$p(\bar x)$

$\bar x$ — $\square$ — $p^*(\bar x) \propto p(\bar x)$

$\bar x \sim p(\bar x)$

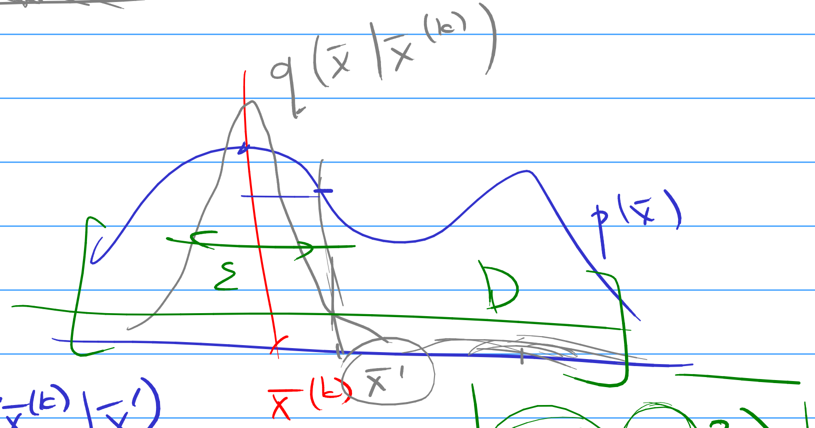
$n - O(\sqrt{n})$

MCMC - методы

Metropolis-Hastings

$\bar x' \sim q(\bar x' | \bar x^{(k)})$

$p(\bar x^{(k)})$ $\quad \frac{}{p(\bar x')}$



$$a(\bar x'; \bar x^{(k)}) = \frac{p^*(\bar x')}{p^*(\bar x^{(k)})} \cdot \frac{q(\bar x^{(k)} | \bar x')}{q(\bar x' | \bar x^{(k)})}$$

$$\boxed{O\left(\left(\frac{D}{\varepsilon}\right)^2\right)}$$

if $a > 1 \Rightarrow$ accept

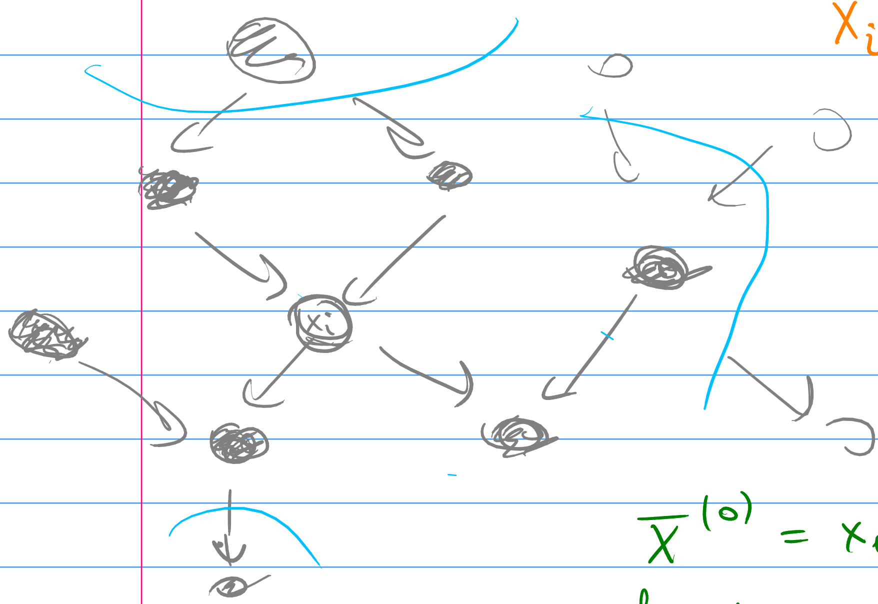
$a < 1 \Rightarrow$ accept с вер. a, else $\bar x^{(k+1)} = \bar x^{(k)}$

