$s_1, s_2, \dots, s_N$ - skill

$q_1, q_2, \dots, q_M$ - questions

Team	1	2	...	36
$s_1, s_2, \dots s_{16}$	+	—	—	+
$s_{21}, \dots, s_{26}$	+	+		—
s_{31}, s_{32}, s_{33}	+	—		+

$$\boxed{p(s_i \text{ отв. на } q_j)} \approx \sigma(\mu + s_i + q_j)$$

((обучаем логист. регрессию

μ, s_i, q_j

$X_{tj} = [\text{отв. команда } t \text{ на } j]$

$z_{ij} = [\text{игрок } i \text{ отв. на вопрос } j]$

E-шаг: $i_1, \dots, i_L \in t$

Если $\forall l \; z_{i_l j} = 0$, то $x_{tj} = 0$

-,,- $\exists l \; z_{i_l j} = 1$, то $x_{tj} = 1$

$$\underset{i \in t}{\mathbb{E}[z_{ij}]} = \begin{cases} 0, & x_{tj} = 0 \\ p(z_{ij} = 1 \mid \exists i' \in t : z_{i'j} = 1, \theta^{(m)}), & x_{tj} = 1 \end{cases}$$

$$\| \quad \frac{\sigma(\mu + s_i + q_j)}{1 - \prod_{i' \in t} (1 - \sigma(\mu + s_{i'} + q_j))}$$

M-шаг:

обуч. лог. регрессию

$$\sigma(\mu + s_i + q_j) \approx \mathbb{E} z_{ij}$$

$$Q(\theta, \theta^{(m)}) = \mathbb{E}_{\theta^{(m)}}[\log p(X, Z|\theta)]$$

Markov chain:

$X_1, X_2, \dots, X_t, X_{t+1}, \dots$

$p(X_{t+1} | X_1, X_2, \dots, X_t) = p(X_{t+1} | X_t)$ ← Markov property

$\lambda = (\pi, A)$ $\quad D = (d_1 \dots d_T)$

$p(\lambda) p(D|\lambda) \to \max_\lambda$

$X_t \in (1, \dots, n)$

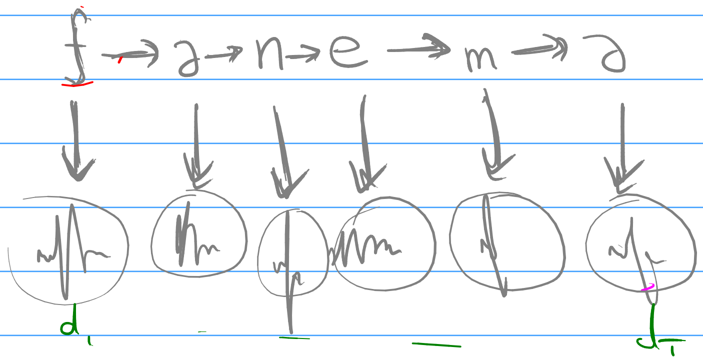
A, π, $\quad a_{ij} = p(X_{t+1} = j | X_t = i) = \frac{p(X_{t+1} = j, X_t = i)}{p(X_t = i)}$

$\pi_i = p(X_1 = i)$

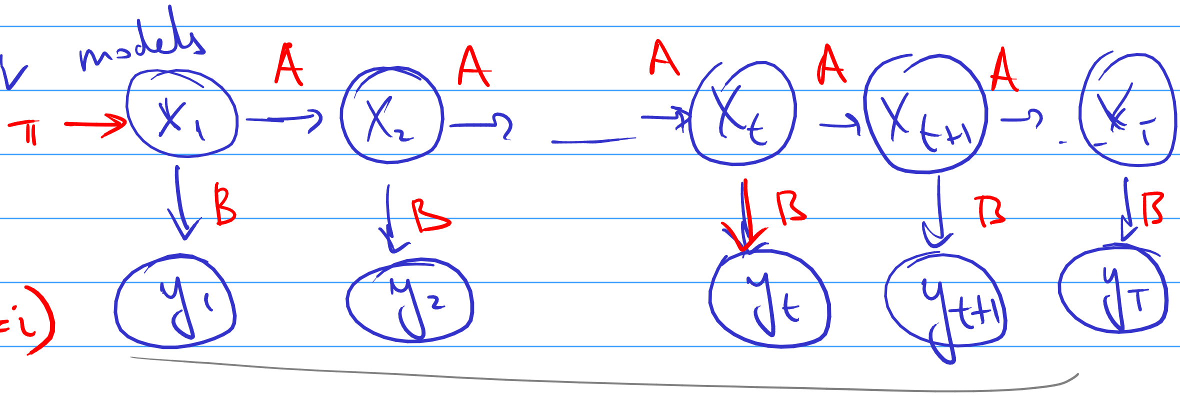
$$D = \{ \underset{X_t = d_{nt}}{\underbrace{(d_{n1}, \dots, d_{nT})}} \}_{n=1}^N \quad \prod_n p(d_{n1} \dots d_{nT} | A, \pi) = \prod_n \pi_{d_{n1}} \cdot a_{d_{n1} d_{n2}} \dots a_{d_{nT-1} d_{nT}} \to \max_{\pi, A}$$

