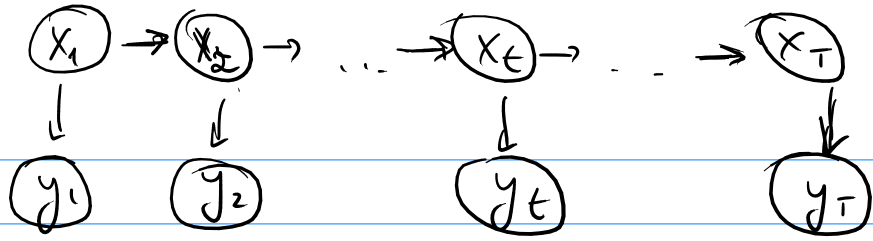
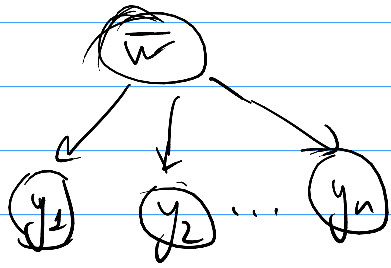


lower. perp.

mult - perp.

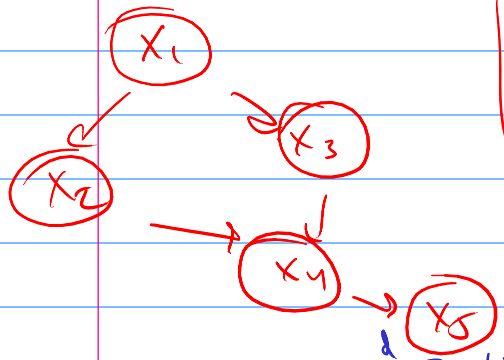
$$p(y_n | X, \bar{y}_{-n}, \bar{w}) = p(y_n | \bar{x}_n, \bar{w})$$



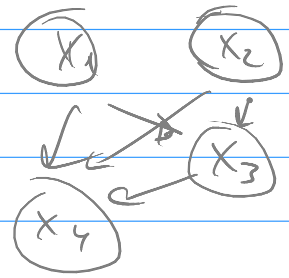
Directed graphical models

$$p(\bar{x}) = p(x_1, \dots, x_n) = p(x_1 | \dots)\, p(x_2 | \_)\, \dots\, p(x_n | \dots)$$

$$p(x_1 \dots x_n) = p(x_1)\,p(x_2|x_1)\,p(x_3|x_1,x_2) \dots p(x_n | x_1 \dots x_{n-1})$$

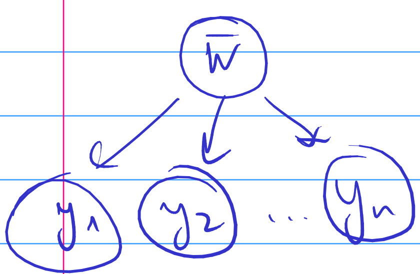


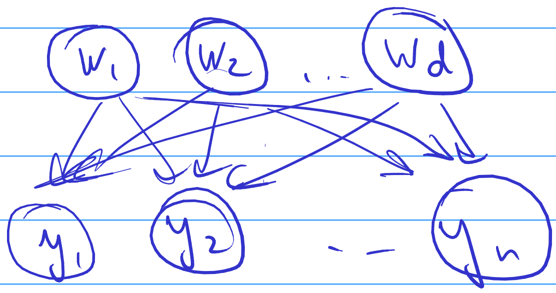
$$\boxed{p(x_1 \dots x_n) = \prod_{i=1}^{n} p(x_i | par(x_i))}$$

