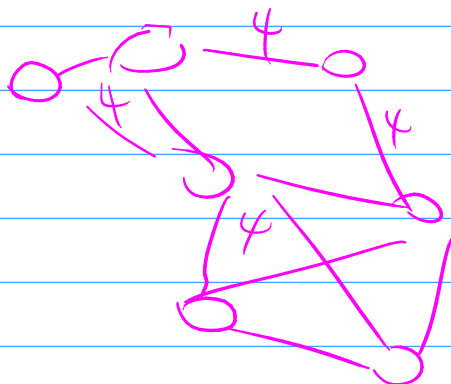
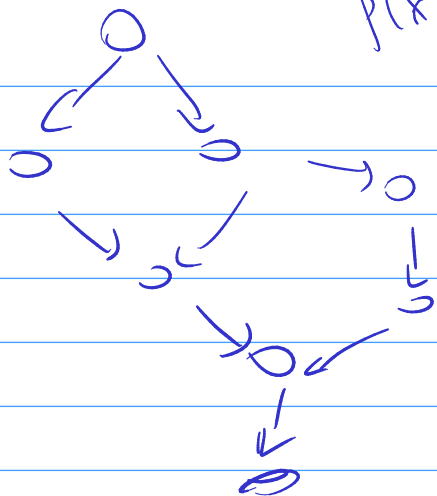
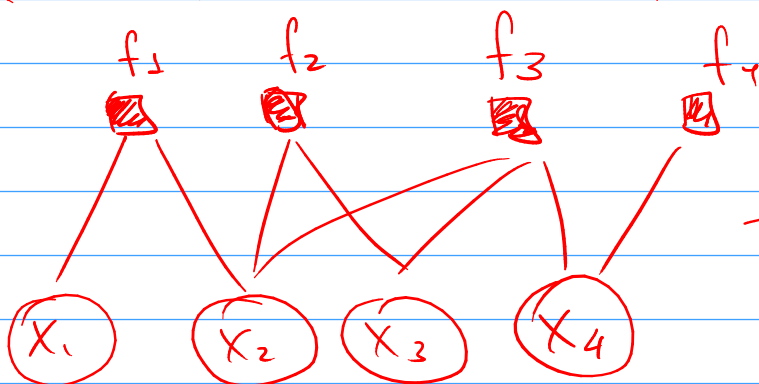


$$p(\bar{X}) = \prod p(x_i | \text{par}(x_i))$$



factor graphs

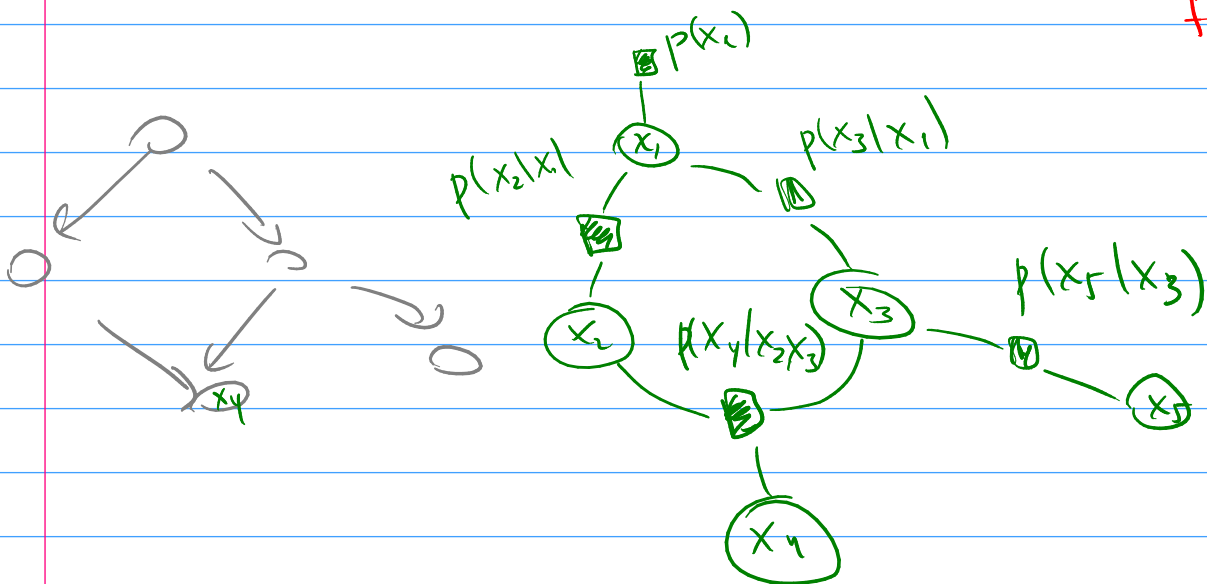
$$f(x_1, \dots, x_n) = \prod f_i(x_i), \quad x_i \in X$$



