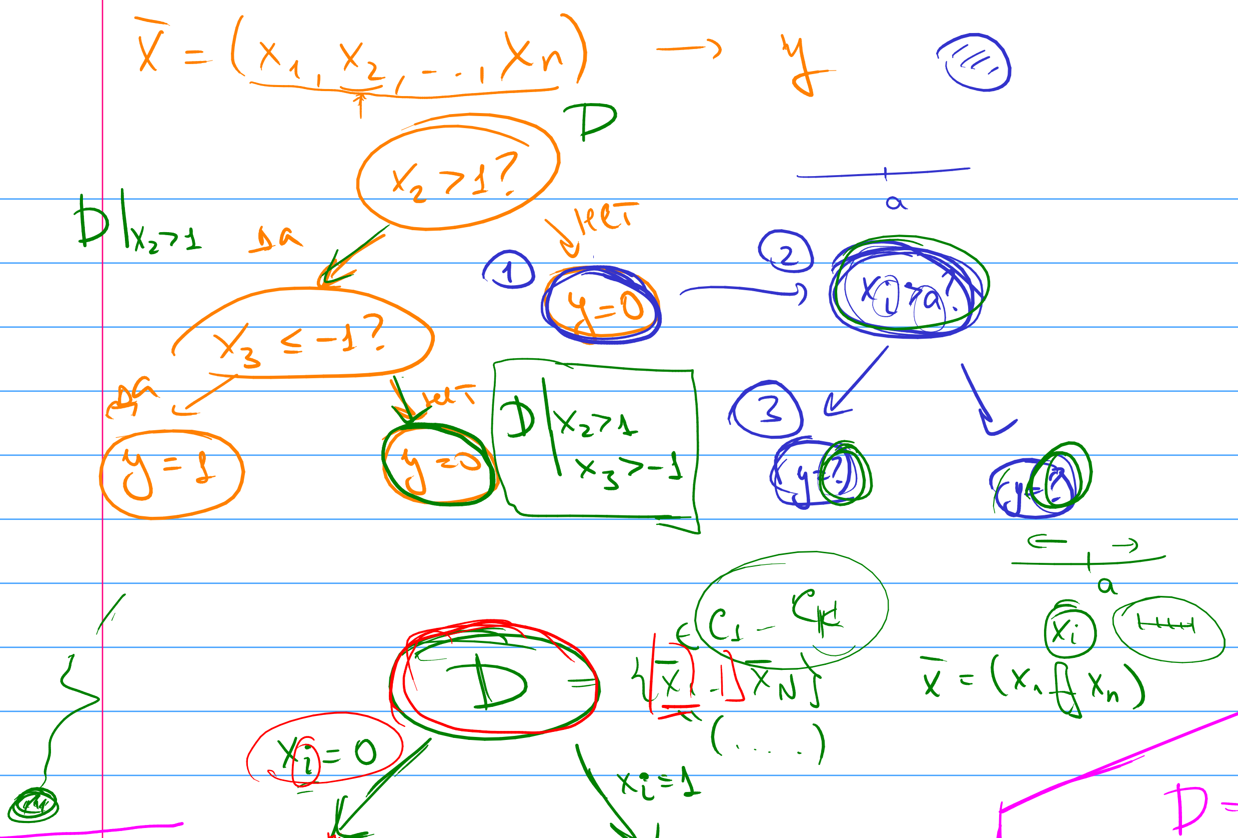
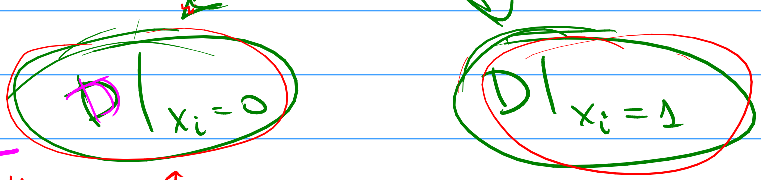
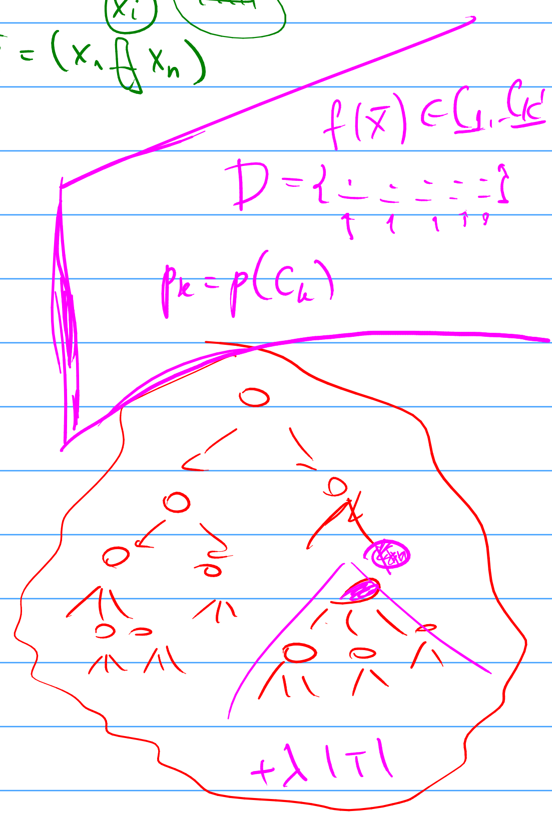
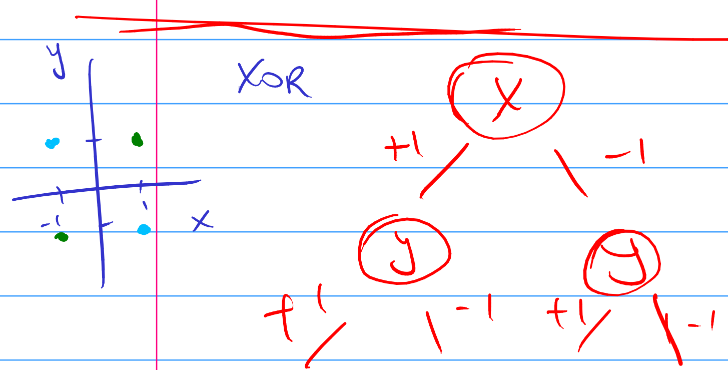




