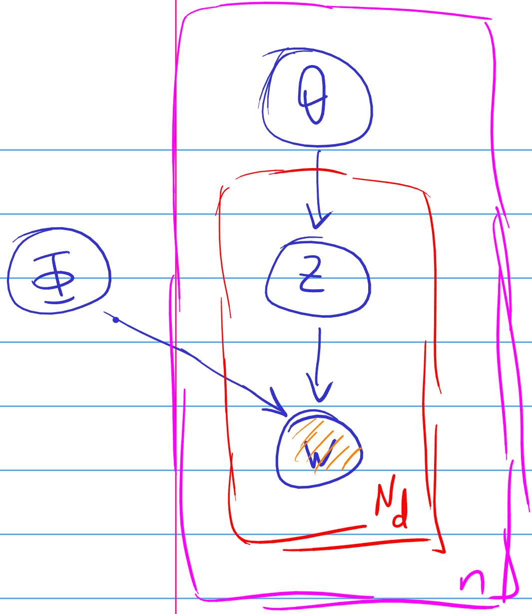
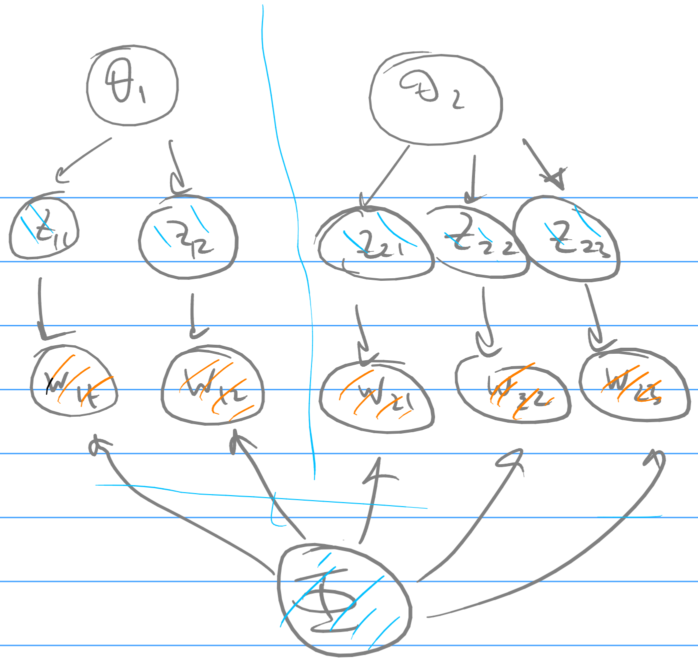




$\theta_{td} = p(t|d)$

$\varphi_{wt} = p(w|t)$

$p(w|d) = \sum_t \theta_{td}\varphi_{wt}$

$p(w|d) = \sum_t p(w,t|d) = \sum_t p(t|d)\,p(w|t)$

$\#\{w \in d\}$ $\|$ n_{dw}

pLSA
LSI

$$p(D|\Theta,\Phi) = \prod_d \prod_w \left(\sum_t \theta_{td}\varphi_{wt}\right) \xrightarrow[\Theta,\Phi]{} \max$$

$\boxed{n_{tdw} = \#\{w \in d \text{ из } t\}}$

$$n_{tdw} = n_{dw} \cdot \frac{\theta_{td}\varphi_{wt}}{\sum_{t'} \theta_{t'd}\varphi_{wt'}}$$

$n_{t*w} = \sum_d n_{tdw}$

$n_{td*} = \sum_w n_{tdw}$

add. reg. for top. models

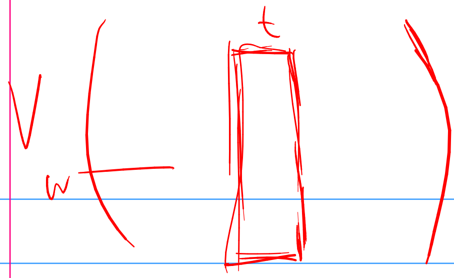
ARTM — Константин Воронцов

$$\ln p(D|\Theta,\Phi) = \sum_d \sum_w n_{dw} \ln\left(\sum_t \theta_{td}\varphi_{wt}\right) + \underbrace{\sum_i \lambda_i R_i(\Phi,\Theta)}_{R}$$

$$n_{t*w} = \left[\sum_d n_{tdw} + \varphi_{wt}\frac{\partial R}{\partial \varphi_{wt}}\right]_+$$

$$n_{td*} = \left[\sum_w n_{tdw} + \theta_{td}\frac{\partial R}{\partial \theta_{td}}\right]_+$$

