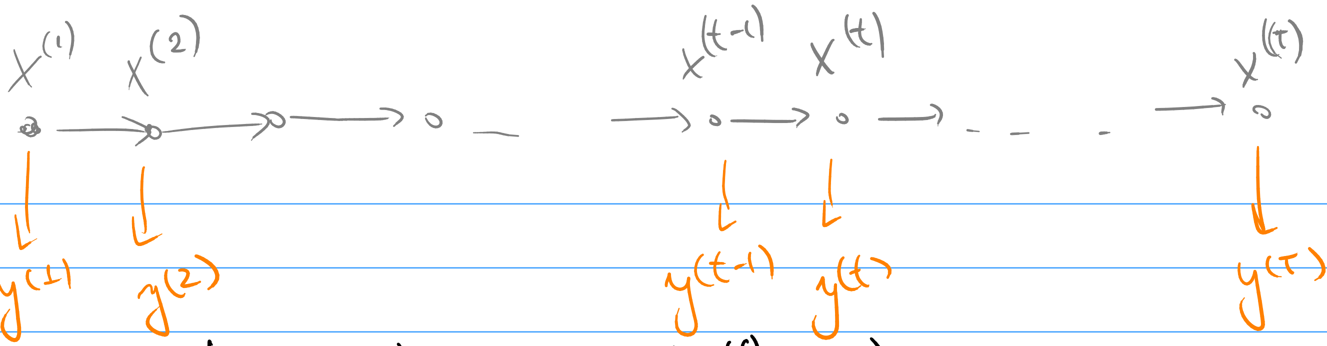
SIR

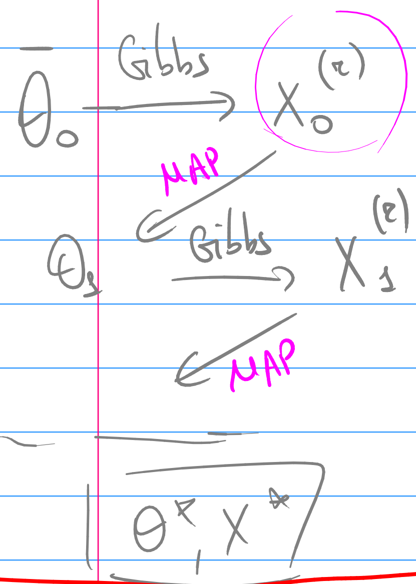


$$p(x, y \mid \bar{\theta}) = \left(\prod_{i=1}^{N} \pi^{[x_i^{(1)} = I]} (1-\pi)^{[x_i^{(1)} = S]} \right) \times$$

$$\times \prod_{t=1}^{T-1} \prod_{i=1}^{N} p\left(x_i^{(t+1)} \mid I^{(t)}, \beta, \mu\right) \times$$

$$\bar{\theta} = (\pi, \beta, \mu, \rho) \qquad \times \prod_{t=1}^{T} \binom{I^{(t)}}{y^{(t)}} \rho^{y^{(t)}} (1-\rho)^{I^{(t)} - y^{(t)}}$$

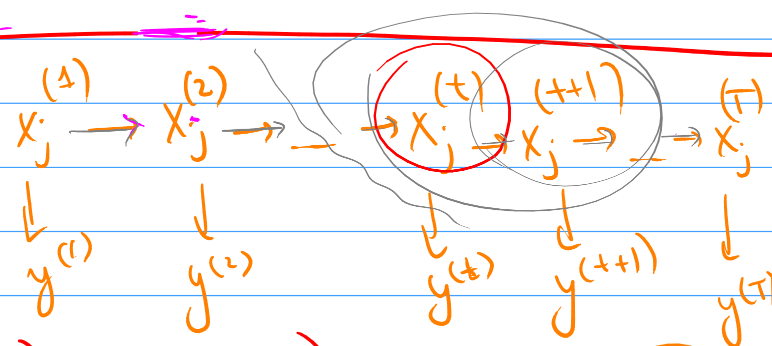
$$p\left(x_i^{(t+1)} \mid I^{(t)}, \beta, \mu\right) \qquad \begin{array}{c|ccc} & S & I & R \\ \hline S & (1-\beta)^{I^{(t)}} & 1-(1-\beta)^{I^{(t)}} & 0 \\ I & 0 & 1-\mu & \mu \\ R & 0 & 0 & 1 \end{array}$$