Gini index: $\sum p_k(1-p_k)$

entropy: $-\sum_{k=1}^{K} p_k \ln p_k$ $\quad \frac{|\bar x \in C_k|}{|\bar x|}$





$$\hat y(\bar x) = \sum_{k=1}^{K} f_k(\bar x),$$ f_k – деревья

$$\hat y^{(0)}(\bar x) = 0$$

$$\hat y^{(1)}(\bar x) = f_1(\bar x) = \hat y^{(0)}(\bar x) + f_1(\bar x)$$

$$\hat y^{(2)}(\bar x) = \hat y^{(1)}(\bar x) + f_2(\bar x)$$

$$L=\sum_{i=1}^{N} l(\underset{\text{прав.}}{y_i},\underset{\text{модель}}{\hat y_i})+\sum_{j=1}^{k}\Omega(f_j)\xrightarrow[f_k]{}\min$$

$l(y_i,\hat y_i)=(y_i-\hat y_i)^2$

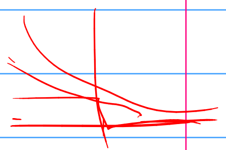
$l(y_i,\hat y_i) = \sum y_{ij}\ln \hat y_{ij}$

$$L^{(k)}=\sum_{i=1}^{N} l\left(y_i,\ \hat y_i^{(k-1)}+f_k(\bar x_i)\right)+\Omega(f_k)+\text{const}\xrightarrow[f_k]{}\min$$

$l=(\cdot-\cdot)^2$

$$L^{(k)}=\sum_{i=1}^{N}\left(f_k(\bar x_i)^2+2f_k(\bar x_i)(\hat y_i^{(k-1)}-y_i)+\cancel{(\hat y_i^{(k-1)}-y_i)^2}\right)+\Omega(f_k)$$

$f_k^*(\bar x_i)$ $\quad (\bar x_i,\ f_k^*(\bar x_i))$



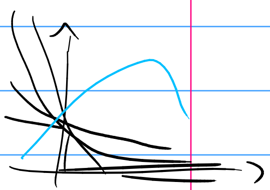
$$l(y_i,\hat y_i^{(k-1)}+f_k(\bar x_i))\approx$$

$(y_i,\ \hat y_i^{(k-1)})$

$$\approx l(\ldots)+f_k(\bar x_i)g_i+\frac{1}{2}h_i f_k(\bar x_i)^2$$

$$g_i=\frac{\partial l}{\partial \hat y}\Big|_{(y_i,\hat y_i)}$$

$$h_i=\frac{\partial^2 l}{\partial \hat y^2}\Big|_{(y_i,\hat y_i)}$$



$$L^{(k)}\approx\sum_{i=1}^{N}\left(\cancel{l(y_i,\hat y_i^{(k-1)})}+g_i f_k(\bar x_i)+\frac{h_i}{2}f_k(\bar x_i)^2\right)+\Omega(f_k)+\text{const}$$

