} \frac{1}{2}\sum w_i^2$, $\quad \forall n \quad t_n(\bar{w}^T \bar{x}_n + w_0) \geq 1$

$$L(\bar{w}, w_0, \bar{\alpha}) = \frac{1}{2}\|\bar{w}\|^2 - \sum_n \alpha_n \left[t_n(\bar{w}^T\bar{x}_n + w_0) - 1\right] \longrightarrow \min_{\bar{w}, w_0, \bar{\alpha}}$$

$\forall n \ \alpha_n \geq 0$

$$\nabla_{\bar{w}} L = \bar{w} - \sum_n \alpha_n t_n \bar{x}_n$$

$$\boxed{\bar{w} = \sum_n \alpha_n t_n \bar{x}_n}$$

$$\frac{\partial L}{\partial w_0} = -\sum_n \alpha_n t_n \qquad \boxed{\sum_n \alpha_n t_n = 0}$$

$$L(\bar{\alpha}) = \underbrace{\frac{1}{2}\Big(\sum_n \alpha_n t_n \bar{x}_n\Big)^T \Big(\sum_m \alpha_m t_m \bar{x}_m\Big)}_{=\ \frac{1}{2}\sum_{n,m} \alpha_n \alpha_m t_n t_m \bar{x}_n^T \bar{x}_m} - \sum_n \Big(\alpha_n t_n \Big(\sum_m \alpha_m t_m \bar{x}_m\Big)^T \bar{x}_n\Big) + \sum_n \alpha_n$$

$$\frac{1}{2}\sum_{n,m} \alpha_n \alpha_m t_n t_m \bar{x}_n^T \bar{x}_m - \sum_{n,m}(\_)$$

(3) $$L(\bar{\alpha}) = \sum_n \alpha_n - \frac{1}{2}\sum_n \sum_m \alpha_n \alpha_m t_n t_m (\bar{x}_n^T \bar{x}_m) \longrightarrow \min$$

$\forall n \ \alpha_n \geq 0, \quad \sum_n \alpha_n t_n = 0$

$\alpha_n \qquad y(\bar{x}) = ?$

$$y(\bar{x}) = \sum_n \alpha_n t_n (\bar{x}_n^T \bar{x}) + w_0$$

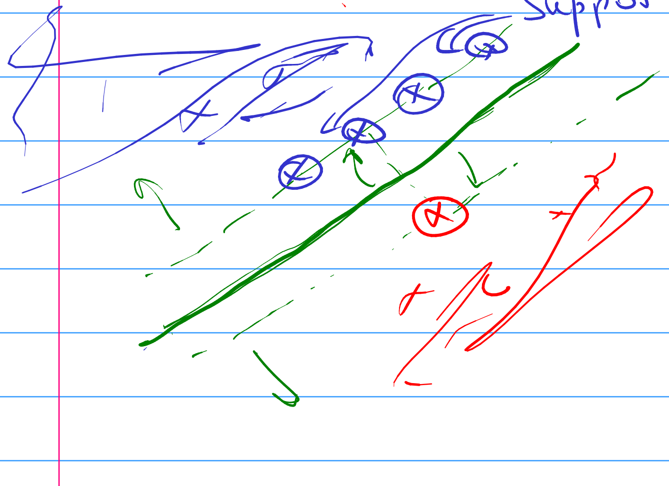
KKT

$\alpha_n \geq 0$

$t_n y(\bar{x}_n) - 1 \geq 0$

$\forall n \quad \alpha_n\big(t_n y(\bar{x}_n) - 1\big) = 0$

support vectors



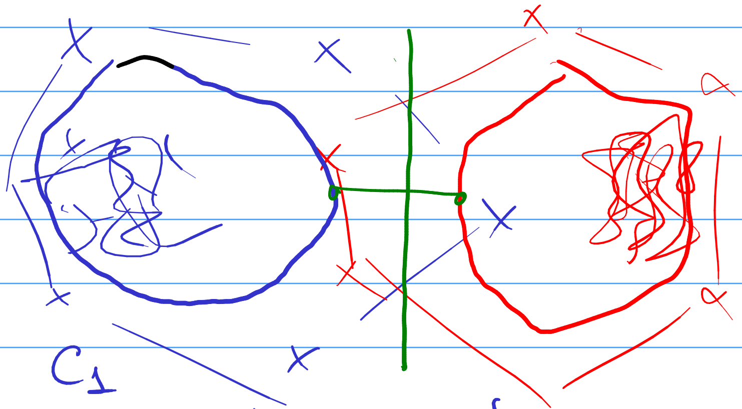
$$\min \frac{1}{2}\|\bar{w}\|^2$$

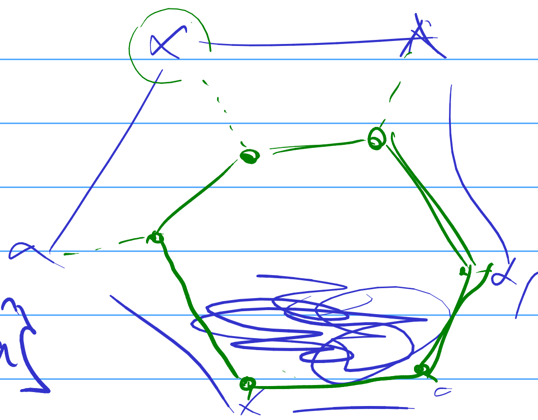
$$\forall n \quad t_n(\bar{w}^T \bar{x}_n + w_0) \geq 1$$