$$f(x_1, \dots, x_n) = f_1(x_1, x_2)$$

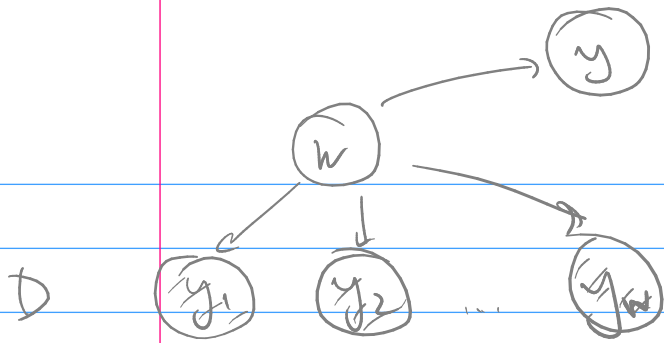
$$f_2(x_2, x_3) \quad f_3(x_2, x_3, x_4)$$

$$f_4(x_4)$$



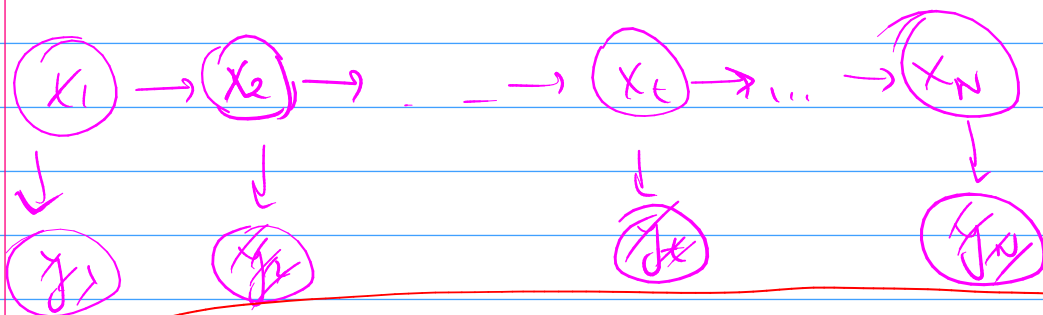
$$f(x_1, x_2, \dots, x_n)$$

$$f(x_i) = \int f(x_1, \dots, x_n) dx_1 \dots dx_{i-1} dx_{i+1} \dots dx_n$$



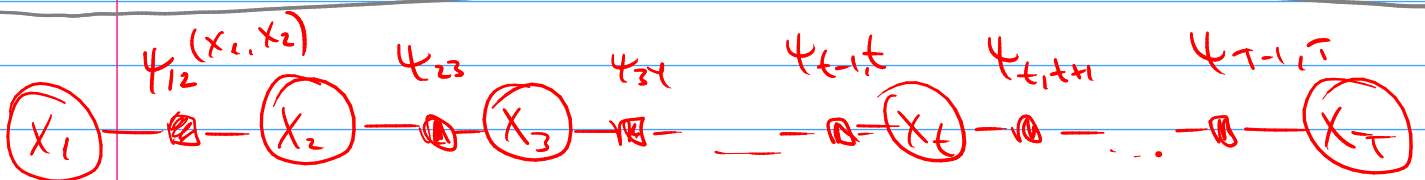
$$p(\bar{w} | D) \propto p(\bar{w}) \prod p(y_n | \bar{w})$$

$$p(y | y_1, \dots, y_N) = \int p(y, \bar{w} | y_1, \dots, y_N) d\bar{w}$$



$$p(x_t | y_1, \dots, y_N) = \int p(x_1, \dots, x_N | y_1, \dots, y_N) d\bar{x}_{-t}$$