$j = 1, \dots, j = N$

$$\bar{x}_j \sim p(\bar{x}_j \mid X_{-j}, y, \bar{\theta})$$

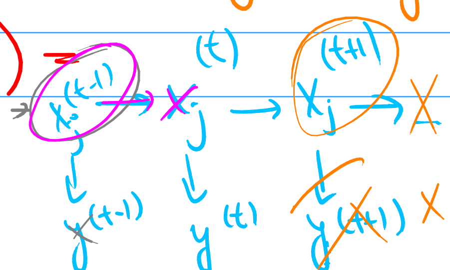
$p(\bar{x}_j \mid \dots)$



Stochastic Viterbi algorithm

$$p\left(x_j^{(t)} \mid x_j^{(t+1)}, X_{-j}, x_j^{(t+2)}, \dots, x_j^{(T)}, \bar{y}, \bar{\theta}\right) =$$

$$= p\left(x_j^{(t)} \mid x_j^{(t+1)}, X_{-j}, y^{(1)} \dots y^{(t)}, \bar{\theta}\right)$$



$$q^{(t)}_{j,s',s} = p\left(x_j^{(t)} = s,\ x_j^{(t-1)} = s' \mid y^{(1)} \dots y^{(t)}, X_{-j}, \bar\theta\right)$$

$$p(x_1^{(t)} = s \mid \dots) \propto \pi(s) \cdot p(y^{(1)} \mid x_1^{(t)} = s, \dots)$$

$$q^{(t)}_{j,s',s} \propto p\left(x_j^{(t)} = s,\ x_j^{(t-1)} = s',\ y^{(t)} \mid \bar y_{<t}, X_{-j}, \bar\theta\right) =$$

$$= p\left(x_j^{(t-1)} = s' \mid \bar y_{<t}, X_{-j}, \theta\right) p\left(x_j^{(t)} = s \mid x_j^{(t-1)} = s', \theta\right) p\left(y^{(t)} \mid x_j^{(t)} = s, \dots\right)$$

$$= \left(\sum_{s''} q^{(t-1)}_{j,s'',s'}\right) \cdot p\left(x_j^{(t)} = s \mid x_j^{(t-1)} = s', \theta, X_{-j}\right) \cdot \mathrm{Binom}\left(y^{(t)} \mid I^{(t)}_{-j} + [s=I], p\right)$$

Gibbs

1) Проход вперед. $Q_j^{(t)} = {}_{s'}\left(q^{(t)}_{j,s',s} \right)$ norm

2) Сэмплир. $x_j^{(T)}, x_j^{(T-1)}, \dots, x_j^{(1)}$

$$p(x_j^{(T)} = s \mid Y, X_{-j}, \theta) = \sum_{s'} q^{(T)}_{j,s',s}$$

$$p(x_j^{(t)} = s \mid x_j^{(t+1)} = s', Y, \dots) = p(\dots \mid \dots, y^{(1)\dots(t)}, X_{-j}, \theta) \propto q^{(t+1)}_{j,s,s'}$$

Outputs $\bar x_j \mid X_{-j}$

Алг.

1) Init θ, X так, чтобы Y могли получиться

2) Loop по $j = 1 \dots N, 1 \dots N$

- строим $Q_j^{(t)}$
- сэмпл. $\bar x_j \sim p(\bar x_j \mid X_{-j}, Y, \theta)$
- зам. $\bar x_j$
- кажд. M шагов:
  - обновл. θ по тек. сэмплу X