Φ



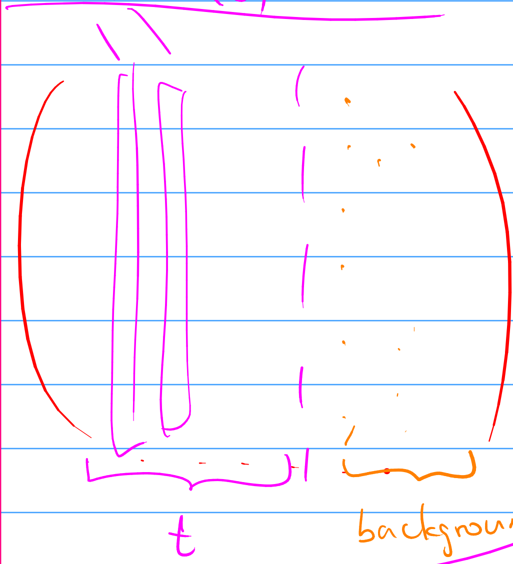
$$KL(\bar{\varphi}_{*t} \,\|\, Unif) \to \max$$

$$\sum_w \frac{1}{V} \cdot \ln \frac{1}{V\varphi_{wt}} = -\sum_w \ln \varphi_{wt}$$

$$KL(\bar{\Theta}_{xd} \,\|\, Unif) \to \max$$

$$KL(\varphi_t \,\|\, \varphi_{t'}) \to \max$$

Φ



$$KL(\varphi \,\|\, Unif) \to \max$$

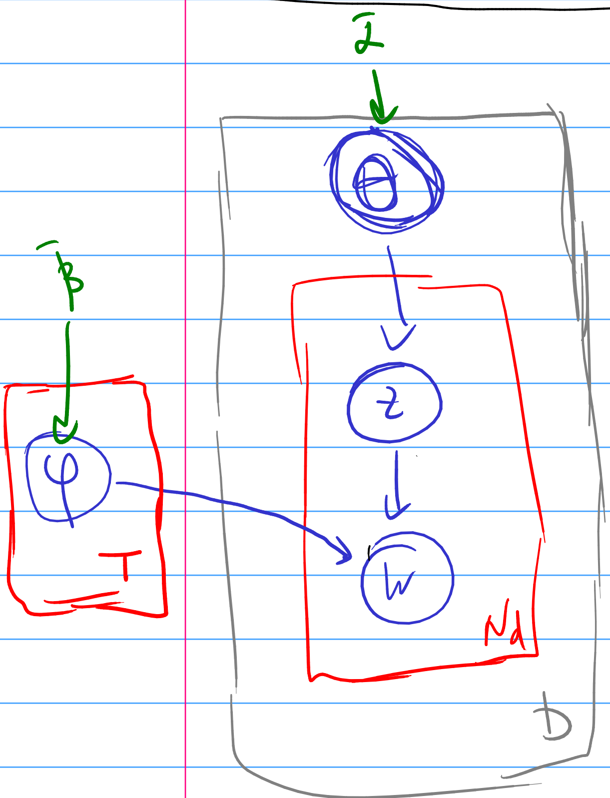
$$KL(\varphi \,\|\, Unif) \to \min$$

Когерентность

d $[t_1\, t_2\, t_3\, t_1\, t_2\, t_3]$

$[t_1\, t_1\, t_1 \;\; t_2\, t_2 \;\; t_1 \ldots]$

Latent Dirichlet Allocation



$$p(W, \Theta, \Phi, Z) = \left(\prod_t \underbrace{p(\bar{\varphi}_t)}_{Dir(\bar{\varphi}_t | \bar{\beta})}\right) \times \left(\prod_j \underbrace{p(\bar{\Theta}_j)}_{Dir(\bar{\Theta}_d | \bar{\alpha})}\right) \times$$

$$\times \prod_{j=1}^{M} \prod_{n=1}^{N_j} \left[p(z_{jn} | \bar{\Theta}_j) \cdot p(w_{jn} | z_{jn}, \Phi) \right]$$

$$p(\bar{\Theta}_j | \bar{\alpha}) = \frac{1}{B(\bar{\alpha})} \prod_{t=1}^{T} \Theta_{jt}^{\alpha_t - 1}$$

$$p(\bar{\varphi}_t | \bar{\beta}) = \frac{1}{B(\bar{\beta})} \prod_{s=1}^{V} \varphi_{ts}^{\beta_s - 1}$$