$$\min \left[\frac{1}{2}\|\bar{w}\|^2 + C \cdot \sum_n z_n\right]$$

$$\forall n \; t_n(\bar{w}^T \bar{x}_n + w_0) + z_n \geq 1$$

$$\forall n \; z_n \geq 0$$

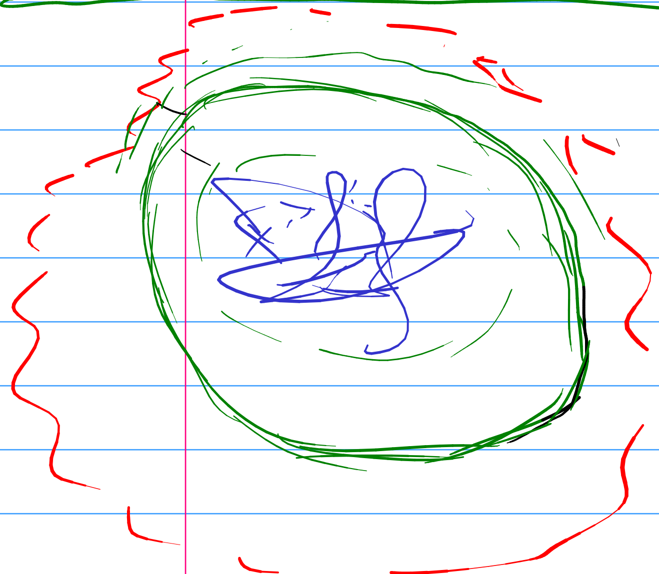




$$\mathrm{Conv}(C_1) = \left\{ \sum \alpha_n \bar{x}_n \;\middle|\; \sum \alpha_n = 1, \alpha_n \geq 0 \right\}$$

$$\mathrm{Conv}_a(C_1) = \left\{ \sum_n \alpha_n \bar{x}_n \;\middle|\; \sum_n \alpha_n = 1, \; \alpha_n \geq 0, \; \alpha_n \leq a \right\}$$

max-margin classifiers



$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x^2 \\ y^2 \\ xy \\ x \\ y \end{pmatrix}^T \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_5 \end{pmatrix}$$

$$\mathbb{R}^2 \longrightarrow \mathbb{R}^5$$

$$\mathbb{R}^d \longrightarrow \mathbb{R}^{\frac{d(d+1)}{2}}$$

$$\mathbb{R}^d \longrightarrow \mathbb{R}^{O(d^k)}$$

$$100 \qquad \boxed{100^4}$$

$$L(\bar\alpha) = \sum \alpha_n - \frac{1}{2}\sum_{n,m} \alpha_n\alpha_m t_n t_m (\bar x_n^T \bar x_m)$$

$+ \bar x^T \bar y$

$$k(\bar x, \bar y) = (\bar x^T \bar y)^2 = (x_1y_1+x_2y_2)^2 =$$

$\begin{pmatrix}x_1\\x_2\end{pmatrix}$ x_3 $\quad$ $\begin{pmatrix}y_1\\y_2\end{pmatrix}$ y_3

$$= x_1^2y_1^2 + 2x_1y_1x_2y_2 + x_2^2y_2^2 =$$

$$= \begin{pmatrix}x_1^2\\ \sqrt{2}\, x_1x_2\\ x_2^2\end{pmatrix}^T \begin{pmatrix} y_1^2\\ \sqrt{2}\, y_1y_2\\ y_2^2\end{pmatrix}$$

x_1 x_2 $\quad$ y_1 y_2

$$(\bar x^T\bar y)^2 = \begin{pmatrix}x_1^2\\x_2^2\\x_3^2\\\sqrt{2}\,x_1x_2\\\sqrt{2}\,x_2x_3\\\sqrt{2}\,x_1x_3\end{pmatrix}\begin{pmatrix} \\ y \\ \\ \end{pmatrix}$$

$$L(\bar\alpha) = \sum_n\alpha_n - \frac12\sum_{n,m}\alpha_n\alpha_m t_n t_m k(\bar x_n,\bar x_m)$$

$\forall n \quad \alpha_n \geq 0$

$\sum_n \alpha_n t_n = 0$

$$k(\bar x,\bar y) = (\bar x^T\bar y + 1)^m - 1$$

$$y(\bar x) = \sum_n \alpha_n t_n k(\bar x,\bar x_n) + w_0$$

$$k(\bar x,\bar x_n) = e^{-\frac{1}{2\sigma^2}(\bar x-\bar x_n)^2}$$

$$\sum_n \boxed{t_n} (\alpha_n k(\bar x, \bar x_n))$$