M-шаг

M-step:

$$p(\theta|y,X) \propto p(\theta)\, p(y,X|\theta)$$

$$p(\theta) = p(\rho)\, p(\mu)\, p(\beta)\, p(\pi)$$

$p(\rho) = \mathrm{Beta}(\rho | a_\rho, b_\rho)$

$$\log p(\theta|y,X) = \mathrm{Const} + (a_\rho - 1)\log \rho + (b_\rho - 1)\log(1-\rho) +$$
$$+ (a_\mu - 1)\log \mu + (b_\mu - 1)\log(1-\mu) + \ldots \quad a_\pi, b_\pi$$
$$+ \sum_{i=1}^{N} \left([x_i^{(1)} = I] \log \pi + [x_i^{(1)} = S] \log(1-\pi) \right) +$$
$$+ \sum_{t=1}^{T} \left(y^{(t)} \log \rho + (I^{(t)} - y^{(t)}) \log(1-\rho) \right) +$$
$$+ \sum_{t=1}^{T-1} \sum_{i=1}^{N} \Big([x_i^{(t)} = S, x_i^{(t+1)} = S] \cdot \log (1-\beta)^{I^{(t)}} +$$
$$+ [x_i^{(t)} = S, x_i^{(t+1)} = I] \cdot \log\left(1 - (1-\beta)^{I^{(t)}}\right) +$$
$$+ [x_i^{(t)} = I, x_i^{(t+1)} = R] \log \mu +$$
$$+ [x_i^{(t)} = I, x_i^{(t+1)} = I] \log(1-\mu) \Big) \longrightarrow \max_{\rho, \mu, \pi, \beta}$$

$$\log \pi \cdot \left(a_\pi + \sum_i [x_i^{(1)} = I] - 1\right) + \log(1-\pi)\left(b_\pi + \sum_i [x_i^{(1)} = S] - 1\right)$$

$$a_\pi' = a_\pi + \sum_i [x_i^{(1)} = I]$$
$$b_\pi' = b_\pi + \sum_i [x_i^{(1)} = S]$$

$$a_\rho' = a_\rho + \sum_{t=1}^{T} y^{(t)}$$
$$b_\rho' = b_\rho + \sum_{t=1}^{T} \left(I^{(t)} - y^{(t)}\right)$$

$$a'_\mu = a_\mu + \sum_{t=1}^{T-1} \sum_{i=1}^{N} [x_i^{(t)} = I,\ x_i^{(t+1)} = R]$$

$$b'_\mu = b_\mu + \sum_{t=1}^{T-1} \sum_{i=1}^{N} [x_i^{(t)} = I,\ x_i^{(t+1)} = I]$$

$$\sum_{t=1}^{T-1} \sum_{i=1}^{N} [x_i^{(t)} = S] \cdot \left(P_i^{(t)} \cdot \log \beta + N_i^{(t)} \log(1-\beta) \right)$$

$P_i^{(t)}$ — # конт., от к-рых x_i заразился

$N_i^{(t)}$ — -//- не заразился

$$P_i^{(t)} + N_i^{(t)} = I^{(t)}$$

- если $x_i^{(t+1)} = S \Rightarrow P_i^{(t)} = 0,\ N_i^{(t)} = I$
- если $x_i^{(t+1)} = I \Rightarrow P_i^{(t)} \geq 1,\ N_i^{(t)} = I^{(t)} - P_i^{(t)}$

$$\mathbb{E}\left[P_i^{(t)} \mid x_i^{(t+1)} = I\right] = I^{(t)} \cdot p(\text{зараж.} \mid \geq 1 \text{ зараж.}) = I^{(t)} \cdot \frac{\beta}{1-(1-\beta)^{I^{(t)}}}$$

$$a'_\beta = a_\beta + \sum_{t=1}^{T-1} \sum_{i=1}^{N} [x_i^{(t)} = S] \cdot P_i^{(t)}$$

$$b'_\beta = b_\beta + \sum_t \sum_i [x_i^{(t)} = S] \cdot (I^{(t)} - P_i^{(t)}),$$ где

$$P_i^{(t)} = \begin{cases} 0, & \text{если } x_i^{(t+1)} = S \\ I^{(t)} \dfrac{\beta}{1-(1-\beta)^{I^{(t)}}}, & \text{если } x_i^{(t+1)} = I \end{cases}$$