$$\pi_i^* = \frac{\sum_n [d_{n1}=i] + \ldots}{N + \ldots} \qquad a_{ij}^* = \frac{\sum_n \sum_t [d_{n,t+1}=j,\ d_{n,t}=i]}{\sum_n \sum_t [d_{n,t}=i]}$$

q_1 — q_T



Hidden Markov models
HMM



$\pi_i = p(x_1 = i)$

$a_{ij} = p(x_{t+1} = j \mid x_t = i)$

$b_j(k) = p(y_t = k \mid x_t = j)$

$d_1 d_2 \ldots d_T$ — y

$y_t \in 1, \ldots, m$

$x_t \in 1, \ldots, n$

$\lambda = (\pi, A, B)$

$D = \{ \bar{d} = d_1 \ldots d_T \}$

$$p(Q, D \mid \lambda) = p(q_1 \ldots q_T,\ d_1 \ldots d_T \mid \pi, A, B) =$$

$$= \pi_{q_1} b_{q_1}(d_1)\, a_{q_1 q_2} b_{q_2}(d_2) \ldots a_{q_{T-1} q_T} b_{q_T}(d_T)$$

$$p(D \mid \lambda) = \sum_Q p(Q, D \mid \lambda) = \sum_{q_1 \ldots q_T} \pi_{q_1} \ldots b_{q_T}(d_T)$$