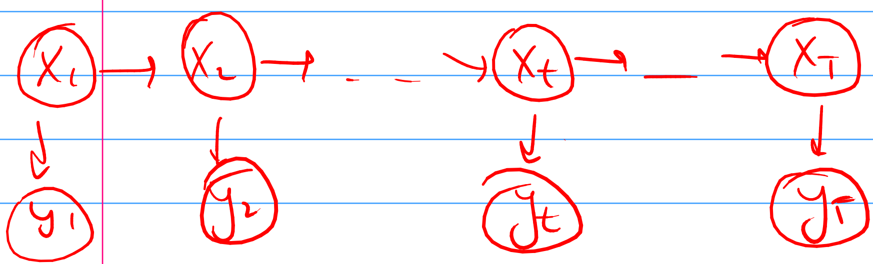


$$\prod_{j=1}^{d} p(w_j)$$

$$p(\bar{y}, \bar{w} | X) = p(\bar{w}) \prod_n p(y_n | \bar{x}_n, \bar{w})$$







$$p(\bar{x}, \bar{y}) = p(x_1)\,p(y_1|x_1)\,p(x_2|x_1) \dots p(x_T|x_{T-1})\,p(y_T|x_T)$$

$n=1$ $\quad X_1 \qquad p(x_1)=p(x_1)$

$n=2$ $\quad X_1 \quad X_2$

$$p(x_1,x_2)=p(x_1)p(x_2)$$

независимы

$$\begin{tikzcd} X_1 \arrow[r, shift left] & X_2 \arrow[l, shift left] \end{tikzcd}$$

$$p(x_1,x_2)=p(x_1)p(x_2|x_1)=$$
$$=p(x_2)p(x_1|x_2)$$

$n=3$ $\quad X_1 \quad X_2 \quad X_3$

$$p(x_1,x_2,x_3)=p(x_1)p(x_2)p(x_3)$$

$X_1 \qquad X_2 \to X_3$

$$p(x_1,x_2,x_3)=p(x_1)p(x_2)p(x_3|x_2)$$

$$\begin{tikzcd} X_1 \arrow[r, shift left] \arrow[dr] & X_2 \arrow[l, shift left] \arrow[d, shift left] \\ & X_3 \arrow[u, shift left] \arrow[ul, shift left] \end{tikzcd}$$

$$p(x_1,x_2,x_3)=p(x_1)p(x_3|x_1)p(x_2|x_1,x_3)$$
$$=p(x_2)p(x_3|x_2)p(x_1|x_2,x_3)$$

последоват. связь

① $X_1 \to X_2 \to X_3$

расходящаяся связь

②
$$\begin{tikzcd} & X_1 \arrow[dl] \arrow[dr] & \\ X_2 & & X_3 \end{tikzcd}$$

сходящаяся связь

$$\begin{tikzcd} X_2 \arrow[dr] & & X_3 \arrow[dl] \\ & X_1 & \end{tikzcd}$$

explaining away

①
$$p(x_1,x_2,x_3)=p(x_1)p(x_2|x_1)p(x_3|x_2)$$

$$p(x_1,x_3|x_2) \overset{?}{=} p(x_1|x_2)p(x_3|x_2)$$
$$\| \quad \frac{p(x_1,x_2,x_3)}{p(x_2)}=\frac{p(x_1)p(x_2|x_1)p(x_3|x_2)}{p(x_2)}$$

②
$$p(x_1,x_2,x_3)=p(x_1)p(x_2|x_1)p(x_3|x_1)$$

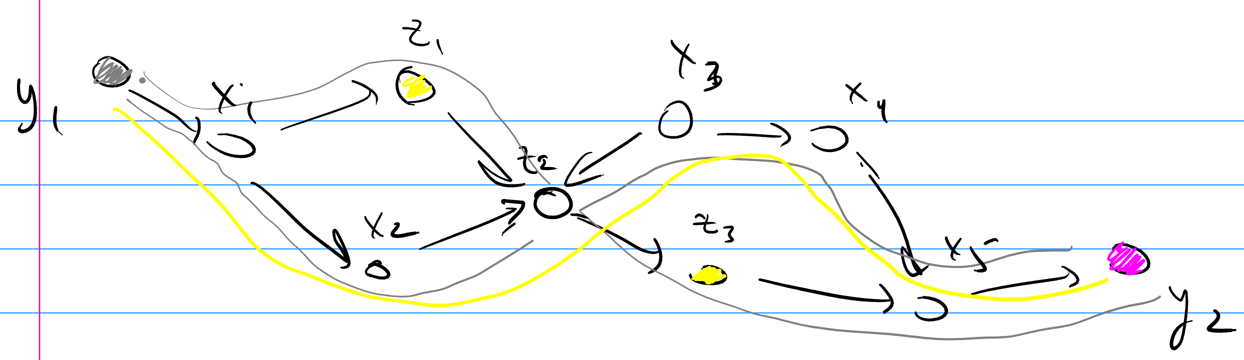
$$p(x_2,x_3|x_1) \overset{?}{=} p(x_2|x_1)p(x_3|x_1)$$
$$\| \quad \frac{p(x_1,x_2,x_3)}{p(x_1)}=\frac{\cancel{p(x_1)}\,p(x_2|x_1)p(x_3|x_1)}{\cancel{p(x_1)}}$$

