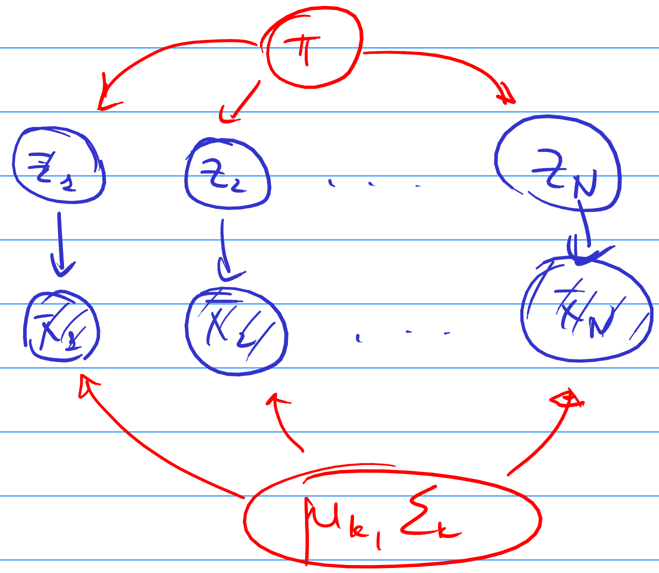
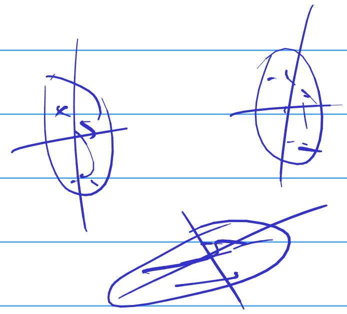
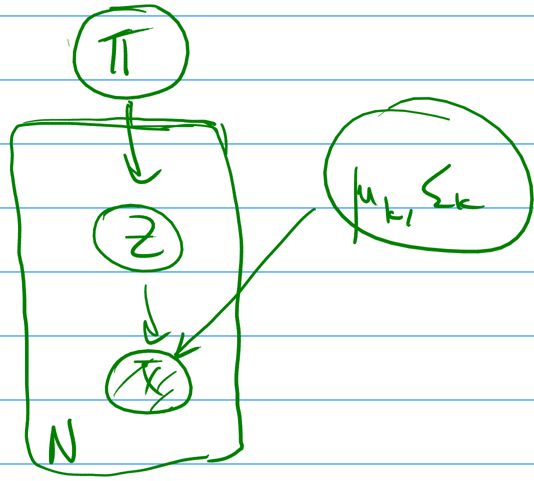
$$p(\bar{x}) = \sum_{k=1}^{K} \pi_k \, p(\bar{x}|\bar{\theta}_k) = \sum_{k=1}^{K} \pi_k \, \mathcal{N}(\bar{x}|\bar{\mu}_k, \Sigma_k)$$