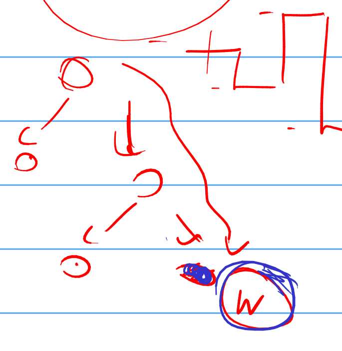
$$L^{(k)}\approx\sum_{i=1}^{N}\left(g_i f_k(\bar x_i)+\frac{h_i}{2}f_k^2(\bar x_i)\right)+\Omega(f_k)$$

$q:\bar x\in\mathbb{R}^d\to\{1,\ldots,T\}$

$f_k(\bar x)=w_{q(\bar x)}$ $\quad \bar w\in\mathbb{R}^T$

$(w_1\ldots w_T)$

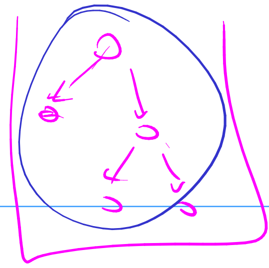
$$\Omega(f_k)=\gamma T+\frac{1}{2}\lambda\sum_{j=1}^{T}w_j^2$$



$$L^{(k)}\approx\sum_{i=1}^{N}\left(g_i f_k(\bar x_i)+\frac{1}{2}h_i f_k^2(\bar x_i)\right)+\gamma T+\frac{\lambda}{2}\sum_{j=1}^{T}w_j^2$$

$$=\sum_{i=1}^{N}\left(g_i w_{q(\bar x_i)}+\frac{1}{2}h_i w^2_{q(\bar x_i)}\right)+\gamma T+\frac{\lambda}{2}\sum_{j=1}^{T}w_j^2$$

$$\sum_{i=1}^{N} g_i w_{q(\bar{x}_i)} = \sum_{j=1}^{T} w_j \left(\sum_{i: q(\bar{x}_i)=j} g_i \right)$$

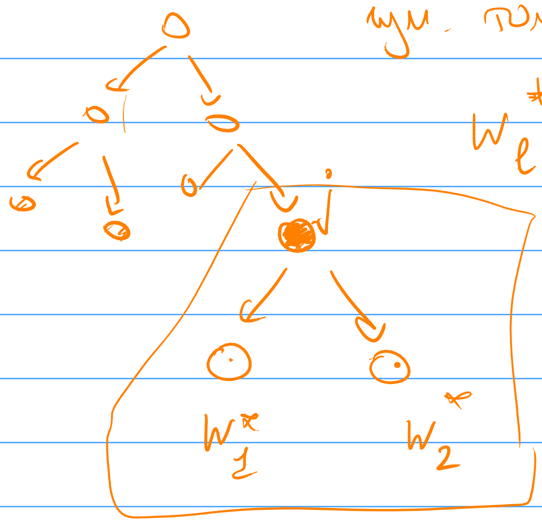


$$= \sum_{j=1}^{T} \left(w_j \underbrace{\left(\sum_{q(\bar{x}_i)=j} g_i \right)}_{= G_j} + \frac{1}{2} w_j^2 \underbrace{\left(\sum_{q(\bar{x}_i)=j} h_i \right)}_{= H_j} \right) + \gamma T + \frac{\lambda}{2} \sum_{j=1}^{T} w_j^2$$

$$= \sum_{j=1}^{T} \left(G_j w_j + \frac{1}{2} (H_j + \lambda) w_j^2 \right) + \gamma T$$

$$w_j^* = - \frac{G_j}{H_j + \lambda} \qquad L^{(t)} = -\frac{1}{2} \sum_{j=1}^{T} \frac{G_j^2}{(H_j + \lambda)} + \gamma T$$

уж. только ~~w_j^*~~



w_ℓ^* сохр.

$$L^{(t)'} - L^{(t)} =$$

$$= \frac{1}{2} \left(\frac{G_j^2}{H_j + \lambda} - \frac{G_{j1}^2}{H_{j1} + \lambda} - \frac{G_{j2}^2}{H_{j2} + \lambda} \right) + \gamma$$