③
$$p(x_1,x_2,x_3)=p(x_2)p(x_3)p(x_1|x_2,x_3)$$

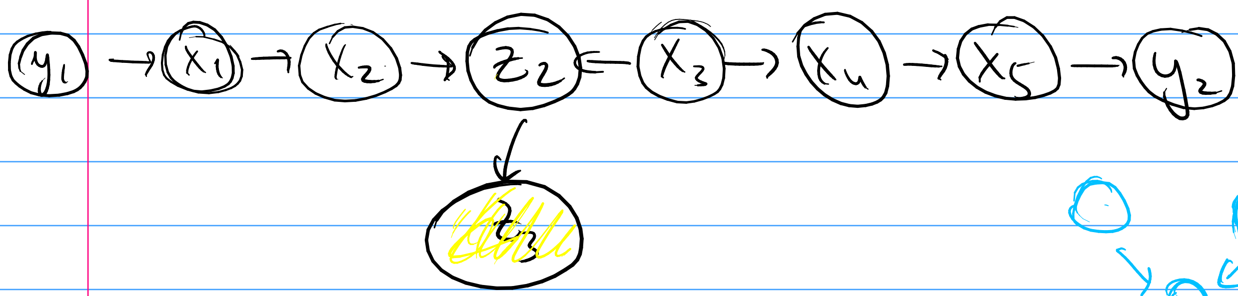
$$p(x_2,x_3|x_1) \;?\; p(x_2|x_1)p(x_3|x_1)$$

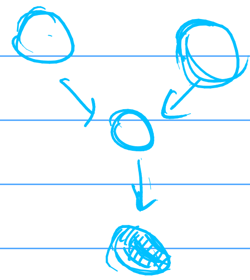
$$p(x_2,x_3)=\int p(x_1,x_2,x_3)dx_1$$
$$=p(x_2)p(x_3)\underbrace{\int p(x_1|x_2,x_3)dx_1}_{=1}$$

$$p(y_1, y_2, x_1, \ldots, x_5, z_1, \ldots z_3) = \ldots$$



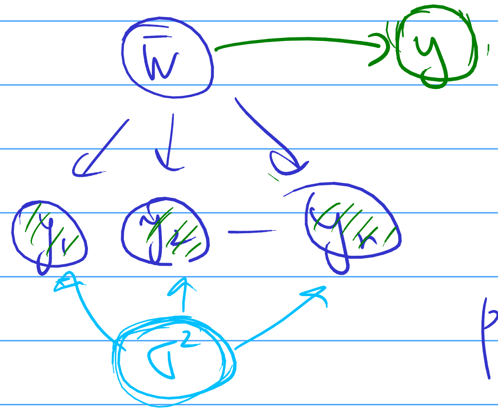
$$p(y_1, y_2 \mid z_1, z_2, z_3) = ?$$





Лин. регр.