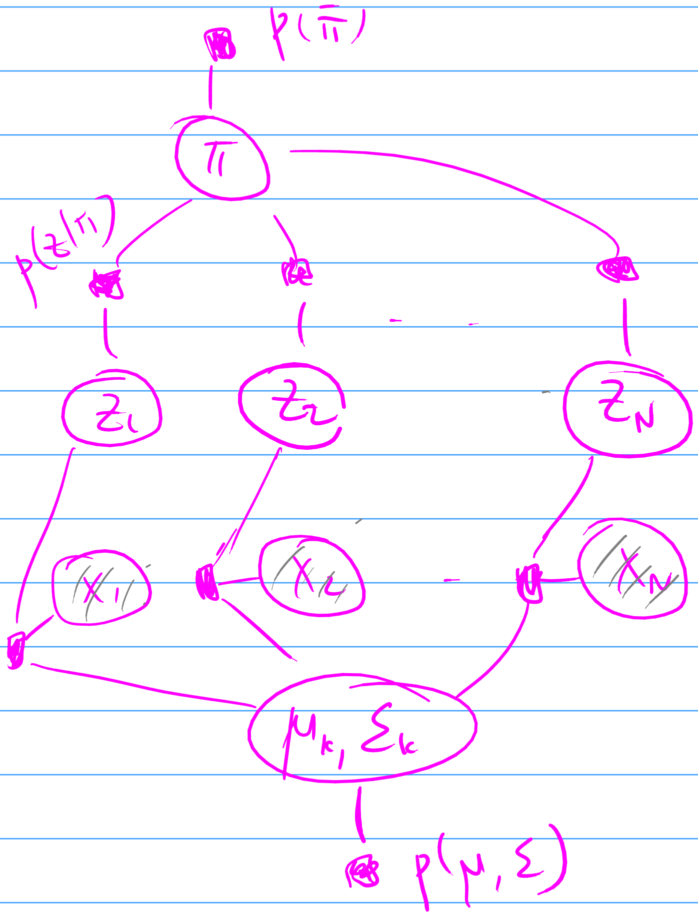


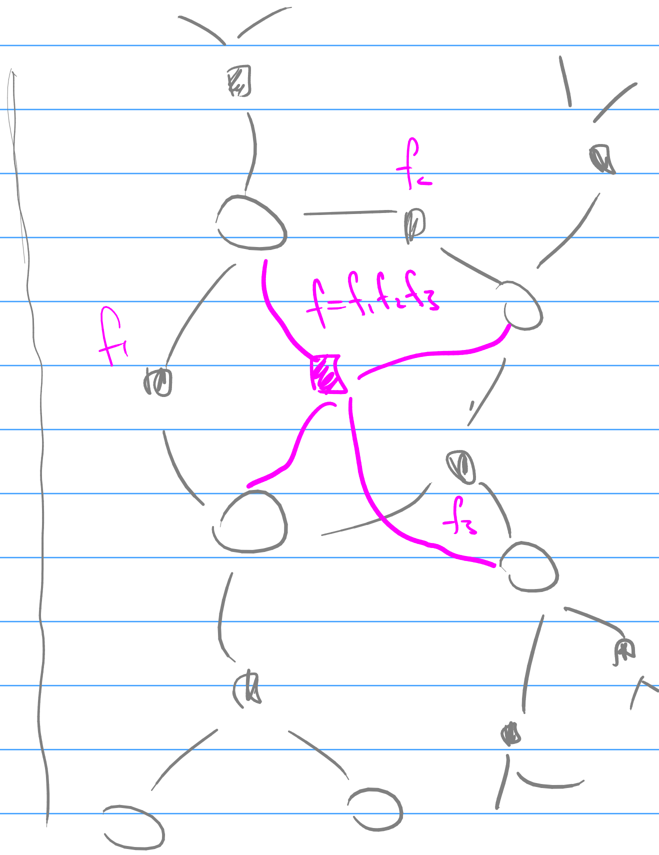


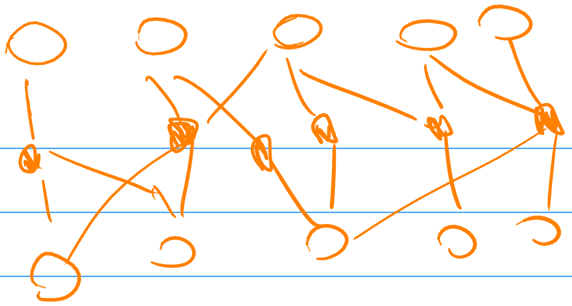


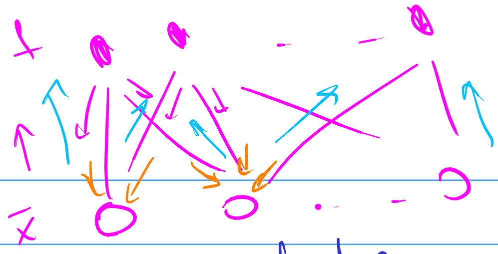
$$p(\pi, \bar{\mu}_k, \Sigma_k, \bar{z}, \bar{x}) = p(\pi)\, p(\bar{\mu}_k, \Sigma_k) \times$$

$$\times \prod_{n=1}^{N} p(z_n|\bar{\pi})\, p(\bar{x}_n | z_n, \{\bar{\mu}_k, \Sigma_k\})$$