1) Variational approx.
2) Gibbs sampling

$$p(\Theta, \Phi \mid W, \bar\alpha, \bar\beta) \xrightarrow[\Theta, \Phi]{} \max$$

$$p(W, \Theta, \Phi \mid \bar\alpha, \bar\beta) \xrightarrow[\Theta, \Phi]{} \max$$

$$p(W, \Theta, \Phi \mid \bar\alpha, \bar\beta) = \sum_{Z} p(W, Z, \Theta, \Phi \mid \bar\alpha, \bar\beta)$$

Variational approx.

$$q(Z, \Theta, \Phi) \approx p(Z, \Theta, \Phi \mid W, \bar\alpha, \bar\beta)$$

$$\log p(W \mid \bar\alpha, \bar\beta) = \log \iint_{\Phi\,\Theta} \sum_{Z} p(W, Z, \Theta, \Phi \mid \bar\alpha, \bar\beta) \frac{q(Z, \Theta, \Phi)}{q(Z, \Theta, \Phi)}$$

Jensen's ineq.

$$\geq \iint_{\Phi, \Theta} \sum_{Z} \log \frac{p(W, Z, \Theta, \Phi \mid \bar\alpha, \bar\beta)}{q(Z, \Theta, \Phi)} q(Z, \Theta, \Phi)\, d\Phi\, d\Theta =$$

$\xrightarrow[q]{} \max$

$$= \iint \sum \log \frac{p(Z, \Theta, \Phi \mid W, \bar\alpha, \bar\beta)}{q(Z, \Theta, \Phi)} q(Z, \Theta, \Phi)\, d\Phi\, d\Theta + \log p(W \mid \bar\alpha, \bar\beta)$$

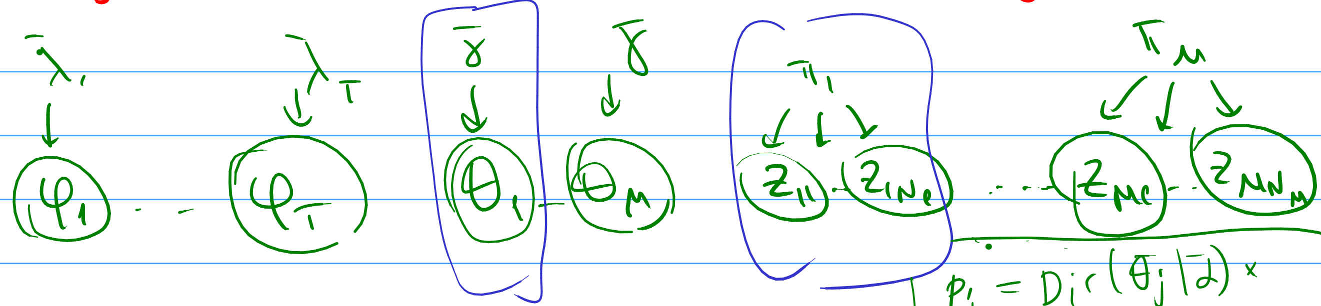
$\xrightarrow[q]{} \max$ $\quad$ $= -KL(q \| p)$

$$q(Z, \Theta, \Phi) = \prod_{t} q_t(\bar\varphi_t) \cdot \prod_{j=1}^{M} \left[q_j(\bar\theta_j) \prod_{n=1}^{N_j} q_j(z_{jn}) \right]$$

$$q_t(\bar\varphi_t \mid \bar\lambda_t) = Dir(\bar\varphi_t \mid \bar\lambda_t) \propto \prod_{s} \varphi_{ts}^{\lambda_{ts} - 1}$$

$$q_j(\bar\theta_j \mid \bar\gamma_j) = Dir(\_) \propto \prod_{t} \theta_{jt}^{\gamma_{jt} - 1}$$

$$q_j(z_{jn}|\bar{\pi}_j) = \mathrm{Mult}(z_{jn}|\bar{\pi}_j) \propto \prod_t \pi_{jt}^{[z_{jn}=t]}$$



$$p_j = \mathrm{Dir}(\theta_j|\bar{\alpha}) \times \prod_{n=1}^{N_j} \prod_t \theta_{jt}^{[z_{jn}=1]}$$

$$q(z, \Theta | \Gamma, \Pi) \approx p(\Theta, z | W, \bar{\alpha}, \bar{\beta}, \Phi)$$

$$KL(q_j \| p_j) = \int_{\bar{\theta}_j} \sum_{\bar{z}_j} q(\bar{z}_j, \bar{\theta}_j | \bar{\gamma}_j, \bar{\pi}_j) \cdot \log \frac{q(\bar{z}_j, \theta_j | \bar{\gamma}_j, \bar{\pi}_j)}{p(\bar{z}_j, \bar{\theta}_j | \bar{w}_j, \Phi, \bar{\alpha}, \bar{\beta})} d\bar{\theta}_j \longrightarrow \min$$