Логист. регр



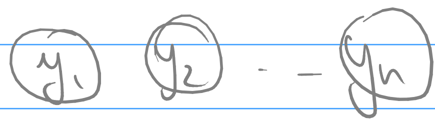
empirical Bayes

$$p(\bar w, y_1, \ldots, y_n) =$$

$$= p(\bar w) \cdot \prod_{i=1}^{n} \boxed{p(y_i \mid \bar w)}$$

$$p(y_i, y_j \mid \bar w) = p(y_i \mid \bar w)\, p(y_j \mid \bar w)$$

$$p(y \mid \bar y, \bar x) = \int p(\bar w, y \mid \bar y)\, d\bar w$$



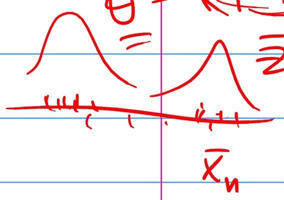
$$p(y_i \mid \bar w) = \mathcal{N}(y_i \mid \bar w^T \bar x_i, \boxed{\sigma^2})$$

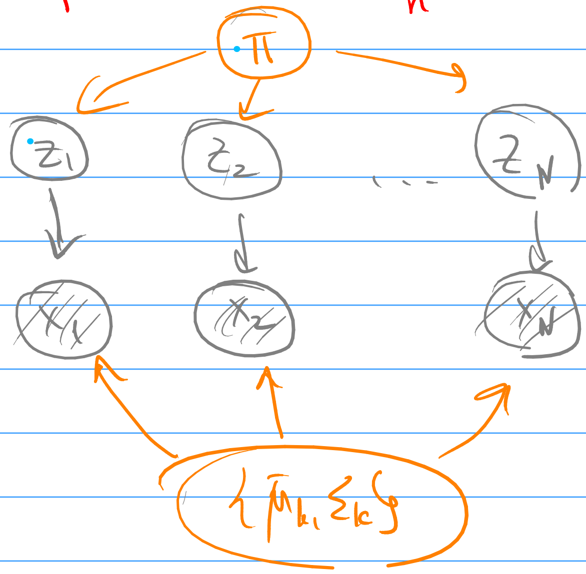
Кластеризация

$$\bar\theta = (\{\mu_k, \Sigma_k\}, k=1..K, \bar\pi)$$

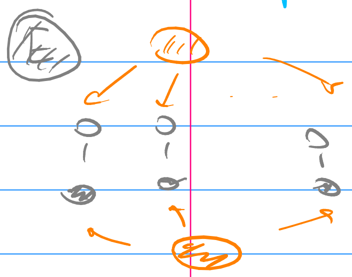
$(\pi, \theta - \pi)$

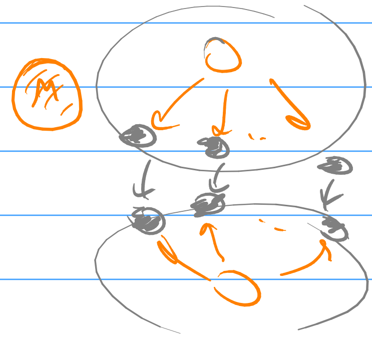
$$p(X, Z|\bar\theta) = \prod_n p(\bar z_n|\bar\pi)\, p(\bar x_n|\bar z_n, \bar\theta)$$