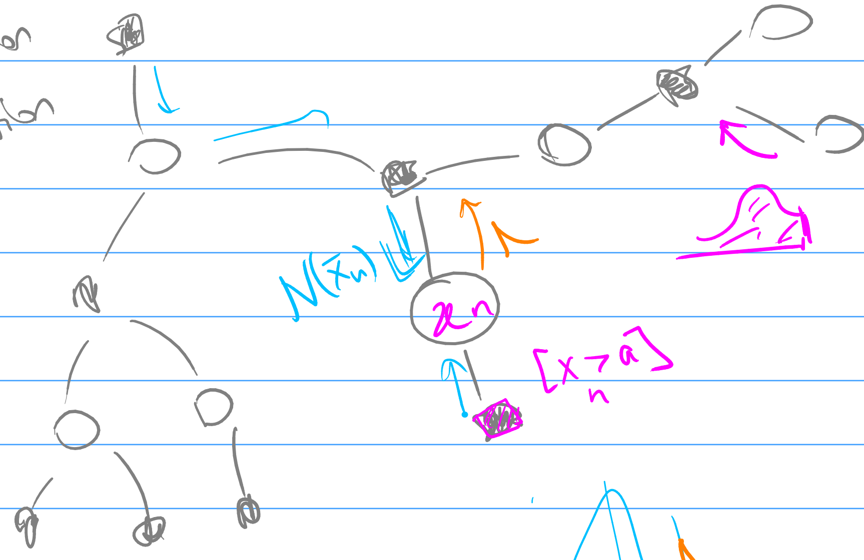






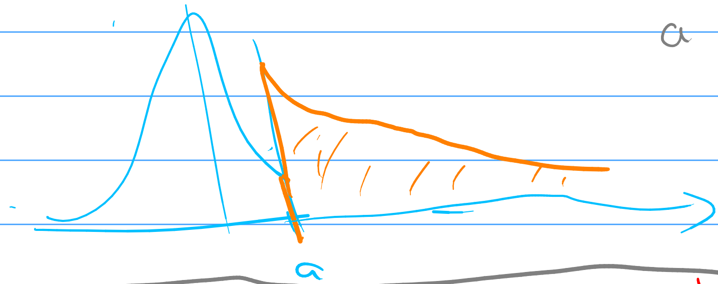
Loopy Belief Propagation

Expectation Propagation



$$\int p(x_1, \dots, x_n, \dots)\, d\bar{x}$$

$$= \int_a^\infty (\dots)\, dx_n$$



Variational approximations

$$p(\bar{x}) \approx q(\bar{x}) = \prod q_i(x_i)$$

$$KL(p\|q) = \int p(\bar{x}) \ln \frac{p(\bar{x})}{q(\bar{x})}\, d\bar{x} \to \min_q$$

$$\neq KL(q\|p)$$

$$p(x|D) = \int p(x|\bar{\theta})\, p(\bar{\theta}|D)\, d\bar{\theta} = E_{p(\bar{\theta}|D)}[p(x|\bar{\theta})]$$

$$\boxed{\bar{\theta}^{(z)} \sim p(\bar{\theta}|D)}$$

$$\boxed{Ef \approx \frac{1}{R}\sum_{1}^{R} f(x^{(z)})}$$

$$p(x|D) \approx \frac{1}{R}\sum_{z=1}^{R} p(x|\bar{\theta}^{(z)})$$

BPMF

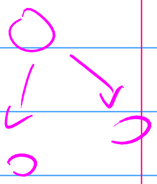
$$p(\bar\theta|D) \propto p(\bar\theta)\cdot\prod_{n=1}^{N} p(x_n|\bar\theta)$$

$\bar x$ — $\square$ — $p(\bar x)$

$\bar x$ — $\square$ — $p^*(\bar x) \propto p(\bar x)$

$\bar x \sim p(\bar x)$?