Topic modeling

Gibbs sampling

$$p(\bar x) = p(x_1, x_2, \dots, x_n)$$

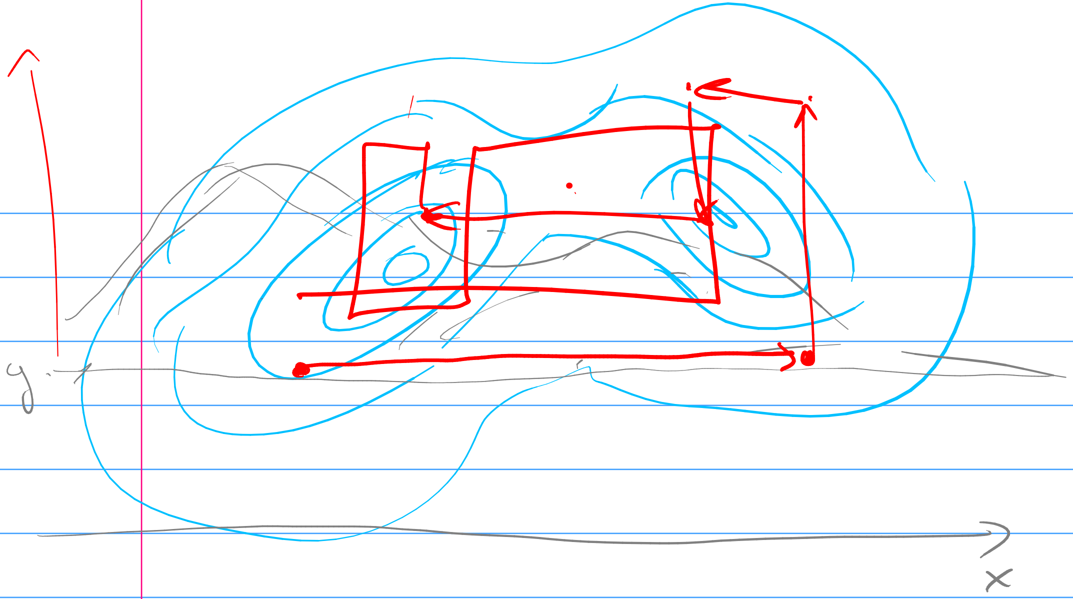
$$x_i \sim p(x_i | x_1 \dots x_{i-1}, x_{i+1}, \dots, x_n)$$
$$= p(x_i | \bar x_{-i})$$

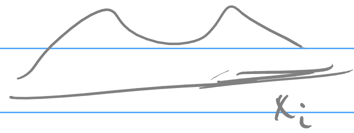


$$\bar x^{(0)} = x_1^{(0)}, \dots, x_n^{(0)}$$

- loop:

$$x_i^{(k+1)} \sim p(x_i^{(k+1)} | x_1^{(k+1)}, \dots, x_{i-1}^{(k+1)}, x_{i+1}^{(k)}, \dots, x_n^{(k)})$$





$$q(\bar{x}' \mid \bar{x}) = \begin{cases} 0, & \bar{x}'_{-i} \neq \bar{x}_{-i} \\ p(x'_i \mid \bar{x}_{-i}), & \bar{x}'_{-i} = \bar{x}_{-i} \end{cases}$$

$\bar{x} = (x_i, \bar{x}_{-i})$

$\bar{x}' = (x'_i, \bar{x}_{-i})$

$$a(\bar{x}', \bar{x}) = \frac{p(\bar{x}')}{p(\bar{x})} \cdot \frac{q(\bar{x} \mid \bar{x}')}{q(\bar{x}' \mid \bar{x})} =$$

$$= \frac{p(\bar{x}')}{p(\bar{x})} \cdot \frac{p(x_i \mid \bar{x}_{-i})}{p(x'_i \mid \bar{x}_{-i})} = \frac{p(x'_i \mid \bar{x}_{-i})\, p(\bar{x}_{-i})}{p(x_i \mid \bar{x}_{-i})\, p(\bar{x}_{-i})} \frac{p(x_i \mid \bar{x}_{-i})}{p(x'_i \mid \bar{x}_{-i})}$$