EM-алгоритм

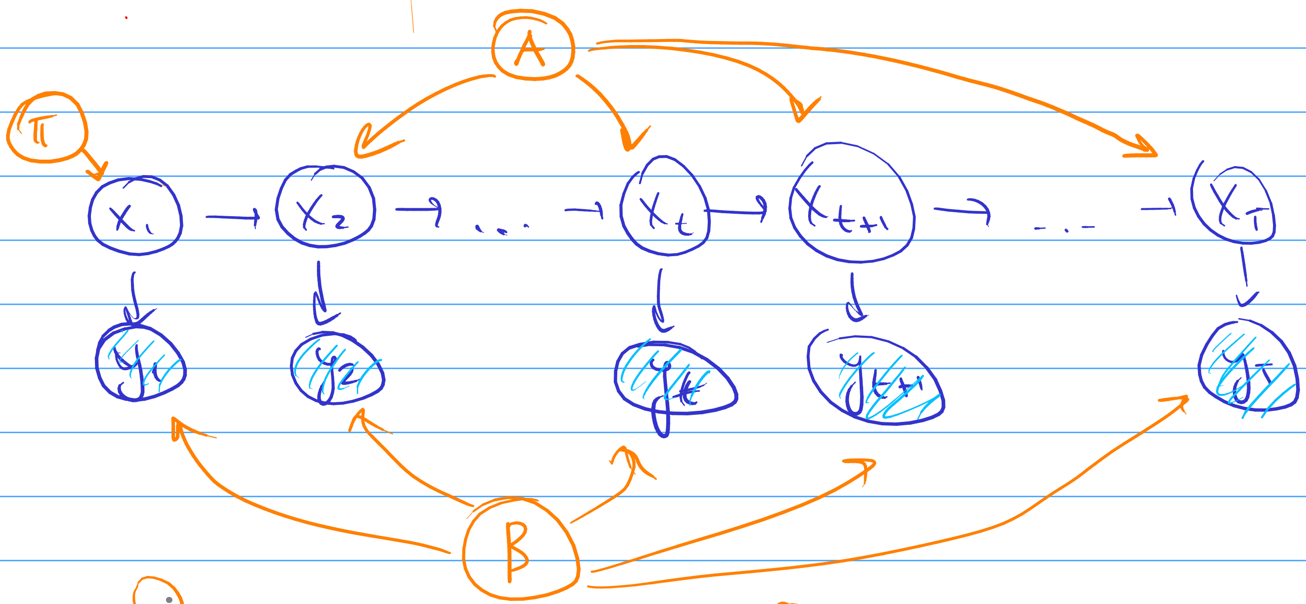


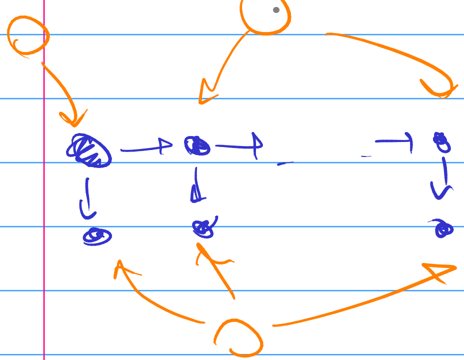


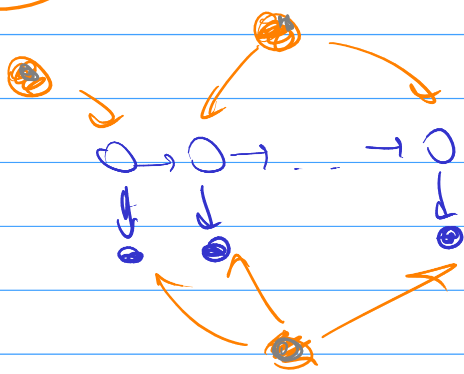
$$p(X, Z, \theta) = p(\pi)\, p(\{\mu_k, \Sigma_k\}) \cdot \prod_n p(z_n|\pi)\, p(x_n|z_n, \{\mu_k, \Sigma_k\})$$