Generative

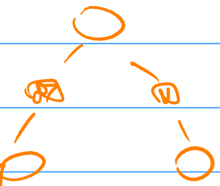


$p(\bar x, y|\bar\theta)$

$p(\bar x, y|\theta_{ML})$

Discriminative

$p(y|\bar x)$



$\boxed{\bar\theta \sim p(\bar\theta|D)}$

EM: $Q(\bar\theta,\bar\theta^{(m)}) = \int p(z|x,\bar\theta^{(m)}) \ln p(x,z|\bar\theta)\,dz =$

$= E_{p(z|x,\theta^{(m)})}\left[\ln p(x,z|\bar\theta)\right] \approx$

$\boxed{z^{(r)} \sim p(z|x,\theta^{(m)})}$

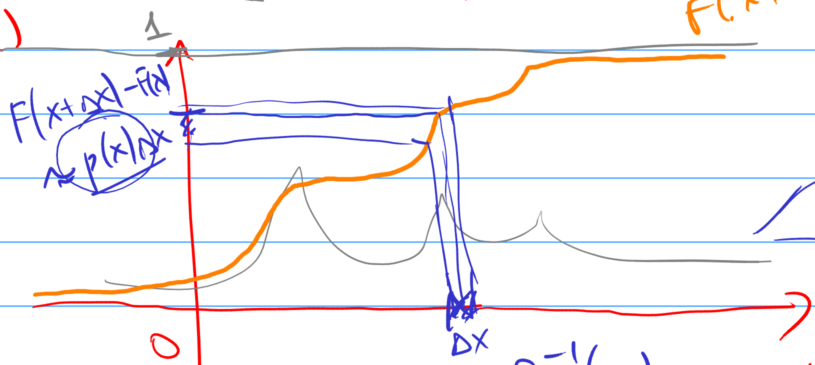
$\approx \frac{1}{R}\sum_{r=1}^{R} \ln p(x,z^{(r)}|\bar\theta)$

Monte-Carlo EM

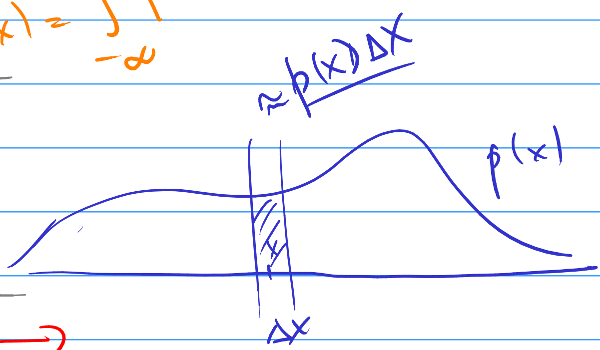
1) $\boxed{z \sim Unif[0,1]}$ rand()