$\bar{\gamma}_j, \bar{\pi}_j \longrightarrow \max$

$$\mathcal{L}(\bar{\gamma}_j, \bar{\pi}_j) = \int_{\theta_j} \sum_{z_j} \log \frac{p(\bar{w}_j, \bar{z}_j, \bar{\theta}_j | \Phi, \alpha, \beta)}{q(\_\_)} q(\_) \, d\theta_j = \mathbb{E}_q\left[\log \frac{\dots}{\dots}\right]$$

$$= \mathbb{E}_q\Big[\log \underbrace{p(\bar{w}_j, \bar{z}_j, \bar{\theta}_j | \Phi, \alpha, \beta)}_{= p(\theta|\alpha)\, p(z|\theta)\, p(w|z,\Phi)}\Big] - \mathbb{E}_q\Big[\log \underbrace{q(\bar{z}_j, \bar{\theta}_j | \bar{\gamma}_j, \bar{\pi}_j)}_{= q(\theta|\gamma) \cdot q(z|\pi)}\Big]$$

$$= \underline{\mathbb{E}_q[\log p(\bar{\theta}_j|\alpha)]} + \mathbb{E}_q[\log p(\bar{z}_j|\bar{\theta}_j)] + \mathbb{E}_q[\log p(\bar{w}_j|\bar{z}_j, \Phi)] -$$
$$- \mathbb{E}_q[\log q(\bar{\theta}_j|\bar{\gamma}_j)] - \mathbb{E}_q[\log q(\bar{z}_j|\bar{\pi}_j)]$$

$$\underline{\mathbb{E}_q[\log \underbrace{p(\bar{\theta}_j|\bar{\alpha})}_{= \frac{1}{B(\bar{\alpha})} \prod \theta_{jt}^{\alpha_t - 1}}]} = \mathbb{E}_q\left[\log \frac{\Gamma(\sum \alpha_t)}{\prod \Gamma(\alpha_t)} \cdot \prod_t \theta_{jt}^{\alpha_t - 1}\right] =$$

$$B(\bar{\alpha}) = \frac{\prod \Gamma(\alpha_t)}{\Gamma(\sum \alpha_t)}$$

$$= \log \Gamma(\textstyle\sum \alpha_t) - \sum_t \log \Gamma(\alpha_t) + \sum_t (\alpha_t - 1) \times \mathbb{E}_q[\log \theta_{jt}]$$

$\mathbb{E}_q[\log\theta_{jt}]$ – ? $q(\bar\theta_j|\bar\gamma_j) = Dir(\bar\theta_j|\bar\gamma_j)$ $t(\bar\theta_j) =$

$\log Dir(\bar\theta_j|\bar\gamma_j) = \sum_t(\gamma_{jt}-1)(\log\theta_{jt}) - \log B(\ldots)$

Экспоненц. семейство

$$p(\bar x|\bar\eta) = h(\bar x)\cdot e^{\bar\eta^T t(\bar x) - a(\bar\eta)}$$

$a(\bar\eta) = \log B(\bar\gamma)$

$$\int h(\bar x) e^{\bar\eta^T t(\bar x) - a(\bar\eta)}\, d\bar x = 1$$

$$\nabla_\eta \int \_\_\_ = 0$$

$$\int h(\bar x)\left(t(\bar x) - \nabla_\eta a(\bar\eta)\right) e^{\bar\eta^T t(\bar x) - a(\bar\eta)}\, d\bar x = 0$$

$$\int h(\bar x) t(\bar x) e^{\cdots}\, d\bar x = \int \nabla_\eta a(\bar\eta)\, h(\bar x) e^{\cdots}\, d\bar x = \nabla_\eta a(\bar\eta)$$

$$\boxed{\mathbb{E}_{p(\bar x)}[t(\bar x)] = \nabla_{\bar\eta} a(\bar\eta)}$$

$$p(\bar x) = Dir(\bar x|\bar\alpha) = \frac{1}{B(\bar\alpha)}\prod x_i^{\alpha_i - 1}$$