HMM

$\lambda = (\pi, A, B)$







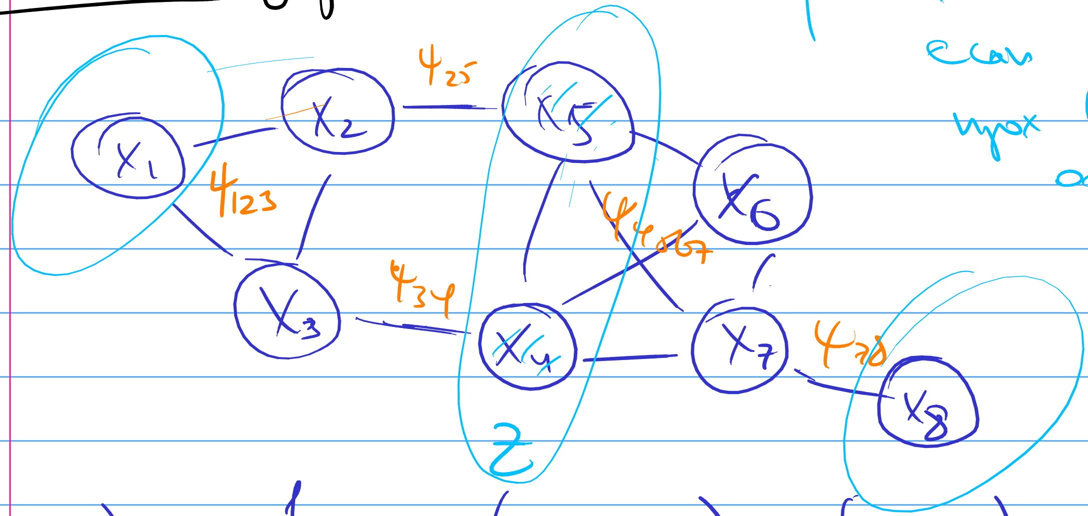
Undirected graphical models

$$p(x,y|z) = p(x|z)\,p(y|z)$$

если знаю z

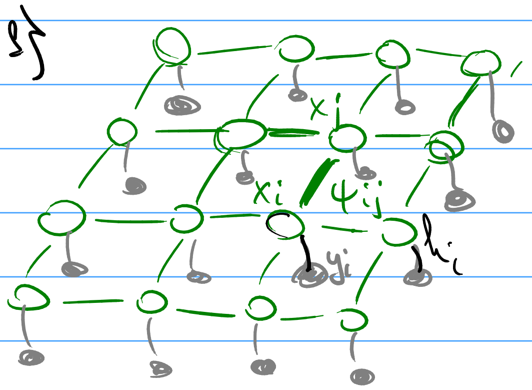
[illegible] все [illegible]

x и y

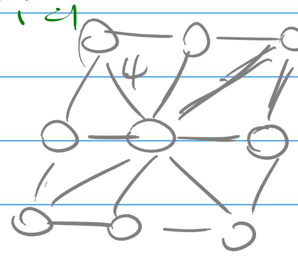


$$p(x_1 \dots x_8) = \frac{1}{Z}\, \psi_{123}(x_1,x_2,x_3)\, \psi_{34}(x_3,x_4)\, \psi_{25}(x_2,x_5)$$
$$\psi_{4567}(x_4,x_5,x_6,x_7)\, \psi_{78}(x_7,x_8)$$

$x_i, y_i \in \{\pm 1\}$



$$\bar{y} = (y_i)_{i=1}^{n}$$
$$\Downarrow$$
$$\bar{x} = (x_i)_{i=1}^{n}$$

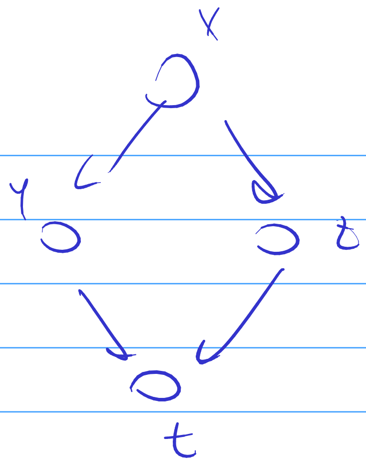


$$f(\bar{x},\bar{y}) = \prod_i h_i(x_i,y_i) \cdot \prod_{(i,j)\in Ne_i} \psi_{ij}(x_i,x_j) =$$

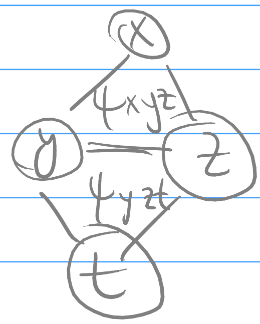
min

$$= \prod_i e^{-\alpha x_i y_i} \cdot \prod_{i,j\in Ne_i} e^{-\beta x_i x_j} \longrightarrow \min$$

$$\left[\alpha \sum_i x_i y_i + \beta \sum_{i,j} x_i x_j \xrightarrow[x_i]{} \max \right]$$



$p(x)\, p(y|x)\, p(z|x)\, p(t|y,z)$



$\psi_{xyz} \stackrel{?}{=} \psi_{xy}\psi_{xz}$