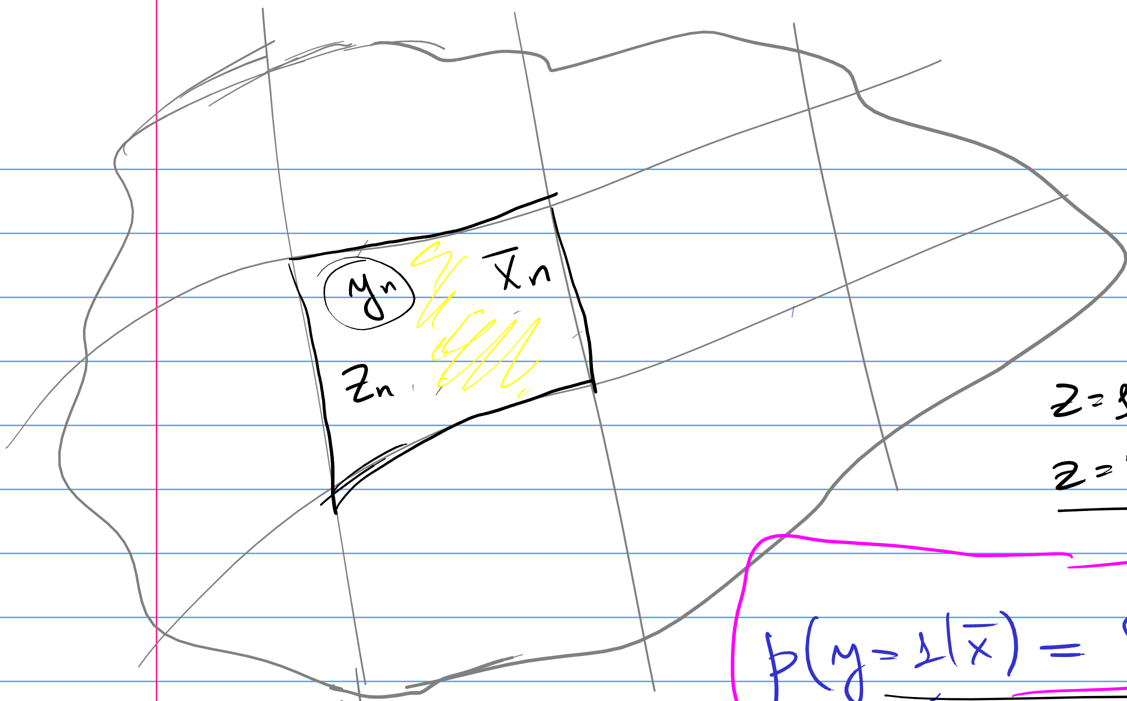


y - был ли суслик

z - видели ли суслика

$z=1 \Rightarrow y=1$

$z=0 \rightarrow y=?$

$$\boxed{p(y=1|\bar{x}) = \sigma(\eta(\bar{x}))}, \quad \eta(\bar{x}) = \bar{w}^T\bar{x}$$

$s=1$ — набл. в нашем датасете

$$\gamma_1 = p(s=1|y=1)$$
$$\gamma_0 = p(s=1|y=0)$$

$\pi = p(y=1)$ $\quad \pi N$

n_1 — сколько с сусликом

n_0 — -//- без -//-

$$\gamma_1 = \frac{n_1}{\pi N}, \quad \gamma_0 = \frac{n_0}{(1-\pi)N}$$

$$\boxed{p(y=1|s=1,\bar{x})} = \frac{p(s=1|y=1,\bar{x})p(y=1|\bar{x})}{\text{-//- } y=1 \text{ -//- } + \text{ -//- } y=0} = \frac{\gamma_1 e^{\eta(\bar{x})}}{\gamma_0 + \gamma_1 e^{\eta(\bar{x})}}$$

полные данные

$$= \sigma(\eta^*(\bar{x}))$$

$$\eta^*(\bar{x}) = \eta(\bar{x}) + \log\frac{\gamma_1}{\gamma_0} = \eta(\bar{x}) + \log\frac{n_1}{n_0} - \log\frac{\pi}{1-\pi}$$