$\log B(\bar\alpha) = \sum \log\Gamma(\alpha_i) - \log\Gamma(\sum\alpha_i)$

$$\mathbb{E}_p[\log x_i] = \frac{\partial \log B(\bar\alpha)}{\partial \alpha_i} = \Psi(\alpha_i) - \Psi(\sum\alpha_i)$$

где $\Psi(x) = \dfrac{\partial \log\Gamma(x)}{\partial x}$

дигамма-функция

$$\mathbb{E}_q[\log p(\bar\theta_j|\bar\alpha)] = \log\Gamma(\sum\alpha_t) - \sum\log\Gamma(\alpha_t) + \sum_t(\alpha_t - 1)\left(\Psi(\gamma_{jt}) - \Psi(\sum_s \gamma_{js})\right)$$

$$\mathbb{E}_q[\log p(\bar z_j|\bar\theta_j)] = \mathbb{E}_q\Big[\sum_{n=1}^{N_j}\sum_{t=1}^{T}[z_{jn}=t]\cdot\log\theta_{jt}\Big] =$$

$$= \sum_{n=1}^{N_j} \sum_{t=1}^{T} \underbrace{\mathbb{E}_q [[z_{jn}=t]]} \cdot \underbrace{\mathbb{E}_q [\log \theta_{jt}]} =$$

$$\mathbb{E}_{\underline{q(z,\theta)}} [[z_{jn}=1] \cdot \log \theta_{jt}) \quad = \sum_n \sum_t \underline{\pi_{jnt}} \left(\psi(\gamma_{jt}) - \psi\left(\sum_s \gamma_{js}\right) \right)$$

$$= \underline{q(z) q(\theta)}$$

$$\mathbb{E}_{\underline{p(x,y)}} [f(x) g(y)]$$

$$= \prod_n \prod_t^T \prod_{s=1}^V \varphi_{ts}^{[z_{jn}=t][w_{jn}=s]}$$

$$\mathbb{E}_q [\log p(\bar{w}_j | \bar{z}_j, \Phi)] = \mathbb{E}_q \left[\sum_n \sum_t \sum_s [z_{jn}=t][w_{jn}=s] \underline{\log \varphi_{ts}} \right] =$$

$$= \sum_{n=1}^{N_j} \sum_{t=1}^{T} \sum_{s=1}^{V} [w_{jn}=s] \pi_{jnt} \cdot \log \varphi_{ts}$$

$$\mathbb{E}_q [\log q(\bar{\theta}_j | \bar{\gamma}_j)] = \log \Gamma\left(\sum \gamma_{jt}\right) - \sum \log \Gamma(\gamma_{jt}) + \sum_t (\gamma_{jt}-1) \mathbb{E}_q [\log \theta_{jt}]$$

$$= \log \Gamma\left(\sum_t \gamma_{jt}\right) - \sum_t \log \Gamma(\gamma_{jt}) + \sum_t (\gamma_{jt}-1) \left(\psi(\gamma_{jt}) - \psi\left(\sum_s \gamma_{js}\right) \right)$$

$$\mathbb{E}_q [\log \underline{q(\bar{z}_j | \bar{\pi}_j)}] = \mathbb{E}_q \left[\sum_n \sum_t [z_{jn}=t] \log \pi_{jnt} \right] =$$

$$\prod_n \prod_t \pi_{jnt}^{[z_{jn}=t]}$$

$$= \sum_n \sum_t \underline{\pi_{jnt}} \log \underline{\pi_{jnt}}$$

$$L(\bar{\gamma}_j, \bar{\pi}_j) = (1) + (2) + (3) - (4) - (5) \xrightarrow[\gamma, \pi]{} \max$$

π_{jn}: условие $\boxed{\forall j, n \quad \sum_t \pi_{jnt} = 1}$

$$L(\bar{\pi}_j) = \sum_{n,t} \pi_{jnt}\left(\psi(\gamma_{jt}) - \psi\left(\sum_s \gamma_{js}\right)\right) +$$

$$+ \sum_{n,t} \sum_{v} [w_{jn} = v]\, \pi_{jnt} \log \varphi_{tv} - \sum_{n,t} \pi_{jnt} \log \pi_{jnt}$$

$$\sum_{n,t} \pi_{jnt}\left(\psi(\gamma_{..}) - \psi(\Sigma\gamma) + \log \varphi_{t,w_{jn}} - \log \pi_{jnt}\right) +$$