$$= 1$$

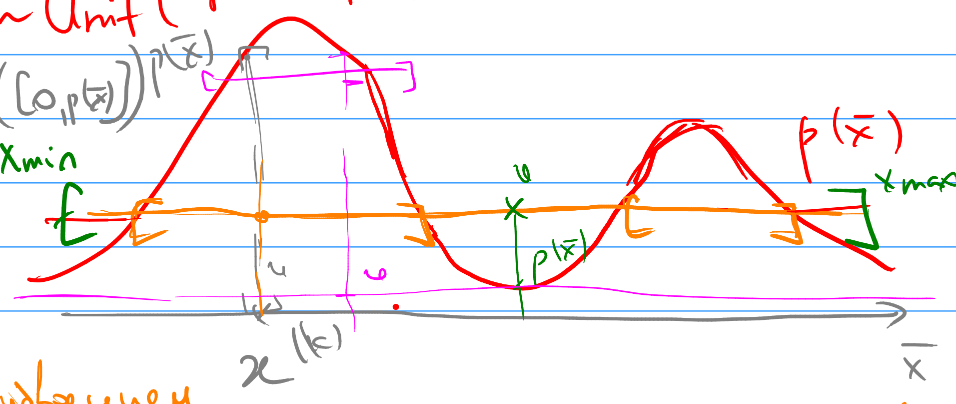
Slice sampling

MCMC

$\bar{x} \sim p(\bar{x}) \iff (\bar{x}, u) \sim \mathrm{Unif}(\text{график } p(\bar{x}))$ *

$u \mid \bar{x} = \mathrm{Unif}([0, p(\bar{x})])$

$\bar{x} \mid u$



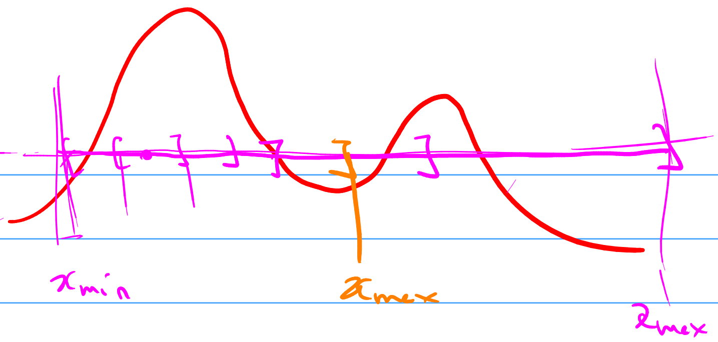
Алгоритм: $x^{(k)} \to x^{(k+1)}$:

- $u \sim \mathrm{Unif}([0, p(\bar{x}^{(k)})])$
- строим $[x_{min}, x_{max}]$ удовлетвор.
- $x^{(k+1)} \sim \mathrm{Unif}([x_{min}, x_{max}] \cap \text{граф. } p(\bar{x}))$ при пом. rejection sampling

— sample направл.

— в этом напр. slice sampling

— slice sampling поочер. $x_1, x_2, \dots, x_n$



Hamiltonian MCMC

$(\bar z, \bar r)$ phase space t

моменты

$$r_i = \frac{dz_i}{dt}$$

потенц. энергия

$$\boxed{p(\bar z) = \frac{1}{Z} e^{-E(\bar z)}}$$

$$\frac{dr_i}{dt} = -\frac{\partial E(\bar z)}{\partial z_i}$$

$$K(\bar r) = \frac{1}{2}\|\bar r\|^2 = \frac{1}{2}\sum r_i^2$$

$$H(\bar z, \bar r) = E(\bar z) + K(\bar r)$$

Hamiltonian dynamics

$$\begin{cases} \dfrac{dz_i}{dt} = r_i = \dfrac{\partial H}{\partial r_i} \\ \dfrac{dr_i}{dt} = -\dfrac{\partial H}{\partial z_i} \end{cases}$$

$$\frac{dH}{dt} = \sum_i \left(\frac{\partial H}{\partial z_i}\frac{dz_i}{dt} + \frac{\partial H}{\partial r_i}\frac{dr_i}{dt} \right) = 0$$