$p(x) = ?$

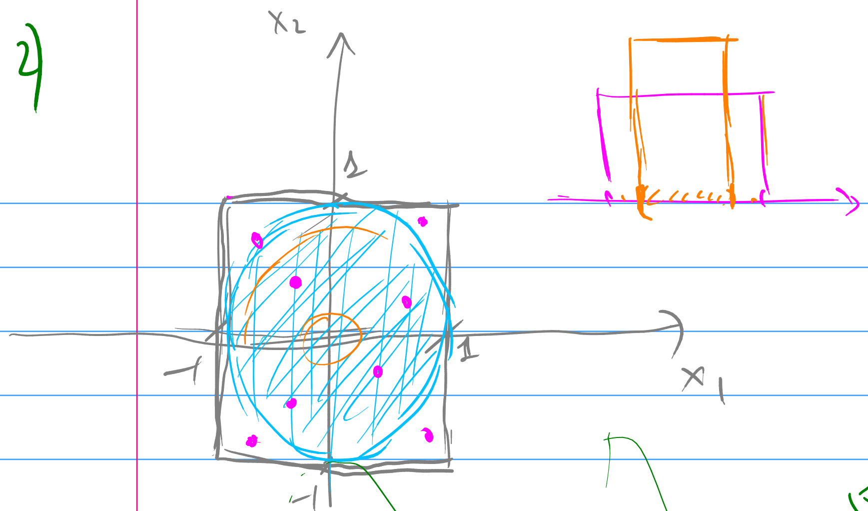
$F(x) = \int_{-\infty}^{x} p(a)\,da$



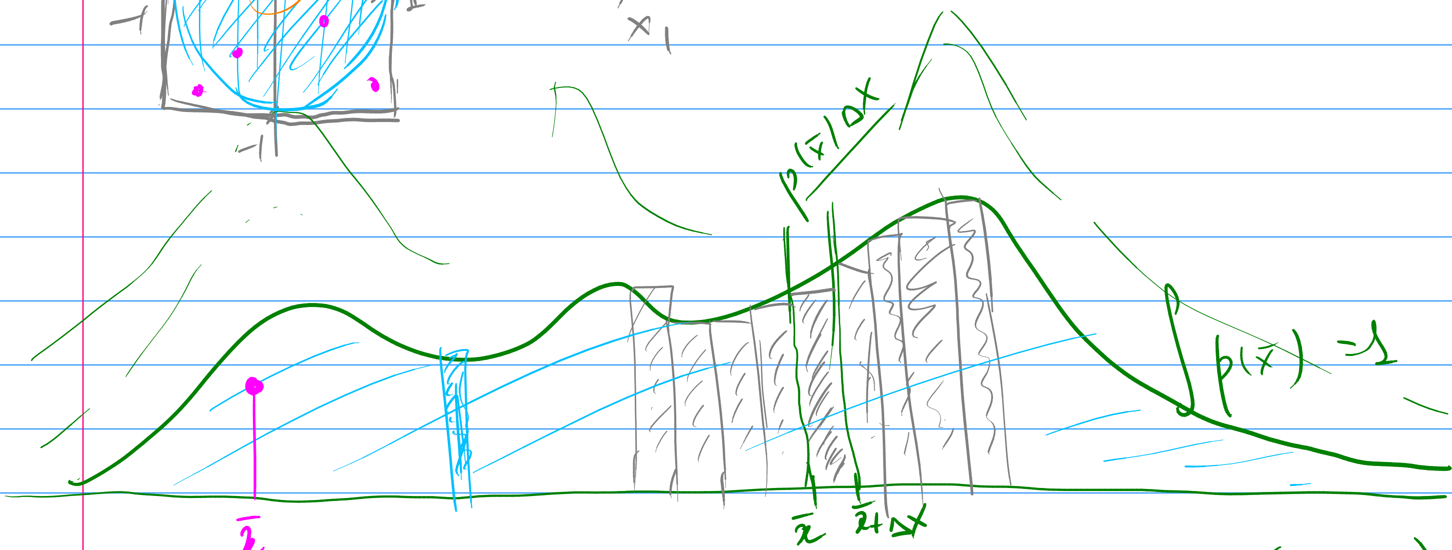
$x = F^{-1}(z)$



2)



3)