✓

1) $p(D \mid \lambda) = ?$

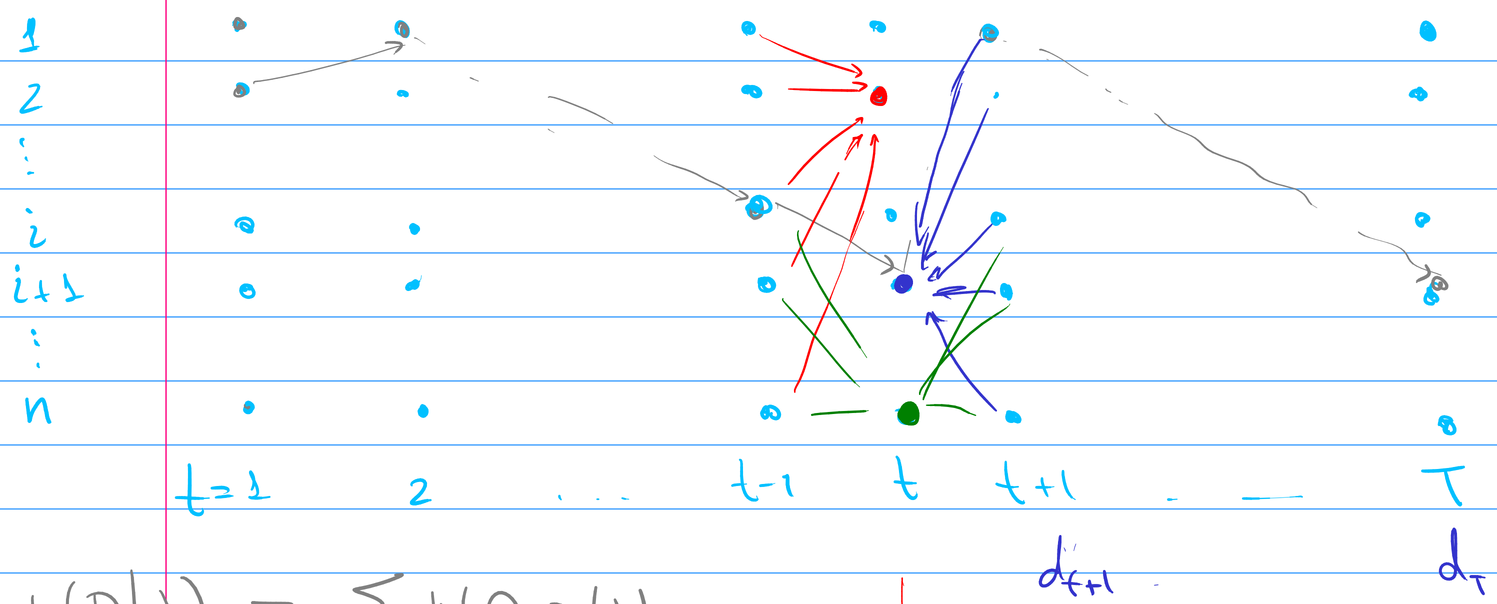
✓

2) Inference: $q_t^* = \arg\max_{q_t} p(q_t \mid D, \lambda)$?

$Q^* = \arg\max_Q p(Q \mid D, \lambda)$?

3) Learning: $\lambda^* = \underset{\lambda}{\arg\max}\ p(D|\lambda)$

$$p(Q,D) = \sum \ln \pi, a, b$$



$$p(D|\lambda) = \sum_Q p(Q,D|\lambda)$$

$d_1 .. d_T$

$$\alpha_t(i) = p(d_1,\dots,d_t, q_t = i \mid \lambda)$$

$$p(D|\lambda) = \sum_i \alpha_T(i)$$

$$\alpha_t(i) = p(d_1 .. d_t, q_t = i|\lambda) = \sum_j p(d_1 .. d_t, q_t = i, q_{t-1} = j|\lambda) =$$

$$= \sum_j \underline{p(d_1 .. d_{t-1}, q_{t-1} = j|\lambda)} \cdot p(q_t = i | q_{t-1} = j, \cancel{d_1 .. d_{t-1}}, \lambda) \cdot$$

$= a_{ji}$

$$\cdot\ p(d_t | q_t = i, \cancel{q_{t-1} = j, d_1 .. d_{t-1}}, \lambda) =$$

$= b_i(d_t)$

$$\boxed{\begin{aligned} \alpha_t(i) &= \sum_{j=1}^{n} a_{ji}\, b_i(d_t) \cdot \alpha_{t-1}(j) \\ \alpha_1(i) &= p(d_1, q_1 = i|\lambda) = \pi_i \cdot b_i(d_1) \end{aligned}}$$

$$\beta_t(i) = p(\underbrace{d_{t+1}, \dots, d_T}\mid q_t = i, \lambda) \qquad \beta_1(i) = p(d_2, \dots, d_T \mid q_1 = i, \lambda)$$

$$\beta_t(i) = p(d_{t+1} \dots d_T \mid q_t = i, \lambda) = \sum_j p(d_{t+1} \dots d_T, q_{t+1} = j \mid q_t = i, \lambda)$$

$$= \sum_j p(q_{t+1} = j \mid q_t = i, \lambda) \cdot p(d_{t+1} \mid q_{t+1} = j, \dots) \cdot p(d_{t+2} \dots d_T \mid q_{t+1} = j, \dots)$$

$$\boxed{\begin{array}{l} \beta_t(i) = \sum_j a_{ij}\, b_j(d_{t+1}) \cdot \beta_{t+1}(j) \\ \beta_T(i) = 1 \end{array}} \qquad \boxed{p(D \mid \lambda) = \sum_i \pi_i\, b_i(d_1)\, \beta_1(i)}$$

2) $p(q_t = i \mid D, \lambda) = ?$

$$\gamma_t(i) = p(q_t = i \mid d_1 \dots d_T, \lambda)$$

$$\gamma_t(i) = p(q_t = i \mid \underbrace{d_1 \dots d_t}, d_{t+1} \dots d_T, \lambda) = \frac{p(q_t = i, d_1 \dots d_t, d_{t+1} \dots d_T, \lambda)}{p(D \mid \lambda)} =$$

$$= \frac{\overbrace{p(d_1 \dots d_t, q_t = i \mid \lambda)} \cdot \overbrace{p(d_{t+1} \dots d_T \mid q_t = i, \cancel{d_1 \dots d_t}, \lambda)}}{p(D \mid \lambda)}$$

$$\gamma_t(i) = \frac{\alpha_t(i)\, \beta_t(i)}{p(D \mid \lambda)} \propto \alpha_t(i)\, \beta_t(i)$$