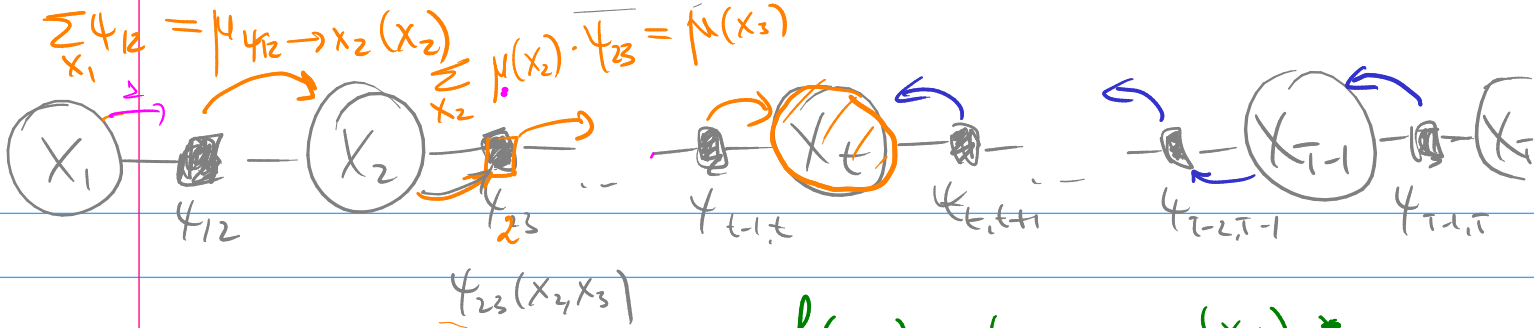
$$Q^* = \text{argmax}_{\bar{x}} p(x_1, \dots, x_N | y_1, \dots, y_N)$$



$$f(x_1, \dots, x_T) = \psi_{12}(x_1, x_2) \dots \psi_{T-1,T}(x_{T-1}, x_T)$$

$$f(x_t) = \sum_{x_1, \dots, x_{t-1}, x_{t+1}, \dots, x_T} f(x_1, \dots, x_T) =$$

$$= \sum_{x_1} \sum_{x_2} \dots \sum_{x_{t-1}} \sum_{x_{t+1}} \dots \sum_{x_T} \left(\psi_{12} \psi_{23} \dots \psi_{t-1,t} \psi_{t,t+1} \dots \psi_{T-1,T} \right)$$

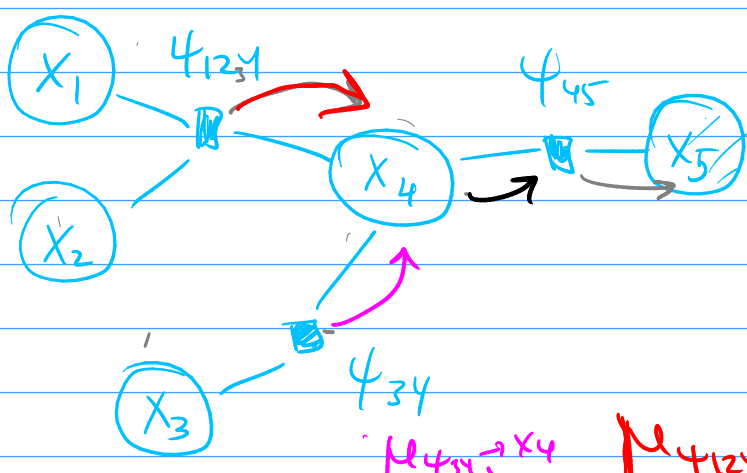
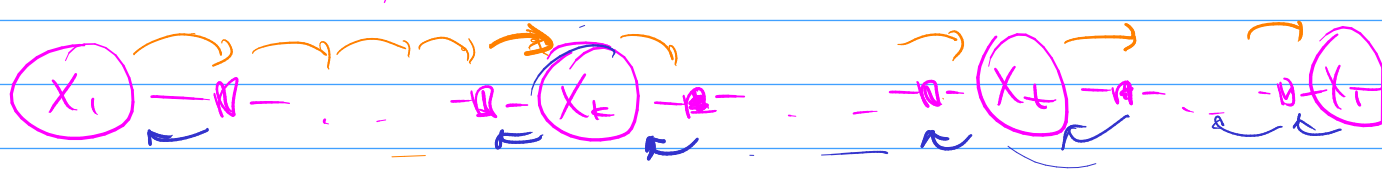