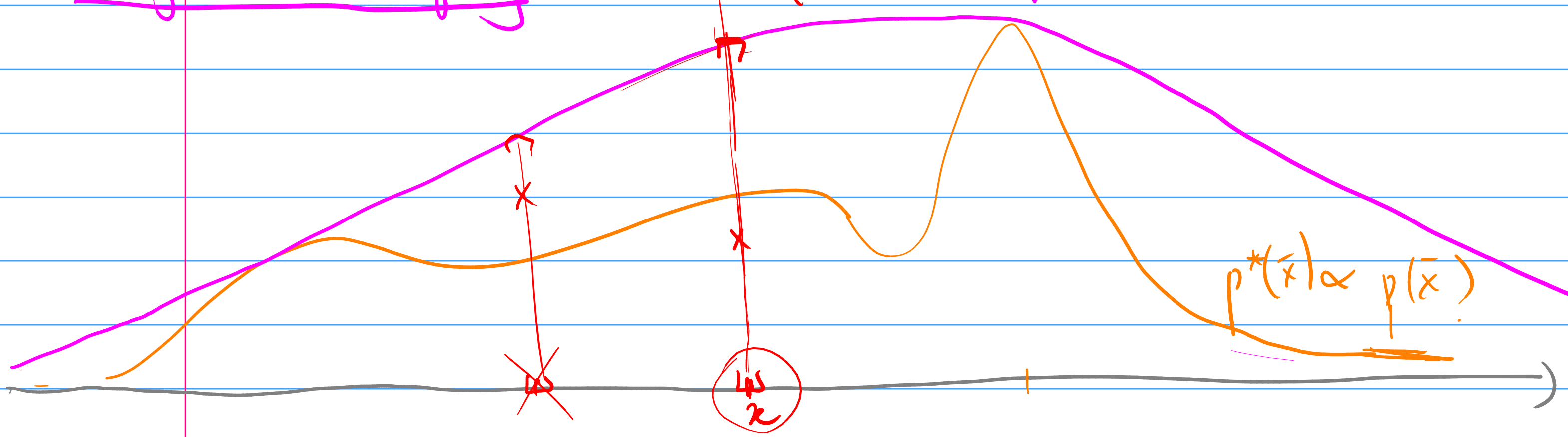


$$\bar{x} \sim p(\bar{x}) \iff (\bar{x}, y) \sim \mathrm{Unif}(\ldots)$$

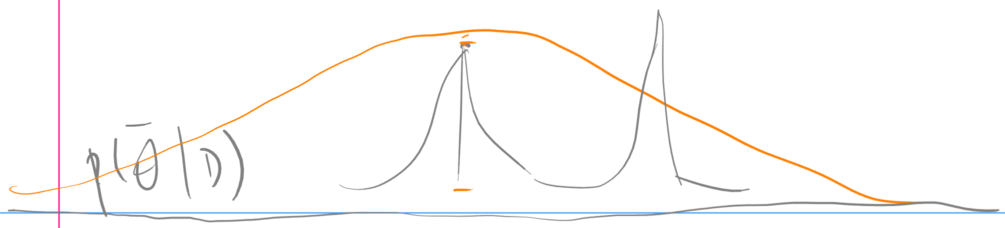
$$p^*(\bar{x}) \propto p(\bar{x})$$

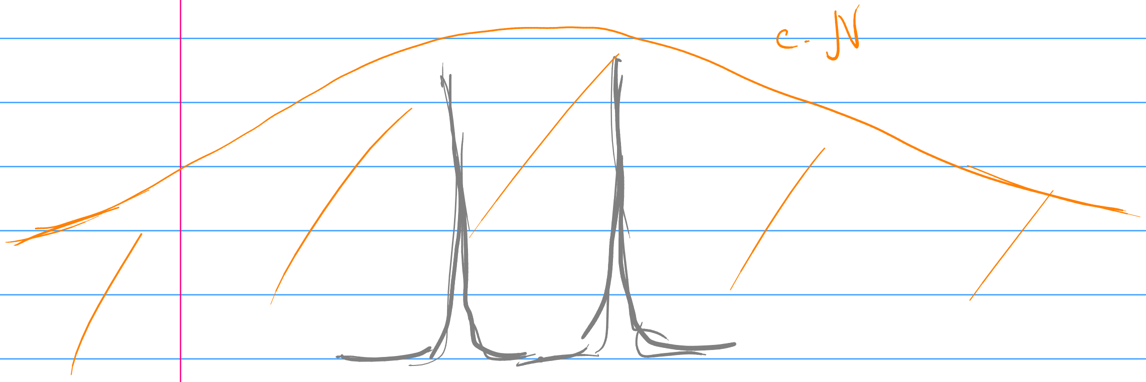
Rejection sampling

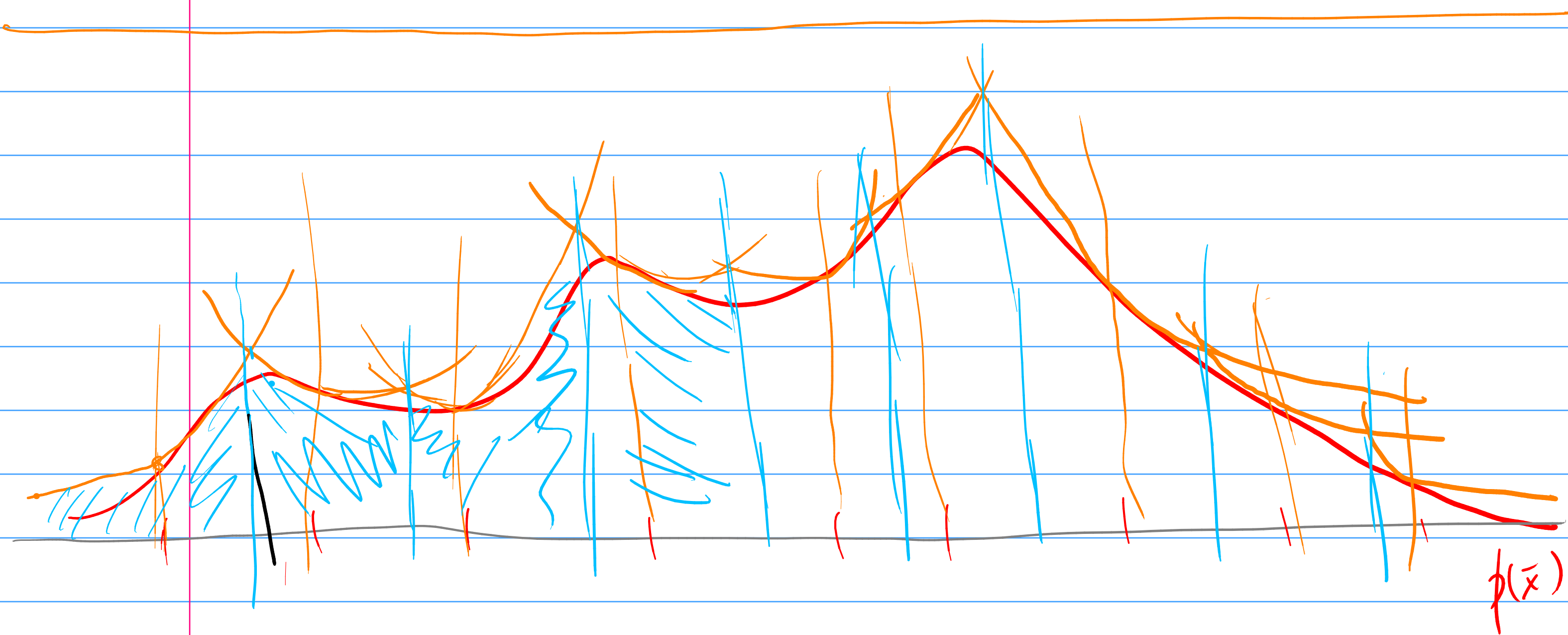
$c \cdot q(\bar{x}) \geq p(\bar{x})$



- sample $\bar{x} \sim q(\bar{x})$
- sample $y \sim \mathrm{Unif}([0, c \cdot q^*(\bar{x})])$
- if $y \leq p(\bar{x})$ accept $\bar{x}$
  else reject $\bar{x}$


p(θ̄|D)


c·N


p(x̄)


ln p(x̄)
ln p(x̄)
≐ X

$d \to \infty$

$p = N_1 + N_2$