Теорема Лиувилля

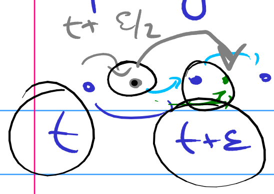
$$(\bar z, \bar r) \xrightarrow{t}$$

t

$$\boxed{p(\bar z, \bar r) = \frac{1}{Z_H} e^{-H(\bar z, \bar r)}}$$

$$\boxed{p(\bar z) = \frac{1}{Z} e^{-E(\bar z)}}$$

1) leapfrog



$$\hat{r}_i\left(t+\frac{\varepsilon}{2}\right) = \hat{r}_i(t) - \frac{\varepsilon}{2}\frac{\partial E}{\partial z_i}\left(\hat{z}(t)\right)$$

$$\hat{z}_i(t+\varepsilon) = \hat{z}_i(t) + \varepsilon\,\hat{r}_i(t+\varepsilon/2)$$

$$\hat{r}_i\left(t+\frac{3\varepsilon}{2}\right) = \hat{r}_i\left(t+\frac{\varepsilon}{2}\right) - \varepsilon\,\frac{\partial E}{\partial z_i}\left(\hat{z}(t+\varepsilon)\right)$$

2) Как меняем H?

loop:

- $(\bar{z}, \bar{r})$
- $\underbrace{\rightsquigarrow (\bar{z}^{(t)}, \bar{r}^{(t)})}$ $\}\, H$
- $\bar{r} \sim p(\bar{r}\,|\,\bar{z}) = \mathcal{N}(\bar{r}\,|\,0, I)$

$$p(\bar{z}, \bar{r}) = \frac{1}{Z_H} e^{-H(\bar{z}, \bar{r})} =$$

$$= \cancel{\frac{1}{Z_H} e^{-E(\bar{z})}} \cdot e^{-K(\bar{r})}$$

$$p(\bar{r}\,|\,\bar{z}) \propto e^{-K(\bar{r})} = e^{-\frac{1}{2}\sum_i r_i^2}$$

$$p(\bar{r}\,|\,\bar{z}) = \mathcal{N}(\bar{r}\,|\,0, I)$$

3) Hybrid Monte-Carlo

$$\frac{p(\bar{z}, \bar{r})}{p(\bar{z}', \bar{r}')}$$

$$(\bar{z}, \bar{r}) \xrightarrow{t} (\bar{z}', \bar{r}')$$

$$\left[\text{accept } (\bar{z}', \bar{r}') \text{ с вер-тью } \min\left(1, e^{H(\bar{z}, \bar{r}) - H(\bar{z}', \bar{r}')}\right)\right]$$

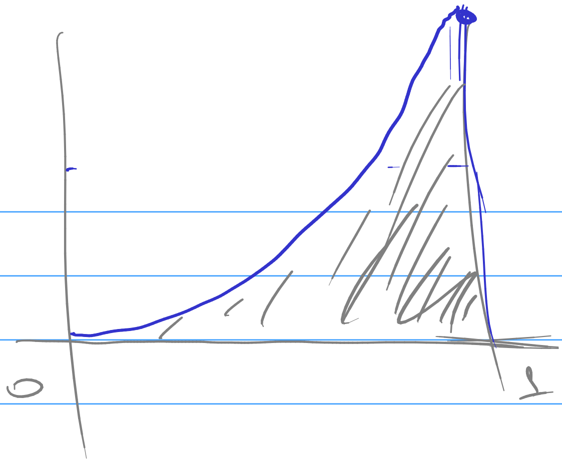
PMF $\quad U\ V \quad \underset{n\times m}{X} \approx \underset{n\times k}{U}\,\underset{m\times k}{V}^T$

$p(U)\ p(V)$

$$p(U, V\,|\,X) \propto p(U)\,p(V)\,p(X\,|\,U, V)$$

BPMF $\quad U^{(1)}, V^{(1)}, \ldots, U^{(50)}, V^{(50)} \sim p(U, V\,|\,X)$

$$\hat{r}_{i,a} = \frac{1}{50}\sum_{1}^{50} \bar{U}_i^{(r)\,T}\,\bar{V}_a^{(r)}$$



З — Зрелка

$$\boxed{p(x|D) = \mathbb{E}_{p(\theta|D)}\left[p(x|\theta)\right]}$$

$$\approx p(x|\theta^*)$$