$$\sum_{x_3} \psi_{34} \dots (x_3)$$

(x_n)

$$f(x_t) = \mu_{\psi_{t-1,t} \rightarrow x_t}(x_t) * \mu_{\psi_{t,t+1} \rightarrow x_t}(x_t)$$

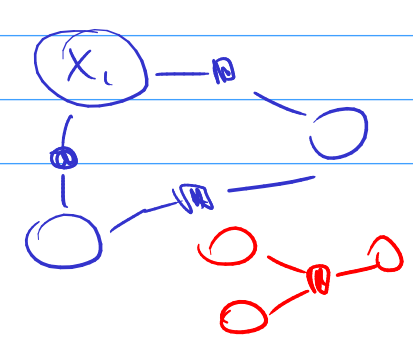
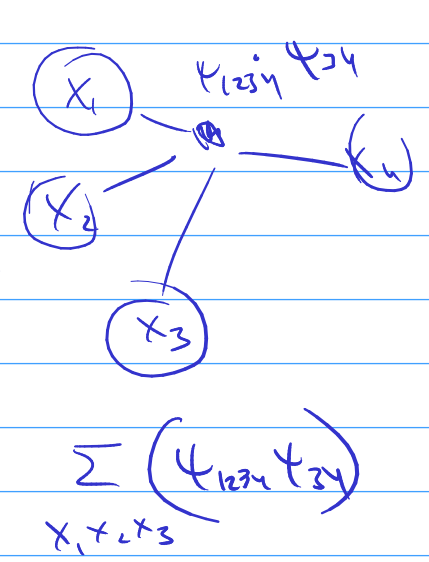
$$f(x_k) = \sum_{x_1 \dots x_n} f(x_1 \dots x_n) \quad f(x_t)$$



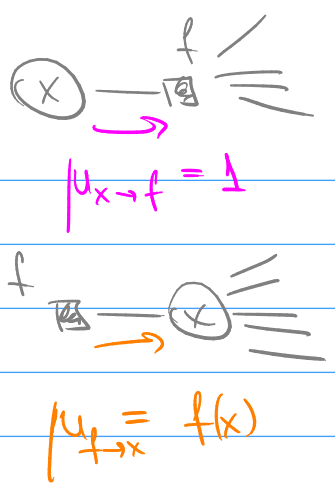
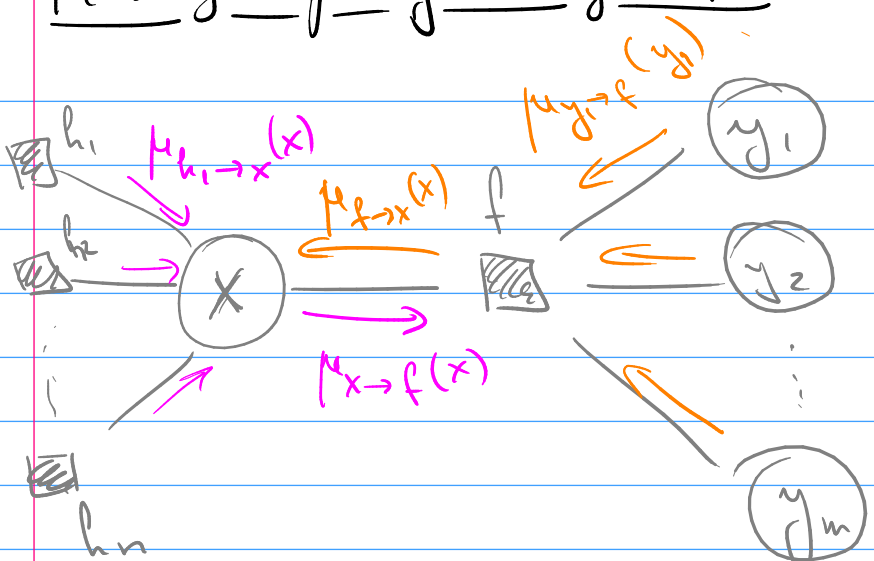
$$f(x_5) = \sum_{x_1 \dots x_4} \psi_{1234} \psi_{34} \psi_{45}$$