$$+ \sum_n \lambda_n\left(\sum_s \pi_{jns} - 1\right)$$

$$\frac{\partial L}{\partial \pi_{jnt}} = \psi(\gamma) - \psi(\Sigma\gamma) + \log \varphi_{t,w_{jn}} - \log \pi_{jnt} - 1 + \lambda_n = 0$$

$$\pi_{jnt} = e^{\lambda_n - 1} \cdot e^{\psi(\gamma) - \psi(\Sigma\gamma) + \log \varphi_{t,w_{jn}}}$$

$$\pi_{jnt} \propto e^{\psi(\gamma_{jt}) - \psi\left(\sum_s \gamma_{js}\right) + \log \varphi_{t,w_{jn}}}$$

γ:

$$L(\gamma_{jt}) = \sum_t (\alpha_t - 1)\left(\psi(\gamma_{jt}) - \psi\left(\sum_s \gamma_{js}\right)\right) + \sum_t \sum_n \pi_{jnt}(\cdot\cdot) -$$

$$- \log\Gamma\left(\sum_s \gamma_{js}\right) + \sum_t \log\Gamma(\gamma_{jt}) - \sum_t (\gamma_{jt} - 1)(\cdot\cdot) =$$

$$\sum_t \psi(\gamma_{jt})(\ldots - \gamma_{jt})$$

$$= \sum_t \log\Gamma(\gamma_{jt}) - \log\Gamma\left(\sum_s \gamma_{js}\right) + \sum_t \left(\psi(\gamma_{jt}) - \psi\left(\sum_s \gamma_{js}\right)\right) \times$$

$$\times \left(\alpha_t - \cancel{1} + \sum_n \pi_{jnt} - \gamma_{jt} + \cancel{1}\right)$$

$$\frac{\partial L}{\partial \gamma_{jt}} = \cancel{\psi(\gamma_{jt})} - \cancel{\psi\left(\sum_s \gamma_{js}\right)} + \left(\alpha_t + \sum_n \pi_{jnt} - \gamma_{jt}\right) \cdot \psi'(\gamma_{jt}) - \cancel{\psi(\gamma_{jt})}$$

$$- \psi'\left(\sum_s \gamma_{js}\right) \cdot \sum_s \left(\alpha_s + \sum_n \pi_{jns} - \gamma_{js}\right) + \cancel{\psi\left(\sum_s \gamma_{js}\right)} =$$

$$= \psi'(\gamma_{jt})\left(\underline{\alpha_t + \sum_n \pi_{jnt} - \gamma_{jt}}\right) - \psi'\left(\sum_s \gamma_{js}\right) \cdot \sum_s \left(\underline{\alpha_s + \sum_n \pi_{jns} - \gamma_{js}}\right) = 0$$

$$\boxed{\gamma_{jt} = \alpha_t + \sum_n \pi_{jnt}}$$

$\bar{\pi}_j^{(0)}, \bar{\gamma}_j^{(0)}, \ldots$ —

LDA: EM-алгоритм, по Φ, Z

- E-шаг: $\forall j$ $\underbrace{KL(q_j \| p_j) \to \min_{\bar{\gamma}_j, \bar{\pi}_j}}$ для фикс. Φ q_j

- M-шаг: $\pi_{jnt} = \mathbb{E}_{q_j}[z_{jnt}]$

$$\varphi_{t\sigma} \propto \sum_{j=1}^{M} \sum_{n=1}^{N_j} [w_{jn} = \sigma] \cdot \pi_{jnt}$$

$$q(Z, \Theta, \Phi \mid \Gamma, \Pi, \Lambda) \approx p(Z, \Theta, \Phi \mid W, \bar{\alpha}, \bar{\beta})$$

$$\mathcal{L}(\ldots) = \int \ldots \log \frac{p(Z, \Theta, \Phi, W \ldots)}{q(\_\_ \mid \ldots)}$$

$q(\bar{\varphi}_t \mid \bar{\lambda}_t)$

$$\boxed{\lambda_{t\sigma} = \beta_\sigma + \sum_{j=1}^{M} \sum_{n=1}^{N_j} [w_{jn} = \sigma] \pi_{jnt}}$$

LDA:

$$\begin{cases} \bar{\gamma}^{(t)} := \ldots \gamma, \pi, \lambda^{(t-1)} \\ \bar{\pi}^{(t)} := \ldots \_\_ \\ \lambda^{(t)} := \_\_ \end{cases}$$