? хотим оценить

$\eta(\bar{x}) + \log\frac{n_1}{n_0} = \bar{w}^T\bar{x} + w_0$

$n_1 = n_p + \pi n_u$

$n_0 = (1-\pi)n_u$

n_p - $\#[z=1]$

presence

n_u - $\#[z=0]$ $\rightarrow \pi n_u \quad y=1$; $\rightarrow (1-\pi)n_u \quad y=0$

unknown

$$p(y=1|s=1) = \frac{n_p + \pi n_u}{n_p + n_u} \qquad p(y=0|s=1) = \frac{(1-\pi)n_u}{n_p+n_u}$$

$$\gamma_1 = p(s=1 \mid y=1) = \frac{p(y=1 \mid s=1)\,p(s=1)}{p(y=1)} = \frac{n_p + \pi n_u}{\pi(n_p+n_u)} \cdot p(s=1)$$

$$\gamma_0 = p(s=1 \mid y=0) = \frac{-\,''\,-\ y=0}{-\,''\,-} = \frac{(1-\pi)n_u}{(1-\pi)(n_p+n_u)} \cdot p(s=1)$$

$$\boxed{\eta^*(\bar{x}) = \eta(\bar{x}) + \log\frac{\gamma_1}{\gamma_0} = \eta(\bar{x}) + \log\frac{n_p + \pi n_u}{\pi n_u}}$$

$$p(Y, Z \mid X, s=1) = \prod_i p(y_i, z_i \mid s_i=1, \bar{x}_i) =$$

M-war

$$= \prod_i p(y_i \mid s_i=1, \bar{x}_i)\; p(z_i \mid y_i, s_i=1, \bar{x}_i)$$

лог. регр на $\eta^*(\bar{x}_i)$

$$p(z_i=0 \mid y_i=0, s_i=1, \bar{x}_i) = 1$$

$$z_i=1 \mid y_i=0 \quad -\,''\,- \quad = 0$$

$$p(z_i=1 \mid y_i=1, s_i=1, \bar{x}_i) = \frac{p(z_i=1, y_i=1 \mid s=1, \bar{x}_i)}{p(y_i=1 \mid s_i=1, \bar{x}_i)} =$$

$p(z_i=1 \mid s_i=1)$

$$= \frac{p(z_i=1, y_i=1 \mid s_i=1)}{p(y_i=1 \mid s_i=1)} = \frac{n_p / n_p+n_u}{n_p+\pi n_u / n_p+n_u} = \frac{n_p}{n_p+\pi n_u}$$

$$p(z_i=0 \mid y_i=1, s_i=1, \bar{x}_i) = \frac{\pi n_u}{n_p+\pi n_u}$$

$$p(Z \mid X, \eta, s=1) \longrightarrow \max$$

$$p(z=1 \mid s=1, \bar{x}) = p(z=1, y=1 \mid -) + \cancel{p(z=1, y=0 \mid \_\,)} =$$

$$= p(z=1, y=1 \mid s=1, \bar{x}) = p(z=1 \mid y=1, s=1) \cdot p(y=1 \mid s=1, \bar{x})$$

$$= \underbrace{\frac{n_p}{n_p+\pi n_u} \cdot \frac{1}{1+e^{-\eta^*(\bar{x})}}}$$

$$p(z|X,\eta,s=1) = \prod_i p(z_i|s_i=1,\bar{x}) = \prod_i \left(\frac{n_p}{(n_p+\pi n_u)}\sigma(\eta^*(\bar{x}))\right)^{z_i}\left(1-\frac{n_p\sigma(\eta^*(\bar{x}))}{n_p+\pi n_u}\right)^{1-z_i}$$

$\eta, \cancel{\pi} \to \max$

EM:

- M-шаг: обуч. $\boxed{\eta^*(\bar{x})}$ как $\sigma(\eta^*(\bar{x}_i)) \sim \hat{y}_i$