$$f(x_5) = \sum_{x_4} \psi_{45} \left(\sum_{x_3} \psi_{34} \right) \left(\sum_{x_1} \sum_{x_2} \psi_{124} \right)$$

$$\mu_{x_4 \rightarrow \psi_{45}}$$



Message passing algorithm

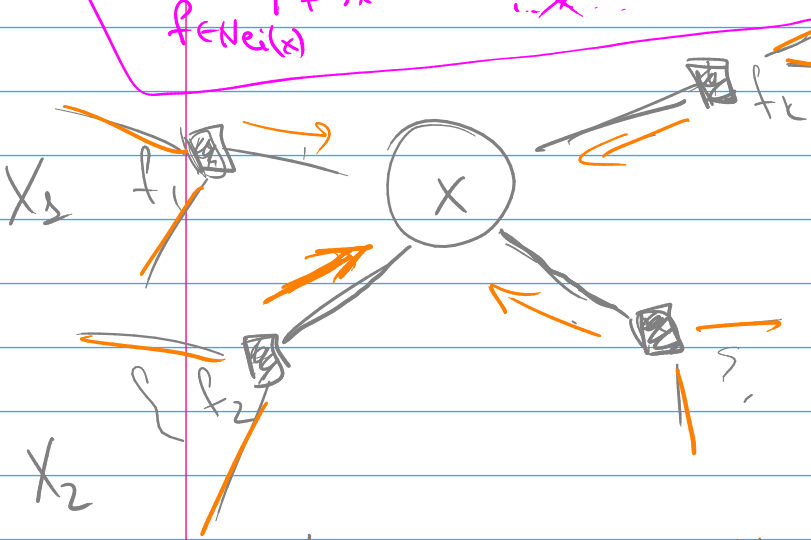


$$\mu_{x \rightarrow f}(x) = \prod_{i=1}^n \mu_{h_i \rightarrow x}(x)$$

$$\mu_{f \rightarrow x}(x) = \int f(x, y_1, \dots, y_m) \cdot \prod_{j=1}^m \mu_{y_j \rightarrow f}(y_j) dy_1 \dots dy_m$$

$$\prod_{f \in N(x)} \mu_{f \rightarrow x}(x) = \sum_{\dots x \dots} F(x_1, \dots, x_n)$$

$$F(x_1, \dots, x_n) = \prod f \dots$$



$$F(x_1, \dots, x_n) = \left(\prod_{k=1}^k f_k(x, x_k) \right) \cdot (\prod \dots)$$

$$X = \{x\} \cup X_1 \cup X_2 \dots \cup X_k$$

$F(x)$

$$\sum_{\dots x \dots} = \left(\sum_{X_1} f_1(x, X_1) \cdot \prod \dots \right) \left(\sum_{X_2} f_2(x, X_2) \prod \dots \right)$$

$$\dots \left(\sum_{X_k} f_k(x, X_k) \prod \dots \right)$$