- E-шаг: $\hat{y}_i = \mathbb{E}[y_i | \hat{\eta}^*, \bar{x}_i] = p(y_i=1|\hat{\eta}^*, \bar{x}_i)$

$$\hat{y}_i = \sigma(\eta(\bar{x}_i)) = \sigma\left(\hat{\eta}^*(\bar{x}_i) - \log\frac{n_p+\pi n_u}{\pi n_u}\right)$$

$$\eta(\bar{x}) = \bar{w}^T\bar{x}$$

$$p(y,z|\eta,\pi,X) = \underbrace{\prod_i p(y_i|s_i=1,\bar{x}_i)} \cdot \boxed{\prod_i p(z_i|y_i,s_i=1,\cancel{\bar{x}_i})}$$

$(\eta^*,\pi) \to \eta^* = f(\eta,\pi) \to \pi$

$z=0|\cancel{y=0}$

$$\prod p(z_i=1|y=1,s=1)\cdot\prod p(z_i=0|\_)^{y=1} = \left(\frac{n_p}{n_p+\pi n_u}\right)^{n_p}\left(\frac{\pi n_u}{n_p+\pi n_u}\right)^{\sum_u y_i}$$

$$\log p(\_) = n_p \cancel{\log n_p} - (n_p+\sum_u y_i)\log(n_p+\pi n_u) + \sum_u y_i \log\pi + \cancel{\sum_u y_i \log n_u}$$

$$\frac{\partial \log p}{\partial \pi} = -(n_p+\sum_u y_i)\frac{n_u}{n_p+\pi n_u} + \frac{\sum_u y_i}{\pi} = 0 \qquad \boxed{\hat{\pi} = \frac{\sum_u y_i}{n_u}}$$

$$\Sigma\cdot(n_p+\pi \cancel{n_u}) = \pi n_u(n_p+\cancel{\Sigma})$$

k

E-step: $$y_i^{(k)} = E\left[y_i \mid x_i,\ \bar{w}^{(k)},\ \pi^{(k)}\right]$$

M-step: $$\pi^{(k+1)} = \frac{\sum_u y_i^{(k)}}{n_u},$$

η^*, π

$$\bar{w}^{(k+1)} = \bar{w}^{*\,(k+1)}$$

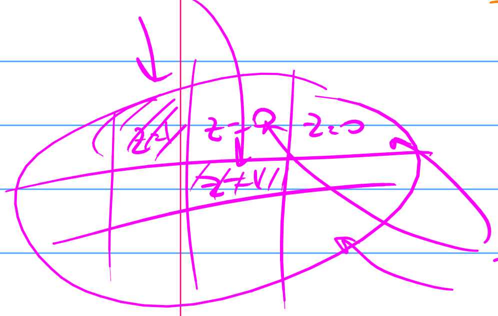
$$w_0^{(k+1)} = w_0^{*\,(k+1)} - \log \frac{n_p + \pi^{(k+1)} n_u}{\pi^{(k+1)} n_u}$$

(Ward, .., Hastie, 2009)

$$p(y=1 \mid \bar{x}) = \frac{p(y=1)\cdot \overbrace{\pi_1(\bar{x})}^{p(\bar{x} \mid y=1)}}{\pi(\bar{x})}$$

$$\pi(\bar{x}) = p(y=1)\,\pi_1(\bar{x}) + \left(1-p(y=1)\right)\pi_0(\bar{x})$$

$\bar{x} \sim \pi_1(\bar{x})$



$\bar{x} \sim \pi(\bar{x})$

$$p(y=1 \mid \bar{x}) = \sigma\left(\bar{w}^T \bar{x}\right)$$

$$p(y=1)\,\pi_1(\bar{x}) = p(y=1 \mid \bar{x})\,\pi(\bar{x})$$

$$\pi_1(\bar{x}_i) = \frac{p(y=1 \mid \bar{x}_i)}{p(y=1)\cdot \pi(\bar{x}_i)} \propto \alpha_i\, p(y=1 \mid \bar{x}_i)$$

$$p(y=1 \mid \bar{x}, \bar{w})$$

$$L(\bar{w}) = \prod p(y=1 \mid \bar{x}, \bar{w}) = \prod \frac{p(y=1 \mid \bar{x}_i, \bar{w})}{\sum_i p(y=1 \mid \bar{x}_i, \bar{w})} \to \max$$